A note on optimal procurement contracts with limited direct cost inflation

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Abstract

Laffont and Tirole’s [4] pioneering analysis identifies the optimal procurement contract when the supplier can readily inflate his innate production cost without detection. When the buyer has some ability to limit such cost inflation, an alternative contract can outperform the contract identified by Laffont and Tirole. The alternative contract induces substantial pooling, discontinuous production costs and effort supply, and rent that varies non-monotonically with innate cost.

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1. Introduction

In Laffont and Tirole’s [4] pioneering analysis of optimal procurement contracts, a monopoly supplier is privately informed about his innate production cost, which is the supplier’s final production cost if he delivers no cost-reducing effort. Laffont and Tirole (LT) implicitly adopt the simplifying assumption that the supplier faces no impediments to direct cost inflation. In other words, in addition to withholding cost-reducing effort, the supplier is free to inflate his final production cost above his innate cost, and the buyer is unable to observe or limit such direct cost inflation. This is a reasonable assumption in many settings. However, egregious cost inflation
often can be detected in practice. For instance, auditors are adept at identifying overpayments for materials and supplies, and excessive numbers of employees and unnecessarily lavish production facilities often are readily detected and punished.  

In this note, we demonstrate that the classic contract identified by LT is no longer optimal when the supplier’s ability to undertake direct cost inflation is sufficiently limited. We identify an alternative contract that outperforms the classic LT contract. Under this alternative contract, the supplier’s rent does not vary monotonically with his innate cost. Furthermore, the supplier’s final production cost is invariant to his innate cost over an extended range (so pooling is optimal). In addition, the final production cost increases discontinuously and cost-reducing effort decreases discontinuously as innate cost increases above a critical level. The discontinuous increase in induced production cost is particularly effective at limiting the supplier’s ability to exaggerate his innate cost when direct cost inflation is sufficiently difficult for the supplier.

The analysis begins in Section 2 with a description of the simple setting under consideration. Section 3 explains when and how the alternative contract can outperform the LT contract in this setting. Section 4 provides concluding observations. The proofs of all findings are provided in the Appendix.  

2. The model

A buyer seeks to procure one unit of a commodity at minimum cost from a monopoly supplier. The supplier’s innate cost \( \beta \in [\beta, \beta] \) is the realization of a random variable with uniform density \( f(\beta) = 1/(2k) \) on the interval \([\beta, \beta]\), where \( \beta = \beta + 2k \) and \( \beta > 2k > 0 \).

The supplier can reduce his final cost of producing the commodity below his innate cost by exerting cost-reducing effort \( e > 0 \). The supplier incurs quadratic personal cost \( \psi(e) = e^2/[4k] \) when he reduces final production cost by \( e \geq 0 \) units. The supplier also can inflate the observed final cost above the realized innate cost by exerting negative effort \( (e < 0) \) at personal cost \( \psi(e) = ze^2/[4k] \). If \( z \) is zero, the supplier can inflate his costs at will, as LT implicitly assume.\(^3\) If \( z \) is strictly positive, the supplier finds it personally costly to engage in direct cost inflation. The supplier’s personal cost might reflect expected penalties associated with detection and punishment of intentional cost inflation, for example.\(^4\)

The supplier learns his innate cost before interacting with the buyer. The buyer never observes \( \beta \) nor can she observe the supplier’s cost-reducing effort \( e \). The supplier can only observe realized cost \( c = \beta - e \), and therefore can only base payments to the supplier on \( c \). Let \( t(\beta) \) denote the transfer payment the buyer delivers to the supplier when the supplier reports his innate cost to be \( \beta \), and therefore is required to realize final cost \( c(\beta) \). Also let \( (t(\beta), c(\beta)) \beta \) denote a contract between the supplier and buyer.\(^5\) In addition, let \( e(\hat{\beta} | \beta) = e(\hat{\beta}) + \beta - \hat{\beta} \) denote the effort the

\(^1\) For two of the many settings in which a supplier and/or its employees have been charged with or convicted for intentionally inflating costs, see [2,8].

\(^2\) A more detailed Technical Appendix is available at http://www.nyu.edu/jet/supplementary.html.

\(^3\) Formally, Laffont and Tirole [4, p. 617] assume \( e \) is non-negative. As the ensuing analysis reveals, this assumption admits superior contracts in settings where direct cost inflation by other means is sufficiently costly for the supplier. However, the contract identified by LT is an optimal contract if \( e < 0 \) is feasible and \( \psi(e) = 0 \) for all \( e < 0 \), as Lemma 1 below reveals.

\(^4\) The supplier’s personal disutility from direct cost inflation parallels the personal cost of misrepresenting private information in [3,6], for example.

\(^5\) The subscript \( \beta \) here denotes the continuum of \( t(\beta), c(\beta) \) pairs over all innate cost realizations \( \beta \in [\beta, \beta] \).
supplier with innate cost $\beta$ must deliver to ensure final cost $c(\hat{\beta}) = \hat{\beta} - e(\hat{\beta})$. Then the buyer’s problem, labeled [BP], can be stated as follows:

\[
\text{Minimize } \int_{\beta} t(\beta) f(\beta) d\beta 
\]

subject to, for all $\beta, \hat{\beta} \in [\underline{\beta}, \bar{\beta}]$:

\[
u(\beta) \equiv t(\beta) - [\hat{\beta} - e(\hat{\beta})] - \psi(e(\hat{\beta})) \geq 0
\]

and

\[
u(\beta) \geq u(\hat{\beta}|\beta) \equiv t(\hat{\beta}) - [\hat{\beta} - e(\hat{\beta})] - \psi(e(\hat{\beta}|\beta)).
\]

Expression (1) reflects the buyer’s goal of minimizing expected procurement cost. Expression (2) ensures that the supplier secures non-negative utility whenever he truthfully reports his innate cost. The incentive compatibility constraints (3) ensure that the supplier always reports his innate cost truthfully.\(^6\)

3. Findings

Lemma 1 reviews the key features of the contract identified by LT (“the LT contract”) and reports that this contract constitutes the solution to [BP] whenever it is more onerous for the supplier to reduce his final production cost than to engage in direct cost inflation.

**Lemma 1.** When $z \leq 1$, the solution to [BP] is the LT contract $(t^L(\beta), c^L(\beta))$ in which $t^L(\beta) = 2\beta - \bar{\beta} + [\hat{\beta} - \beta]^2 / [2k]$ and $c^L(\beta) = 2\beta - \bar{\beta}$ for $\beta \in [\underline{\beta}, \bar{\beta}]$. The LT contract induces the supplier to deliver effort $e^L(\beta) = 2k - [\beta - \bar{\beta}] \geq 0$ and provides rent $u^L(\beta) = [\hat{\beta} - \beta]^2 / [4k]$ for $\beta \in [\underline{\beta}, \bar{\beta}]$.\(^7\)

The buyer’s expected procurement cost under the LT contract is $\hat{\beta} + 2k/3$.

When the supplier can readily inflate his innate cost, the buyer faces a fundamental tradeoff. Any cost reduction ($e(\cdot) > 0$) induced when the supplier’s innate cost is $\beta_1 > \beta$ necessarily provides rent to the supplier for all smaller innate cost realizations, $\beta \in [\underline{\beta}, \beta_1]$. The rent is unavoidable because the buyer must provide a payment $t(\beta_1)$ in excess of realized cost $c(\beta_1)$ in order to induce cost-reducing effort $e(\beta_1) > 0$. A supplier with innate cost $\beta < \beta_1$ can substitute his innate cost advantage for cost-reducing effort—simply inflating his innate cost at little or no personal cost, if necessary—and thereby collect the payment $t(\beta_1)$ and associated rent by producing at cost $c(\beta_1)$.

In contrast, when innate cost inflation is sufficiently onerous for the supplier, the buyer can induce some cost reduction from the supplier with high innate cost without ceding rent to the supplier for all lower innate cost realizations. The buyer can do so, for example, with the following “discontinuous contract”, which constitutes a fairly simple modification of the LT contract.

\[\text{\textsuperscript{6}}\text{The revelation principle (e.g., [7]) ensures this formulation is without loss of generality.}\]

\[\text{\textsuperscript{7}}\text{Notice that } e^L(\beta) > 0 \text{ for all } \beta \in [\underline{\beta}, \bar{\beta}] \text{ because } \bar{\beta} > 2k, \text{ by assumption. Also, } e^L(\beta) > 0 \text{ for all } \beta \in [\underline{\beta}, \bar{\beta}] \text{ because } \bar{\beta} - \beta = 2k. \text{ Section 4 considers a setting where } e(\beta) = 0 \text{ for a non-trivial range of innate cost realizations under the contract identified by LT.}\]
**Definition.** The discontinuous contract \((r^d(\beta), c^d(\beta))\) has the following properties:

For \(\beta \in [\bar{\beta}, \bar{\beta} + 1)\):

(i) \(t^d(\beta) = t^L(\beta) - \lfloor 2k - l \rfloor^2/[4k]\); and
(ii) \(c^d(\beta) = c^L(\beta)\).

For \(\beta \in [\bar{\beta} + 1, \bar{\beta}]\):

(iii) \(t^d(\beta) = \frac{\sqrt{2(\beta + l) + \bar{\beta}}}{1 + \sqrt{z}} + \frac{z[2k - l]^2}{4k(1 + \sqrt{z})^2} \equiv \bar{t}^d\); and
(iv) \(c^d(\beta) = \frac{\sqrt{2(\beta + l) + \bar{\beta}}}{1 + \sqrt{z}} \equiv \bar{c}^d\), where \(l = 2k\left[\frac{3 + 2\sqrt{z} + 2\bar{\beta}}{1 + 2\sqrt{z} + 4\bar{\beta}}\right]\) and \(z > 1\).

The discontinuous contract has two components. For the lower innate cost realizations \((\beta \in [\underline{\beta}, \bar{\beta} + 1)\), the contract parallels the LT contract except that payments to the supplier are reduced systematically by \(\lfloor 2k - l \rfloor^2/[4k]\). The discontinuous contract also allows the supplier to realize a relatively high cost \((\bar{c}^d)\) in return for a relatively high payment \((\bar{t}^d)\). \(\bar{c}^d\) is less than \(\bar{\beta}\), and so the supplier with one of the higher innate cost realizations must deliver cost-reducing effort in order to secure final cost \(\bar{c}^d\). The payment \(\bar{t}^d\) compensates the supplier with innate cost \(\bar{\beta}\) for the effort he must supply to realize final cost \(\bar{c}^d\). Consequently, this payment provides rent to the supplier with innate cost \(\beta \in (\beta + l, \bar{\beta})\) when he realizes final cost \(\bar{c}^d\).

In contrast, \(\bar{c}^d\) and \(\bar{t}^d\) are designed to preclude any rent for the supplier with innate cost below \(\beta + l\) when innate cost exaggeration is sufficiently onerous \((z > 1)\). The personal disutility the supplier with innate cost \(\beta \in [\underline{\beta}, \beta + l)\) would incur in exaggerating his innate cost to realize the relatively high final cost \(\bar{c}^d\) exceeds \(\bar{t}^d - \bar{c}^d\), and so such cost exaggeration is not attractive to the supplier. Consequently, when innate cost exaggeration is sufficiently onerous, the buyer can employ the discontinuous contract to induce cost-reducing effort for the higher \(\beta\) realizations without automatically ceding rent to the supplier for all lower \(\beta\) realizations.

It is not costless for the buyer to avoid ceding such rent. To do so, \(\bar{c}^d\) must be set substantially above \(c((\beta + l)^-), (\beta + l)^- \equiv \lim_{\beta \uparrow \beta + l} \beta\). The corresponding relatively high payment \(\bar{t}^d\) is onerous for the buyer. However, the benefit of the rent reduction exceeds the burden of the high procurement cost for the higher \(\beta\)’s when \(z\) is sufficiently large. This conclusion is recorded formally in Proposition 1, after the supplier’s realized cost, effort, and rent under the discontinuous contract are characterized in Lemma 2.

**Lemma 2.** The discontinuous contract is individually rational and incentive compatible (i.e., it satisfies constraints (2) and (3) for all \(\beta, \bar{\beta} \in [\underline{\beta}, \bar{\beta}]\)). For \(\beta \in [\underline{\beta}, \beta + 1)\), the discontinuous contract induces effort \(e^d(\beta) = e^L(\beta)\) and delivers rent \(u^d(\beta) = \frac{[\bar{\beta} - \beta]^2 - [2k - l]^2}{4k}\). For \(\beta \in [\beta + 1, \bar{\beta}]\), the contract induces effort \(e^d(\beta) = \beta - \sqrt{2(\beta + l) + \bar{\beta}}[1 + \sqrt{z}]\) and delivers rent \(u^d(\beta) = \frac{z[2k - l]^2 - r^2\delta(r)}{4k(1 + \sqrt{z})^2}\), where \(r \equiv \sqrt{\bar{\beta} - (\beta + l)} - \sqrt{\bar{\beta} - \beta}\) and \(\delta(r) = 1\) if \(r \geq 0\) and \(\delta(r) = z\) if \(r < 0\).

Fig. 1 illustrates how the supplier’s effort, final cost, and utility vary with his innate cost under the discontinuous contract. Notice from the bottom panel in Fig. 1 that the supplier with innate cost \(\beta + l\) secures no rent. For innate costs just above \(\beta + l\), the supplier finds it profitable to undertake the cost inflation \((e(\cdot) < 0, \text{as illustrated in the top panel in Fig. 1})\) required to secure final cost \(\bar{c}^d\). Cost inflation is not profitable for the supplier with lower innate costs, though.
Fig. 1. Outcomes under the discontinuous contract.

(Notice that \( e(\beta) \geq 0 \) for all \( \beta \in [\underline{\beta}, \bar{\beta} + I] \) in the top panel in Fig. 1.) The absence of profitable cost exaggeration opportunities allows the buyer to limit the supplier’s rent for all \( \beta \in [\underline{\beta}, \bar{\beta} + I] \) by systematically reducing the payment to the buyer by \( \frac{(2k - I)^2}{4k} \) relative to the LT contract for all \( \beta \in [\underline{\beta}, \bar{\beta} + I] \). This rent reduction provides the reduction in expected procurement cost identified in Proposition 1.
Proposition 1. The buyer’s expected payment to the supplier under the discontinuous contract is $\beta + 2k/3 - G(\alpha)$, where $G(\alpha)$ is a strictly increasing function of $\alpha$, $G(\alpha) > 0$ for all $\alpha > 1$, and $\lim_{\alpha \to \infty} G(\alpha) = k/12$. 8

As Proposition 1 suggests, a discontinuous increase in $c^d(\beta)$ above $\beta$ will only deter the supplier from exaggerating his innate cost when such cost inflation is sufficiently onerous. When $\alpha$ is less than 1, the increases in $\bar{c}^d$ and $\bar{r}^d$ required to deter innate cost inflation with a discontinuous contract are too pronounced to be remunerative for the buyer. More generally, the region of pooling in the discontinuous contract $(\{\beta + l, \beta\})$ becomes negligible as $\alpha$ declines toward unity, and so the extra cost saving the discontinuous contract offers relative to the LT contract declines toward zero. In contrast, as $\alpha$ becomes arbitrarily large so that the supplier is effectively unable to inflate his innate cost, the buyer is able to reduce her expected payment to the supplier below the level achieved with a cost reimbursement contract by 25% more when she employs the discontinuous contract than when she employs the LT contract. 9

4. Conclusion

This research has shown that LT’s implicit assumption that the supplier can inflate his innate cost at will is a substantive assumption. If direct cost inflation (as opposed to a failure to reduce costs efficiently) is sufficiently difficult for the supplier to implement, procurement contracts that entail pooling, discontinuous effort supply and final production costs, and rent that varies non-monotonically with the supplier’s innate cost can outperform the standard LT contract. A discontinuous increase in final production cost can be particularly effective at limiting the supplier’s ability to exaggerate his costs when direct cost inflation entails sufficient personal disutility. The corresponding reduction in supplier rent can more than offset the higher payments required to compensate the supplier for his higher final production cost.

For simplicity, the foregoing analysis has considered a setting where $e(\beta) > 0$ for all $\beta \in [\beta, \bar{\beta}]$ under the LT contract. When $\bar{\beta} - \beta > 2k$, the supplier is optimally induced to deliver no cost-reducing effort over a non-trivial range of the higher innate cost realizations. In this setting, a contract that contains multiple discontinuities can reduce the buyer’s expected payments to the supplier even more substantially than the discontinuous contract analyzed above. 10 Recall that the discontinuous contract partitions the set of innate cost realizations into two regions and induces cost-reducing effort in the upper region without increasing the rent the supplier receives in the lower region. When $\bar{\beta} - \beta > 2k$, a corresponding contract with multiple discontinuities can divide $[\beta, \bar{\beta}]$ into more than two regions, where the rent afforded the supplier in each region is not affected by the cost-reducing effort induced in regions of higher $\beta$ realizations. In particular, the buyer could replicate the performance of the discontinuous contract.

8 $G(\alpha)$ is defined precisely in the proof of Proposition 1.

9 The buyer’s expected payment under a cost reimbursement contract (which induces no cost-reducing effort) is the supplier’s expected innate cost, $\beta + k$. Therefore, from Lemma 1 and Proposition 1, the proportionate reduction in the buyer’s expected payment (relative to cost reimbursement) under the discontinuous contract rather than the LT contract is the ratio of $[\beta + k - (\bar{\beta} + 2k/3 - k/12)]$ to $[\beta + k - (\bar{\beta} + 2k/3)]$, which is $(\frac{\alpha}{12})/\frac{\alpha}{12} = 1.25$.

10 See [1] for details.
on \([\beta, \overline{\beta} + 2k]\) and induce \(e(\beta) > 0\) in a subset of the \([\beta + 2k, \overline{\beta}]\) region, and thereby reduce expected procurement cost in the region of high innate cost realizations.

The discontinuous contract is a relatively simple variant of the LT contract that can reduce expected procurement cost under the identified conditions. Alternative contracts might secure even greater gains for the buyer. Future research should examine the properties of optimal procurement contracts under both the identified conditions and other conditions. Alternative assumptions about the supplier’s ability to undertake direct cost inflation also merit consideration. For example, the supplier might enjoy intrinsic benefits from direct cost inflation, as when inflation entails the consumption of perquisites.\(^{11}\) Future research might also permit stochastic cost reduction and allow the buyer to procure multiple units of the commodity.

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Appendix

Proof of Lemma 1. It is readily shown that \(u(\beta)\) is a continuous function. Therefore, constraint (2) must bind at some \(\tilde{\beta} \in [\beta, \overline{\beta}]\) in any solution to [BP]. LT’s analysis is readily employed to prove that when \(\alpha \leq 1\), the incentive compatible contract that minimizes expected procurement cost on \([\beta, \overline{\beta}]\) is the LT contract, under which no cost inflation occurs.

For \(\beta \in [\tilde{\beta}, \overline{\beta}]\), incentive compatibility requires \(u'(\beta) = -\psi'(e(\beta))\) almost everywhere. Because \(u(\beta) \geq 0\) and \(u(\overline{\beta}) = 0\), it follows that:

\[
0 \leq u(\overline{\beta}) - u(\tilde{\beta}) = \int_{\tilde{\beta}}^{\overline{\beta}} u'(\beta) \, d\beta = - \int_{\tilde{\beta}}^{\overline{\beta}} \psi'(e(\beta)) \, d\beta \leq - \int_{\tilde{\beta}}^{\overline{\beta}} \frac{1}{2k} e(\beta) \, d\beta. \quad (A.1)
\]

The last inequality in (A.1) holds because \(\psi'(e(\beta)) = \frac{1}{2k} e(\beta)\) when \(e(\beta) \geq 0\) and \(\psi'(e(\beta)) = \frac{\alpha}{2k} e(\beta) \geq \frac{1}{2k} e(\beta)\) when \(e(\beta) \leq 0\), since \(\alpha \leq 1\). (A.1) implies \(\int_{\tilde{\beta}}^{\overline{\beta}} e(\beta) \, d\beta \leq 0\). For any \(\beta \in [\tilde{\beta}, \overline{\beta}]\), the reduction in procurement cost relative to the cost reimbursement contract is \(\beta - t(\beta)\). Since \(u(\beta) = t(\beta) - [\beta - e(\beta)] - \psi(e(\beta))\) from (2), this cost reduction is

\[
\beta - t(\beta) = e(\beta) - \psi(e(\beta)) - u(\beta) \leq e(\beta). \quad (A.2)
\]

The inequality in (A.2) holds because \(\psi(e(\beta))\) and \(\mu(\beta)\) are both non-negative for all innate cost realizations. (A.2) implies \(\int_{\tilde{\beta}}^{\overline{\beta}} \beta - t(\beta) \, dF(\beta) \leq \int_{\tilde{\beta}}^{\overline{\beta}} e(\beta) \, d\beta \leq 0\), and so the expected reduction in procurement cost relative to the cost reimbursement contract on \([\tilde{\beta}, \overline{\beta}]\) is at most zero. Therefore, because the LT contract minimizes expected procurement cost when \(\beta = \overline{\beta}\), the optimal value of \(\tilde{\beta}\) is \(\overline{\beta}\), and so the LT contract is the optimal contract when \(\alpha \in [0, 1]\).

\(^{11}\) See [5], for example.
Finally, notice that:
\[
\int_{\bar{\beta}}^{\bar{\beta}} t^L(\beta) \, d F(\beta) = \frac{1}{2k} \int_{\bar{\beta}}^{\bar{\beta}} \left[ 2\beta - \bar{\beta} + \frac{1}{2k} (\bar{\beta} - \beta)^2 \right] \, d\beta
\]
\[
= \frac{1}{2k} \left\{ -\beta^2 + \beta^2 - 2k\bar{\beta} + \frac{1}{6k} (\bar{\beta} - \beta)^3 \right\}
\]
\[
= \frac{1}{2k} \left\{ 2k\bar{\beta} - \frac{4}{3} k^2 \right\} = \bar{\beta} + \frac{2k}{3}.
\]
\[\Box\]

**Proof of Lemma 2.** To demonstrate that the discontinuous contract is individually rational, notice first that for all \(\beta \in [\bar{\beta}, \bar{\beta} + l]\), the supplier can undertake the same effort and receive the same payment less \([2k - l]^2/[4k]\) as under the LT contract. It is readily verified that the LT contract ensures the supplier a utility of at least \([2k - l]^2/[4k]\) for all \(\beta \in [\bar{\beta}, \bar{\beta} + l]\). Therefore, the discontinuous contract ensures non-negative utility for the supplier for all \(\beta \in [\bar{\beta}, \bar{\beta} + l]\).

For \(\beta \in [\bar{\beta} + l, \bar{\beta}]\), the maximum amount of cost inflation the supplier will undertake under the discontinuous contract is the difference between the highest final cost admitted under the discontinuous contract and the smallest innate cost in \([\bar{\beta} + l, \bar{\beta}]\). This difference is \([2k - l]^2/[1 + \sqrt{z}]\). The personal cost the supplier experiences from undertaking this level of cost inflation is \(z[2k - l]^2/[4k(1 + \sqrt{z})^2] = \tilde{r}^d - \tilde{c}^d\).

The maximum amount of cost-decreasing effort the supplier can supply under the discontinuous contract is the difference between the highest innate cost in \([\bar{\beta} + l, \bar{\beta}]\) and the smallest final cost the supplier can realize under the contract. This difference is \([2k - l]^2/[1 + \sqrt{z}]\). The personal cost the supplier incurs from delivering this effort is \(z[2k - l]^2/[4k(1 + \sqrt{z})^2] = \tilde{r}^d - \tilde{c}^d\).

Consequently, the supplier can always secure a utility of at least \(t^d(\beta) - c^d(\beta) - [\tilde{r}^d - \tilde{c}^d] = 0\) for all \(\beta \in [\bar{\beta} + l, \bar{\beta}]\) under the discontinuous contract. Therefore, the contract is individually rational.

To demonstrate that the discontinuous contract is incentive compatible, first notice that if the supplier claims to have innate cost \(\bar{\beta} + l\) when \(\beta \in [\bar{\beta}, \bar{\beta} + l]\), he must realize final cost \(\sqrt{z}[\bar{\beta} + l + \bar{\beta}]/[1 + \sqrt{z}]\). To secure this final cost, the supplier with innate cost \(\beta\) must undertake cost inflation \(\beta + l - \beta + [2k - l]/[1 + \sqrt{z}]\). This level of cost inflation entails personal disutility that exceeds \(z[2k - l]^2/[4k(1 + \sqrt{z})^2]\). Therefore, the supplier’s utility from this action is less than \(\tilde{r}^d - \tilde{c}^d - z[2k - l]^2/[4k(1 + \sqrt{z})^2] = 0\). Consequently, the supplier would receive negative utility from this form of cost exaggeration.

Now suppose the supplier’s innate cost is \(\bar{\beta} + l\). If the supplier understates his innate cost marginally and so claims to have innate cost \((\bar{\beta} + l)^-\) \(\leq \lim_{\beta \to \bar{\beta} + l} \beta\), he secures utility \(u((\bar{\beta} + l)^-|\bar{\beta} + l) = t^d((\bar{\beta} + l)^-) - e^d((\bar{\beta} + l)^-) - e^d((\bar{\beta} + l)|\bar{\beta} + l)^2/[4k] = 0\). Because the LT contract is incentive compatible, \(u(\beta|\beta + l) \leq u((\bar{\beta} + l)|\beta + l) = 0\) for all \(\beta \in [\bar{\beta}, \bar{\beta} + l]\). Therefore, the supplier with innate cost \(\bar{\beta} + l\) will not understate this cost. Furthermore, because \(e^d(\bar{\beta}|\bar{\beta})\) is increasing in \(\bar{\beta}\) for all \(\beta \in [\bar{\beta}, \bar{\beta} + l]\) and \(\bar{\beta} \in [\bar{\beta} + l, \bar{\beta}]\), \(u(\bar{\beta}|\bar{\beta}) < u(\beta|\beta + l) \leq 0\) for any \(\beta \in [\bar{\beta}, \bar{\beta} + l]\), for all \(\bar{\beta} \in (\bar{\beta} + l, \bar{\beta}]\). Therefore, the discontinuous contract is incentive compatible.

The identified expressions for effort and rent follow directly from Lemma 1 and from the facts that \(e^d(\beta) = \beta - e^d(\beta)\) and \(u^d(\beta) = t^d(\beta) - c^d(\beta) - \psi(e^d)\). \[\Box\]
Proof of Proposition 1. The buyer’s expected payment on the interval \([\bar{\beta}, \bar{\beta} + l]\) under the discontinuous contract is
\[
\int_{\bar{\beta}}^{\bar{\beta} + l} \left( 2\beta - \bar{\beta} + \frac{1}{2k} [\bar{\beta} - \beta]^2 - \frac{1}{4k} [2k - l]^2 \right) \frac{d\beta}{2k} = \left[ \beta - k + l - \frac{l^2}{12k} \right] \frac{l}{2k}. \tag{A.3}
\]
The buyer’s corresponding expected payment for \(\beta \in [\bar{\beta} + l, \bar{\beta}]\) is
\[
\left[ \frac{\sqrt{\pi}[\beta + l] + \bar{\beta}}{1 + \sqrt{\pi}} + \frac{\alpha[2k - l]^2}{4k[1 + \sqrt{\pi}]^2} \right] \left[ \bar{\beta} - (\beta + l) \right]. \tag{A.4}
\]
(A.3) and (A.4) imply the buyer’s total expected payment under the discontinuous contract is
\[
\left[ \beta - k + l - \frac{l^2}{12k} \right] \frac{l}{2k} + \left[ \frac{\sqrt{\pi}[\beta + l] + \bar{\beta}}{1 + \sqrt{\pi}} + \frac{\alpha[2k - l]^2}{4k[1 + \sqrt{\pi}]^2} \right] \left[ \frac{2k - l}{2k} \right]. \tag{A.5}
\]
The derivative of the expression in (A.5) with respect to \(l\) is
\[
\left[ \frac{1 + 2\sqrt{\pi} + 4\alpha}{8k^2(1 + \sqrt{\pi})^2} \right] [2k - l] \left( l - 2k \left[ \frac{3 + 2\sqrt{\pi} + 2\alpha}{1 + 2\sqrt{\pi} + 4\alpha} \right] \right). \tag{A.6}
\]
Letting \(\gamma = \frac{3 + 2\sqrt{\pi} + 2\alpha}{1 + 2\sqrt{\pi} + 4\alpha}\) and \(x = 2kl\), (A.6) implies the buyer’s expected payment under the discontinuous contract is
\[
\beta + \frac{2}{3}k - \frac{2}{3}k \gamma \left[ \frac{4x + 2\sqrt{\pi} + 1}{8k^2[1 + \sqrt{\pi}]^2} \right] [2k - l][l - 2k\gamma] dl = \beta + \frac{2}{3}k - G(x),
\]
where \(G(x) \equiv \frac{4x}{3} \left[ \frac{(\sqrt{\pi} - 1)^2(1 + \sqrt{\pi})}{(1 + 2\sqrt{\pi} + 4\alpha)^2} \right]\). It is apparent that \(G(1) = 0\). Furthermore, it is readily shown that \(G'(x) > 0\) if and only if \(3 + 12\sqrt{\pi} + 6\alpha > 0\). Therefore, \(G(x)\) is a strictly increasing function of \(x\) for all \(x \geq 0\). In addition:
\[
\lim_{x \to \infty} G(x) = \left[ \frac{4k}{3} \right] \lim_{x \to \infty} \left[ \frac{(\sqrt{\pi} - 1)^2[1 + \sqrt{\pi}]}{[1 + 2\sqrt{\pi} + 4\alpha]^2} \right] = \left[ \frac{4k}{3} \right] \lim_{x \to \infty} \left\{ \frac{x^2}{16\alpha^2} \right\} = \frac{k}{12}. \tag*{□}
\]

References


