Buildings, spiders, and geometric Satake

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Louis Kauffman found a special description of the Jones polynomial and the representation theory of $U_q(\mathfrak{sl}(2))$ in which each skein space has a basis of planar matchings. There is a similar calculus (discovered independently by myself and the late Francois Jaeger) for each of the three rank 2 simple Lie algebras $A_2$, $B_2$, and $G_2$. These skein theories, called “spiders”, can also be viewed as Gröbner-type presentations of pivotal categories. In each of the four cases (optionally also including the semisimple case $A_1 \times A_1$), the Gröbner basis property yields a basis of skein diagrams called “webs”. The basis webs are defined by an interesting non-positive curvature condition.

I will discuss a new connection between these spiders and the geometric Satake correspondence, which relates the representation category of a simple Lie algebra to an affine building of the Langlands dual algebra. In particular, any such building is CAT(0), which seems to explain the non-positive curvature of basis webs. Despite mysterious limitations, the model seems like it should be related to categorification.