On maximally stabilizing adaptive traffic signal control

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***Abstract*— In this paper, we design adaptive traffic signal control policies for urban traffic networks. Vehicles at the end of an approach to an intersection queue up in separate lanes, with finite flow capacities, corresponding to different possible turn maneuvers according to static route choice behavior. We analyze stability of such traffic networks under signal control policies that, at every intersection, give green light to at most one incoming lane at any time. We particularly focus on a class of *minimalist* distributed policies, under which traffic signal control at an intersection requires information only about the occupancy levels on the lanes incoming at that intersection, and does *not* require information about turn ratios, flow capacities or the external arrival rates to the network. We show that such a minimalist policy that exhibits monotonicity in the green light durations with respect to the occupancy levels is maximally stabilizing for acyclic network topologies, and admits a globally asymptotically stable equilibrium. Our results rely on novel tools developed recently for stability analysis of monotone dynamical systems with conservation of mass. We also present simulations to compare the stability conditions under static and dynamic route choice.**

I. INTRODUCTION

Traffic congestion is a major societal issue faced by many cities. Rapid advancements in traffic sensing technology has made it possible to use real-time traffic information to regulate traffic flow. This has opened up the possibility of replacing traditional fixed-timing traffic signal controllers with adaptive controllers. Motivated by such possibilities, this paper addresses the problem of designing adaptive signal control policies for urban traffic networks.

An overview of the problem and practices of urban traffic signal control can be found in [1], [2]. Classical strategies consist in extensive surveys to obtain network parameters and in design of traffic light plans, either fixed [3] or constantly re-tuned as in SCOOT [4]. Classical control techniques have also been used for signal control, e.g., see [5], [6]. However, these works do not provide any guarantees with respect to performance metrics of interest such as throughput, delay, and robustness to disruptions. Recently, well-known algorithms for routing in data networks, such as the back-pressure algorithm and its throughput analysis, have been adapted to the traffic signal control setting, e.g., see [7], [8]. However, these existing works require the traffic signal controllers to have explicit knowledge about the turn ratios representing the route choice behavior of drivers, as well as the lane flow capacities, which may be impractical especially in dynamic scenarios such as disruptions. In this paper, we address some of these shortcomings of the existing approaches.

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We consider a setup where every intersection of the traffic network is signalized. In our setting, vehicles at the end of an approach to an intersection queue up in separate lanes corresponding to different possible turn maneuvers according to static route choice behavior. Each lane is assumed to have finite capacity on flow but unbounded capacity on occupancy. We analyze stability of the traffic network under signal control policies that, at every intersection, give green light to at most one incoming lane at any time. We particularly focus on a class of *minimalist* distributed policies, under which traffic signal control at an intersection requires information only about the occupancy levels on the lanes incoming at that intersection, and does *not* require information about turn ratios, flow capacities or the external arrival rates to the network. We show that such a minimalist policy that exhibits monotonicity in the green light durations with respect to the occupancy levels is maximally stabilizing for acyclic network topologies, and admits a globally asymptotically stable equilibrium. We conjecture that the result holds true also for cyclic network topologies.

The contributions of this paper are as follows. First, we formulate and analyze a continuous time traffic signal control setting where the lanes have finite flow capacities. Second, we show that distributed routing policies that rely only on information about occupancy levels on incoming lanes at intersections are sufficient to stabilize the traffic network. Finally, through the traffic signal control setting of this paper, we illustrate the utility of novel tools developed recently [9] for stability analysis of monotone dynamical systems with conservation of mass for an important control design problem.

The rest of the paper is organized as follows. In Section II, we describe the problem setup and provide the main results in Section III. The proofs of the main results of the paper are given in Section IV. Section V reports results from a simulation study to compare the stability conditions under static and dynamic route choice behaviors, and to illustrate the stability of cyclic network topologies under monotonic distributed green light policies. Finally, we conclude with remarks on future work in Section VI.

We conclude this section by defining important notations to be used throughout the paper. Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}_+ := \{x \in \mathbb{R} : x \geq 0\}$ be the set of nonnegative real numbers. Let $\mathcal{A}$ and $\mathcal{B}$ be finite sets. Then $|\mathcal{A}|$ denotes the cardinality of $\mathcal{A}$, $\mathbb{R}^{\mathcal{A}}$ (respectively, $\mathbb{R}_+^{\mathcal{A}}$) the space of real-valued (nonnegative-real-valued) vectors whose components are indexed by elements of $\mathcal{A}$, and $\mathbb{R}^{\mathcal{A}\times\mathcal{B}}$ the space of matrices whose real entries are indexed by pairs in $\mathcal{A}\times\mathcal{B}$. If $\mathcal{B}\subseteq\mathcal{A}$ and $x\in\mathbb{R}^{\mathcal{A}}$, then $x_{\mathcal{B}}\in\mathbb{R}^{\mathcal{B}}$ stands for the projection of x on $\mathcal{B}$. A directed multi-graph is $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ stand for the node set and the link set, respectively, and are both finite. They are endowed with two vectors: $\sigma,\tau\in\mathcal{V}^{\mathcal{E}}$. For every $e\in\mathcal{E}$, σ_e and τ_e stand for the tail and head nodes respectively of link e. We shall always assume that there are no self-loops, i.e., $\tau_e\neq\sigma_e$, for all $e\in\mathcal{E}$. On the other hand, we allow for parallel links. For a node $v\in\mathcal{V}$, let $\mathcal{E}_v^+:=\{e:\sigma_e=v\}$ and $\mathcal{E}_v^-:=\{e:\tau_e=v\}$. For a link $e\in\mathcal{E}$, let $\mathcal{E}_e^+:=\mathcal{E}_{\tau_e}^+$ be the set of links downstream to e and $\mathcal{E}_e^-:=\mathcal{E}_{\sigma_e}^-$ be the set of links upstream to e. For a vector $x\in\mathbb{R}^{\mathcal{E}}$, we shall denote by $x^v:=\{x_j:j\in\mathcal{E}_v^-\}$ its projection on $\mathcal{E}_v^-$. For a finite set $\mathcal{A}$, let $\mathcal{S}_{\mathcal{A}}:=\{x\in\mathbb{R}_+^{\mathcal{A}}:\sum_{i\in\mathcal{A}}x_i=1\}$ denote the simplex defined over the elements of $\mathcal{A}$, and $\bar{\mathcal{S}}_{\mathcal{A}}:=\{x\in\mathbb{R}_+^{\mathcal{A}}:\sum_{i\in\mathcal{A}}x_i\leq 1\}$.

II. Problem formulation

Let $\mathcal{N}$ be the set of intersections in the road network, each of which is signalized. These intersections are connected by directed roads. Following standard simplifications (e.g., see Figure 1), we assume that each road has a separate lane for every possible turn maneuver while leaving that road and that, upon entering the road from upstream, the drivers queue up in one of these lanes based on their desired turn at the upcoming intersection. For convenience, we introduce a *virtual* intersection at the point where a road splits into different lanes. Let $\mathcal{U}$ be the set of such virtual nodes. We regard the portion of the road upstream of the virtual intersection as a separate directed link in the underlying network topology than the lanes downstream of the virtual intersection on the road, which correspond to different links. Accordingly, let $\mathcal{R}$ be the set of directed links corresponding to the portion of the roads upstream of the virtual intersections, and $\mathcal{L}$ be the directed links corresponding to the lanes. We model external arrivals to the network to be at the set of links in $\mathcal{R}$, i.e., upon entering a link $e\in\mathcal{R}$, the external traffic homogeneously joins other traffic incoming to e from upstream parts of the network. Let the corresponding arrival rate vector be denoted by $\lambda\in\mathbb{R}_+^{\mathcal{R}}$. Similarly, each link in $\mathcal{R}$ could act as a destination for the drivers, i.e., upon reaching link $e\in\mathcal{R}$, drivers for whom the destination lies on e leave the network. We model the departure rate from link e to be $\theta_e\in[0,1]$ fraction of the inflow to link e. Let $\theta=\{\theta_e:e\in\mathcal{R}\}$ be the vector of departure ratios from all links in $\mathcal{R}$. Note that we do not distinguish between arrival rates for different destinations. Such a distinction will not be necessary in this paper since we consider static route choice behavior, and hence it is sufficient to consider aggregate arrival over all destinations. In summary, the network topology used for the analysis of road network is the directed multi-graph $\mathcal{G}=(\mathcal{V},\mathcal{E})=(\mathcal{N}\cup\mathcal{U},\mathcal{R}\cup\mathcal{L})$. Throughout this paper, we shall restrict ourselves to network topologies satisfying the following:

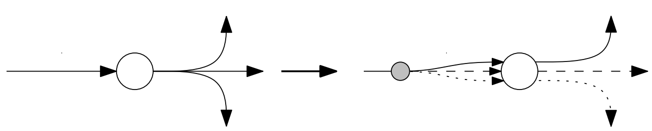

Fig. 1. An approach with three possible turn maneuvers at the intersection is split into three lanes, one for each turn maneuver.

Assumption 1: $\mathcal{G}$ contains no cycles.

For $i\in\mathcal{L}$, let $C_i\in(0,+\infty)$ denote the flow capacity of lane i. We assume that all the lanes have unbounded occupancy capacities. Following the notations established towards the end of Section I, for $u\in\mathcal{U}$, $\mathcal{E}_u^-$ and $\mathcal{E}_u^+$ denote the unique incoming link and the set of lanes coming out of u respectively, and for $v\in\mathcal{N}$, $\mathcal{E}_v^-$ denotes the set of lanes incoming to v. We assume that the links in $\mathcal{R}$ have infinitesimal sizes, and that the state of the system is described by occupancy levels on the lanes in $\mathcal{L}$. Let $\rho\in\mathbb{R}_+^{\mathcal{L}}$ be the corresponding vector of occupancy levels on the lanes. For example, $\rho_\ell(t)$ denotes the total number of cars present on lane ℓ at time t. The dynamics in the occupancy levels is governed primarily by the external inflow into the network, the route choice behavior of the drivers at the virtual intersections $\mathcal{U}$ and the traffic signal control policies at the intersections $\mathcal{N}$. We next formalize the route choice behavior and the traffic signal control policies.

For every virtual intersection $u\in\mathcal{U}$, we associate a route choice vector $G^u\in\mathcal{S}_{\mathcal{E}_u^+}$, sometimes also referred to as *turn ratio* at u, e.g., see [8]. This corresponds to *static* route choice. The choice of lane i in $\mathcal{E}_u^+$ according to G^u uniquely determines the link $e\in\mathcal{E}_{\tau_i}^+$ that the drivers in i will turn into when i is given a green light by the signal control policy at the intersection τ_i. Such a pair (i,e) is referred to as a *phase* in the terminology of traffic signal control. In this paper, we consider a traffic signal control architecture where, at every intersection, at most one phase is given green light at any given time. Accordingly, for an intersection $v\in\mathcal{N}$, we denote the *green light policy* at v by $h^v:\mathbb{R}_+^{\mathcal{E}_v^-}\to\bar{\mathcal{S}}_{\mathcal{E}_v^-}$. Let $G=\{G^u:u\in\mathcal{N}\}$ be the set of route choice vectors at all the intersections. For brevity in notation, we shall drop the superscript on G and h when the referred intersection is clear from the context.

We are now ready to describe the dynamics in occupancy levels:

$$\dot{\rho}_i(t)=f_i^{\text{in}}(\rho)-C_ih_i\left(\rho^{\tau_i}\right),\qquad\forall i\in\mathcal{L},\tag{1}$$

where $f_i^{\text{in}}(\rho)$ is the inflow to lane i. If $e(i)\in\mathcal{R}$ denotes the unique link upstream of lane i, then for all $\rho\in\mathbb{R}_+^{\mathcal{L}}$,

$$f_i^{\text{in}}(\rho)=\Big(\left(1-\theta_{e(i)}\right)f_{e(i)}^{\text{in}}(\rho)+\lambda_{e(i)}\Big)G_i^{\sigma_i},$$

where $f_{e(i)}^{\text{in}}(\rho)$ is the inflow to link $e(i)$ from the upstream part of the network, defined as $f_{e(i)}^{\text{in}}(\rho)=\sum_{j\in\mathcal{E}_{\sigma_{e(i)}}^-:\,(j,e(i))\text{ is a phase}}C_jh_j(\rho^{\sigma_{e(i)}})$ if $\mathcal{E}_{\sigma_{e(i)}}^-\neq\emptyset$ and $f_{e(i)}^{\text{in}}(\rho)=0$ otherwise. In order to ensure positivity of lane occupancies under the dynamics in Equation (1), we implicitly assume the following additional condition on the green light policy: for all $v \in \mathcal{N}$, $i \in \mathcal{E}_v^-$,

$$\rho_i \to 0^+ \quad \implies \quad h_i(\rho^v) \to 0^+ . \tag{2}$$

Remark 1: 1) The traffic signal control policies are typically designed in a discrete time setting, where only a subset of nonconflicting phases are given green light between successive epochs. The green light policies designed in the continuous setting of this paper could be interpreted as time-averaged green light duration in an appropriate discrete time queueing network setting. The rigorous study of this connection is however a topic of investigation in the future.

2) The positivity constraint on ρ_i under the dynamics in (1) can also be obtained by replacing C_i with $C_i \mathbf{1}_{\rho_i > 0}$ in (1), where $\mathbf{1}$ is the indicator function. Our choice of rather using (2) is only for technical convenience.

Assumption 1 implies that one can find a (not necessarily unique) topological ordering of the set of intersections $\mathcal{N}$ (see, e.g., [10]). We shall assume to have fixed one such ordering, and at times we will identify $\mathcal{N}$ with the integer set $\{0, \ldots, |\mathcal{N}|\}$ in such a way that

$$\mathcal{E}_v^- \subseteq \bigcup_{1 \le u < v} \bigcup_{e \in \mathcal{E}_u^+} \mathcal{E}_{\tau_e}^+ , \qquad \forall\, v = 1, \ldots, |\mathcal{N}| .$$

For a given $G \in \Pi_{v \in \mathcal{N}} \bar{\mathcal{S}}_{\mathcal{E}_v^-}$, $\lambda \in \mathbb{R}_+^{\mathcal{R}}$ and $\theta \in [0,1]^{\mathcal{R}}$, let $f_i(G, \lambda, \theta)$ denote the flow *induced* on lane $i \in \mathcal{L}$, defined inductively as:

$$f_i(G, \lambda, \theta) := \left(\left(1 - \theta_{e(i)}\right) g_{e(i)}(G, \lambda, \theta) + \lambda_{e(i)} \right) G_i^{\sigma_i} ,$$

where $g_{e(i)}(G, \lambda, \theta) = \sum_{j \in \mathcal{E}^-_{\sigma_{e(i)}} : (j, e(i)) \text{ is a phase}} f_j(G, \lambda, \theta)$ if $\mathcal{E}^-_{\sigma_{e(i)}} \neq \emptyset$ and zero otherwise. Let $f(G, \lambda, \theta) = \{f_i(G, \lambda, \theta) : i \in \mathcal{L}\}$ be the vector of induced flows on all the lanes. The idea behind induced flow is that it is the load induced on different lanes of the network as a result of external arrival rates λ, route choice vector G and departure ratios θ. The flow induced by a given G, λ and θ is called *feasible* if: (i) for all $i \in \mathcal{L}$, $f_i(G, \lambda, \theta) \in [0, C_i]$; and (ii) for all $n \in \mathcal{N}$ such that $\mathcal{E}_v^+ = \emptyset$, $f_i(G, \lambda, \theta) = 0$ for all $i \in \mathcal{E}_v^-$. Essentially, an induced flow is called feasible, if it is non-negative and satisfies lane-wise capacity constraints, and all the flow departs the network. Throughout this paper, we shall implicitly make the natural assumption that G, λ and θ are such that the induced flow $f(G, \lambda, \theta)$ is feasible. For given G, λ and θ, the traffic network is *stabilizable* if there exists a (not necessarily distributed) green light policy $\{h^v(\rho) : v \in \mathcal{N}\}$ such that, under the dynamics in (1), from any initial condition $\rho^\circ \in \mathbb{R}_+^{\mathcal{L}}$,

$$\limsup_{t \to \infty} \rho_e(t) < +\infty .$$

The primary objective of this paper is to design distributed green light policies for traffic networks that are maximally stabilizing.

III. Main Results

In this paper, we show that the following class of policies, called *monotonic distributed green light policies* is maximally stabilizing.

Definition 1: A distributed green light control policy is a family of Lipschitz-continuous maps $\{h^v\}_{v \in \mathcal{N}}$, $h^v : \mathbb{R}_{\mathcal{E}_v^-}^+ \to \bar{\mathcal{S}}_{\mathcal{E}_v^-}$. We say that the policy is *monotonic* if it satisfies the following for all $v \in \mathcal{N}$, and for almost every $\rho^v \in \mathbb{R}_+^{\mathcal{E}_v^-}$:

a) $\frac{\partial}{\partial \rho_j} h_i(\rho^v) < 0, \quad \forall i, j \in \mathcal{E}_v^-, \, i \neq j$,

b) $\frac{\partial}{\partial \rho_j} \sum_{i \in \mathcal{E}_v^-} h_i(\rho^v) > 0, \quad \forall j \in \mathcal{E}_v^-$.

c) for any $\mathcal{I} \subseteq \mathcal{E}_v^-$, if $\sum_{i \in \mathcal{I}} \rho_i \to +\infty$ and $\frac{\sum_{i \in \mathcal{I}} \rho_i}{\sum_{j \in \mathcal{E}_v^-} \rho_j} \to 1^-$, then $\sum_{i \in \mathcal{I}} h_i(\rho^v) \to 1^-$.

Observe that the Lipschitzianity assumption on the green light policy ensures, by standard analytical results (Picard's Existence Theorem), existence and uniqueness of a solution of the system for every initial condition $\rho(0) \in \mathbb{R}_+^{\mathcal{L}}$.

Example 1: An example of a monotonic green light policy is given by the following: for all $v \in \mathcal{N}$, $i \in \mathcal{E}_v^-$,

$$h_i(\rho^v) = \frac{\rho_i}{\sum_{j \in \mathcal{E}_v^-} \rho_j + \kappa}$$

for a constant $\kappa > 0$.

We are now in a position to state the main result of the paper.

Theorem 1: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a network satisfying Assumption 1 with lane flow capacities $C \in \mathbb{R}_+^{\mathcal{L}}$, route choice vectors $G \in \Pi_{v \in \mathcal{N}} \bar{\mathcal{S}}_{\mathcal{E}_v^-}$, external arrival rates $\lambda \in \mathbb{R}_+^{\mathcal{R}}$, departure ratios $\theta \in [0,1]^{\mathcal{R}}$ and green light policy $\{h(\rho^v) : v \in \mathcal{N}\}$. Let $\rho(t)$ be the solution of (1) starting from initial condition $\rho^\circ \in \mathbb{R}_+^{\mathcal{L}}$. Then:

(i) If the network is stabilizable, then the induced flows satisfy $\sum_{i \in \mathcal{E}_v^-} f_i \left(G, \lambda, \theta\right) / C_i \le 1$ for all $v \in \mathcal{N}$;

(ii) If $\{h(\rho^v) : v \in \mathcal{N}\}$ are monotonic green light policies as per Definition 1 and $\sum_{i \in \mathcal{E}_v^-} f_i \left(G, \lambda, \theta\right) / C_i < 1$ for all $v \in \mathcal{N}$, then there exists a unique equilibrium $\rho^* \in \mathbb{R}_+^{\mathcal{L}}$ such that

$$\lim_{t \to +\infty} \rho(t) = \rho^*$$

for every initial condition $\rho^\circ \in \mathbb{R}_+^{\mathcal{L}}$.

Remark 2: Theorem 1 implies that the class of monotonic distributed green light policies as per Definition 1 is maximally stabilizing within the class of all, not necessarily distributed, green light policies. While maximally stabilizing distributed green light policies inspired by the back pressure routing policies from data networks have been proposed, e.g., in [7], [8], the class of monotonic distributed green light policies have at least two key advantages. First, unlike the policies in [7], [8], they do not require information about the route choice vectors G and the lane flow capacities $C_i, i \in \mathcal{L}$. Moreover, unlike [7], [8] where the green light policy at an intersection requires information about the occupancy levels on the incoming as well as outgoing lanes at that intersection, the class of monotonic distributed routing policies requires information about occupancy levels only

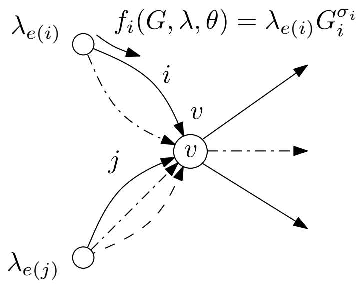

Fig. 2. An example of local system with 5 incoming lanes.

on the lanes incoming to an intersection. Second, the class of monotonic distributed routing policies guarantee global asymptotic stability. However, we also emphasize that, unlike [7], [8], the stability result in Theorem 1 is restricted to acyclic network topologies and to traffic signal architectures that give green light to at most one phase at a time at every intersection. Nonetheless, we conjecture that Theorem 1 also extends to cyclic network topologies. We provide simulations to support this conjecture in Section V.

We prove Theorem 1 through an induction argument. To this aim, first we study a local system consisting of a single intersection in isolation, and then use a cascade argument to extend the analysis to the network case. Moreover, studying the local system first will also allow us to illustrate key steps in our analysis in a simple setup.

A. Base case: the local system

We consider a local system composed of a single intersection v with n incoming lanes, $\mathcal{E}_v^- = \{1,\dots,n\}$, with flow capacities $C_1, C_2, \dots, C_n$, respectively. Each lane i lies downstream link $e(i)$, see Figure 2. We assume for each $e(i)$ that $f_{e(i)}^{\text{in}} = 0$, so that $f_i(G,\lambda,\theta) = \lambda_{e(i)} G_i^{\sigma_i}$, for all $i \in \mathcal{E}_v^-$. The resulting dynamics on the lanes are:

$$\dot\rho_i = f_i(G,\lambda,\theta) - C_i h_i(\rho), \quad i \in \mathcal{E}_v^- . \tag{3}$$

This local system models an intersection whose inflows only come from the external world. In particular, it provides the base case for the induction argument. Indeed, consider the whole network. Recall that since it is acyclic, we can order the nodes in such a way that

$$\mathcal{E}_v^- \subseteq \bigcup_{1\le u<v} \bigcup_{e\in\mathcal{E}_u^+} \mathcal{E}_{\tau_e}^+ , \qquad \forall\, v = 1,\dots,n .$$

Let $\hat v$ be the first node in the topological ordering such that $\mathcal{E}_{\hat v}^- \neq \emptyset$. Notice that if u is such that $1 \le u < \hat v$, then u has no incoming lanes, and thus if $e \in \mathcal{R}$, $\sigma_e = u$, is the unique link of infinitesimal size stemming from u, then $f_e^{\text{in}} = 0$. This implies that the reduced system composed of the links in $\mathcal{E}_{\hat v}^-$ is a particular case of local system as described above.

Next we shall prove Theorem 1 for the local system. First, we need the following technical result, which holds true for the whole network.

Lemma 1: Let $\rho(t)$ denote the solution of (1) with initial condition ρ° and green light policy $\{h_v(\rho^v)\}_{v\in\mathcal{N}}$. If, for some $i \in \mathcal{L}$, $\limsup_{t\to\infty} \rho_i(t) < +\infty$, then

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \left(C_i h_i(\rho(s) - f_i^{\text{in}}(\rho(s)) \right) ds = 0 .$$

Proof: Stability and (1) imply

$$\begin{aligned} 0 &= \lim_{t\to\infty} t^{-1}(\rho_i(t) - \rho_i(0)) \\ &= \lim_{t\to\infty} t^{-1} \int_0^t \left(f_i^{\text{in}}(\rho(s)) - C_i h_i(\rho(s)) \right) ds . \end{aligned}$$

■

We can now prove the necessary condition for the stability of the system, i.e., part (i) of Theorem 1.

Lemma 2: Let $\rho(t)$ denote the solution of (3) with initial condition ρ° and green light policy $h(\rho)$. If

$$\limsup_{t\to\infty} \rho_i(t) < +\infty , \qquad \forall i \in \mathcal{E}_v^- ,$$

then

$$\sum_{i\in\mathcal{E}_v^-} \frac{f_i(G,\lambda,\theta)}{C_i} \le 1 .$$

Proof: For the local network (3), Lemma 1 ensures that

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t C_i h_i(\rho(s)) ds - f_i(G,\lambda,\theta) = 0 , \qquad \forall i \in \mathcal{E}_v^- .$$

It follows that

$$\sum_{i\in\mathcal{E}_v^-} \frac{f_i(G,\lambda,\theta)}{C_i} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \sum_{i\in\mathcal{E}_v^-} h_i(\rho(s)) ds \le 1 ,$$

thus proving the claim. ■

The following proposition establishes instead the sufficient condition for the existence of a globally asymptotically stable equilibrium in the local system, i.e., part (ii) of Theorem 1. The proof is postponed to Section IV.

Proposition 1: Let $\rho(t)$ denote the solution of (3) with initial condition ρ° and green light policy $h(\rho)$. Assume that $h(\rho)$ is monotonic as per Definition 1 and that

$$\sum_{i\in\mathcal{E}_v^-} \frac{f_i(G,\lambda,\theta)}{C_i} < 1 .$$

Then (3) admits a globally asymptotically stable equilibrium.

B. Induction step: a cascade argument

In the previous Section we have proved that Theorem 1 holds on the reduced system defined on the links in $\cup_{1\le u\le\hat v} \mathcal{E}_u^- = \mathcal{E}_{\hat v}^-$, where $\hat v$ is the first node in the topological ordering such that $\mathcal{E}_{\hat v}^-$ is not the empty set.

Let now v be a generic node in the network, $v > \hat v$, and assume, by induction, that Theorem 1 holds on the reduced system defined on the links in $\cup_{1\le u<v} \mathcal{E}_u^-$.

The dynamics on the generic link $i \in \mathcal{E}_v^-$ is given by

$$\dot\rho_i = \left(\left(1 - \theta_{e(i)}\right) f_{e(i)}^{\text{in}}(\rho) + \lambda_{e(i)} \right) G_i^{\sigma_i} - C_i h_i(\rho^v)$$

where $f_{e(i)}^{\text{in}}(\rho) = \sum_{j\in\mathcal{E}_{\sigma_{e(i)}}^- : (j,e(i)) \text{ is a phase}} C_j h_j(\rho^{\sigma_{e(i)}})$ if $\mathcal{E}_{\sigma_{e(i)}}^- \neq \emptyset$, and $f_{e(i)}^{\text{in}}(\rho) = 0$ otherwise. Note that $\sigma_{e(i)} < v$ for all $i \in \mathcal{E}_v^-$ by acyclicity. We can now prove the induction step.

1) Necessity: assume that the network is stabilizable. In particular, this holds true for the reduced system in $\cup_{1\leq u<v}\mathcal{E}_u^-$, and thus the induction assumption implies that

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t C_j h_j(\rho(s))ds = f_j(G,\lambda,\theta)\,,\quad \forall j\in\bigcup_{1\leq u<v}\mathcal{E}_u^-\,.$$

In turn, this implies that

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t f_{e(i)}^{\text{in}}(\rho(s))ds = \sum_{j\in\mathcal{E}_{\sigma_{e(i)}}^-:\,(j,e(i))\text{ is a phase}} f_j(G,\lambda,\theta)$$

and hence

$$\begin{aligned}\lim_{t\to\infty}\frac{1}{t}\int_0^t f_i^{\text{in}}(\rho(s))ds &= \left(\left(1-\theta_{e(i)}\right)g_{e(i)}+\lambda_{e(i)}\right)G_i^{\sigma_i}\\ &= f_i(G,\lambda,\theta)\,,\end{aligned}$$

as $g_{e(i)}=\sum_{j\in\mathcal{E}_{\sigma_{e(i)}}^-:\,(j,e(i))\text{ is a phase}} f_j(G,\lambda,\theta)$, if $\mathcal{E}_{\sigma_{e(i)}}^-\neq\emptyset$, and $g_{e(i)}=0$ otherwise. This implies, by Lemma 1, the key induction step

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t C_i h_i(\rho(s))ds = f_i(G,\lambda,\theta),\quad \forall i\in\mathcal{E}_v^-\,,$$

and the thesis

$$\sum_{i\in\mathcal{E}_v^-}\frac{f_i(G,\lambda,\theta)}{C_i}=\lim_{t\to\infty}\frac{1}{t}\int_0^t h_i(\rho(s))ds\leq 1\,.$$

2) Sufficiency: since $\{h(\rho^u)\,:\,u\in\mathcal{N}\}$ are monotonic and $\sum_{i\in\mathcal{E}_u^-}\frac{f_i(G,\lambda,\theta)}{C_i}<1$ for all $u\in\mathcal{N}$, this holds true in particular for $1\leq u<v$. By the induction assumption, this implies that the system on the links in $\cup_{1\leq u<v}\mathcal{E}_u^-$ admits a globally asymptotically equilibrium. In particular, the density ρ_j on each $j\in\mathcal{E}_u^-$, $1\leq u<v$ will have a finite limit, and hence the quantities $C_j h_j(\rho^u)$ will also converge. Since convergence is towards an equilibrium, it is easy to see that $C_j h_j(\rho^u(t))\overset{t\to\infty}{\longrightarrow} f_j(G,\lambda,\theta)$, for all $j\in\cup_{1\leq u<v}\mathcal{E}_u^-$. In turn, this implies that for all $i\in\mathcal{E}_v^-$,

$$\begin{aligned}f_i^{\text{in}}(\rho(t)) &\overset{t\to\infty}{\longrightarrow}\left(\left(1-\theta_{e(i)}\right)g_{e(i)}(G,\lambda,\theta)+\lambda_{e(i)}\right)G_i^{\sigma_i}\\ &= f_i(G,\lambda,\theta)\,.\end{aligned}$$

We shall now interpret the system on the links $\mathcal{E}_v^-$ as a controlled system

$$\dot\rho_i=\nu_i-C_i h_i(\rho^v)=\Psi_i(\rho,\nu)\,,\quad i\in\mathcal{E}_v^-$$

where $\nu=\{\nu_i\}_{i\in\mathcal{E}_v^-}$ is the vector of external inputs, given by $\nu_i(t)=f_i^{\text{in}}(\rho(t))$.

Notice that $\frac{\partial\Psi_i(\rho,\nu)}{\partial\nu_j}\geq 0$ for all i,j, and that $\frac{\partial\Psi_i(\rho,\nu)}{\partial\rho_j}\geq 0$ for all $j\neq i$. The system with external inputs ν is thus a *monotone control system* in the sense of [11]. For every fixed $\nu(t)\equiv\bar\nu$ such that $\sum_{i\in\mathcal{E}_v^-}\frac{\bar\nu_i}{C_i}<1$ the system admits a unique globally asymptotically equilibrium, thus, using the terminology in [11], the local system admits a *static input-state characteristic*. Also notice that the local system takes values in the cone $\mathbb{R}_+^{\mathcal{E}_v^-}$, which admits a bounded order. Since $\nu_i\to f_i(G,\lambda,\theta)$ for all $i\in\mathcal{E}_v^-$ and $\sum_{i\in\mathcal{E}_v^-}\frac{f_i(G,\lambda,\theta)}{C_i}<1$, we can apply Proposition V.5 in [11] to conclude that $\rho_i(t)\to\rho_i^*$ for all $i\in\mathcal{E}_v^-$, where ρ^* is a globally asymptotically stable equilibrium.

IV. Proof of Proposition 1

In this section, we provide proof for Proposition 1. We start with the following simple adaptation of the ℓ_1 contraction principle for monotone dynamical systems with mass conservation that is proven in [9].

Lemma 3: Let $\varphi:\mathbb{R}_+^m\to\mathbb{R}^m$ be Lipschitz and such that

$$\frac{\partial}{\partial x_j}\varphi_i(x)\geq 0\,,\qquad \forall\, i\neq j\in\{1,\ldots,m\}\qquad(4)$$

$$\sum_{1\leq i\leq m}\frac{\partial}{\partial x_j}\varphi_i(x)\leq 0\,,\qquad \forall\, j\in\{1,\ldots,m\}\qquad(5)$$

for almost every $x\in\mathbb{R}_+^m$. Then

$$\sum_{1\leq i\leq n}\operatorname{sgn}(x_i-y_i)\left(\varphi_i(x)-\varphi_i(y)\right)\leq 0,\qquad \forall x,y\in\mathbb{R}_+^m\,.\qquad(6)$$

Moreover, if

(i) there exists some $j\in\{1,\ldots,n\}$ such that the inequality (5) is strict for almost all $x\in\mathbb{R}_+^m$,

then inequality (6) is strict for all $x,y\in\mathbb{R}_+^m$ such that $x_j\neq y_j$.

If

(ii) for every proper subset $\mathcal{K}\subseteq\{1,\ldots,m\}$, there exist $i\in\mathcal{K}$, and $j\in\{1,\ldots,m\}\setminus\mathcal{K}$ such that inequality (4) is strict for almost all $x\in\mathbb{R}_+^m$,

then inequality (6) is strict for all $x\neq y$ such that $x\not\leq y$ and $y\not\leq x$.

Finally, if (i) and (ii) hold true, then inequality (6) is strict for all $x,y\in\mathbb{R}_+^m$ such that $x\neq y$.

We are now ready to prove Proposition 1. Notice that, under the property (a) in Definition 1, (3) is a *cooperative* dynamical system in the sense of Hirsch [12], [13], i.e,

$$\frac{\partial\Phi_i}{\partial\rho_j}(\rho)\geq 0,\qquad \forall i,j\in\mathcal{E}_v^-,i\neq j\,,\qquad(7)$$

where $\Phi_i(\rho)$ is the right hand side of (3) Then, Kamke's theorem [13, Theorem 1.2], [14] implies that (3) is a monotone system [12], i.e.,

$$\rho(0)\preceq\varrho(0)\quad\Rightarrow\quad\rho(t)\preceq\varrho(t)\,,\quad\forall t\geq 0\,.\qquad(8)$$

Therefore, existence of a limit density is ensured for the initial condition $\rho(0)=\mathbf{0}$. Indeed, let $\phi^t(\rho^\circ)=\rho(t)$ denote the solution of (3) with initial condition $\rho(0)=\rho^\circ\in\mathbb{R}_+^{\mathcal{E}_v^-}$. Then, (8) implies that $\phi^{t+s}(\mathbf{0})=\phi^t(\phi^s(\mathbf{0}))\succeq\phi^t(\mathbf{0})$, for $s,t\geq 0$, i.e., $\phi^t(\mathbf{0})$ is component-wise non-decreasing and hence convergent to some limit, to be denoted by $\rho^*:=\lim_{t\to+\infty}\phi^t(\mathbf{0})$.

We shall now prove the existence of a globally asymptotically stable equilibrium in three steps. First of all, we prove stability for the trajectory with initial condition $\rho(0)=\mathbf{0}$. Let $\mathcal{J}:=\{e\in\mathcal{E}_v^-\,:\,\rho_e^*=+\infty\}$, and $\mathcal{J}^c:=\mathcal{E}_v^-\setminus\mathcal{J}$. Note that $\mathcal{J}^c\neq\emptyset$ because this will imply that $\sum_{i\in\mathcal{E}_v^-}\rho_i/C_i\to+\infty$, which contradicts the fact that

$$\lim_{t\to+\infty}\sum_{i\in\mathcal{E}_v^-}\frac{\dot\rho_i}{C_i}=\lim_{t\to+\infty}\sum_{i\in\mathcal{E}_v^-}\left(\frac{f_i(G,\lambda,\theta)}{C_i}-h_i(\rho)\right)<0.$$

Assume that $\mathcal{J} \neq \emptyset$. Since $\mathcal{J} \neq \mathcal{E}_v^-$, property (c) in Definition 1 implies that, for all $i \in \mathcal{J}^c$, $h_i(\rho) \to 0^+$, and that $\sum_{i\in\mathcal{J}} h_i(\rho) \to 1^-$. This yields

$$\lim_{t\to+\infty} \sum_{i\in\mathcal{J}} \frac{\dot{\rho}_i}{C_i} = \lim_{t\to+\infty} \sum_{i\in\mathcal{J}} \left(\frac{f_i(G,\lambda,\theta)}{C_i} - h_i(\rho) \right) < 0,$$

which contradicts the assumption that $\sum_{i\in\mathcal{J}} \rho_i/C_i \to +\infty$. Therefore, $\mathcal{J} = \emptyset$, i.e., (3) is stable when $\rho(0) = \mathbf{0}$. The system is thus stable, and in particular the limit point ρ^* is such that $\rho_i^* < \infty$ for all $i \in \mathcal{E}_v^-$.

As a second step, we show that the vector of densities ρ^* is an equilibrium. But this is immediate, since Barbalat's Lemma and $\rho_i^* < +\infty$ for all i imply that $\lim_{t\to\infty} \dot{\rho}_i(t) = 0$.

Third, we address the general initial condition $\rho(0) = \rho^\circ \in \mathbb{R}^{\mathcal{E}_v^-}$. We establish stability through the use of Lemma 3 to prove that

$$\|\tilde{\rho}(t) - \tilde{\rho}^*\|_1 \leq \|\tilde{\rho}(0) - \tilde{\rho}^*\|_1 \qquad \forall\, t \geq 0\,, \tag{9}$$

where $\tilde{\rho}_e = \rho_e/C_e$ is a scaled variable and $\rho(t) = \phi^t(\rho^\circ)$. Indeed, if we consider the dynamics of the scaled variable $\dot{\tilde{\rho}}_i = \lambda_i/C_i - h_i(\rho) =: \tilde{h}_i(\tilde{\rho})$. Then $\frac{\partial}{\partial \tilde{\rho}_j}\tilde{h}_i(\tilde{\rho}) = -\frac{\partial}{\partial \tilde{\rho}_j} h_i(\rho) = -C_j \frac{\partial}{\partial \rho_j} h_i(\rho) > 0$ (from property (a) in Definition 1). Also, $\sum_{k\in\mathcal{E}_v^-} \frac{\partial}{\partial \tilde{\rho}_j}\tilde{h}_k(\tilde{\rho}) = \sum_{k\in\mathcal{E}_v^-} -C_j \frac{\partial}{\partial \rho_j} h_k(\rho) < 0 \; \forall j \in \mathcal{E}_v^-$ (from property (b) in Definition 1). Therefore, Lemma 3 implies that

$$\frac{d}{dt}\|\tilde{\rho}(t) - \tilde{\rho}^*\|_1 = \sum_{i\in\mathcal{E}_v^-} \mathrm{s}(\tilde{\rho}_i, \tilde{\rho}_i^*) \left(\tilde{h}_i(\tilde{\rho}(t)) - \tilde{h}_i(\tilde{\rho}^*) \right) \leq 0,$$

where $\mathrm{s}(x,y) = \mathrm{sgn}\,(x-y)$, with strict inequality if $\tilde{\rho}(t) \neq \tilde{\rho}^*$, which implies (9). (9) in turn implies stability for any initial condition $\rho(0) \in \mathbb{R}^{\mathcal{E}_v^-}$ because $\rho_i(t) \leq \rho_i(0) + C_i|\tilde{\rho}_i(t) - \tilde{\rho}_i^*| \leq \rho_i(0) + C_i\|\tilde{\rho}(t) - \tilde{\rho}^*\|_1 \leq \rho_i(0) + C_i\|\tilde{\rho}(0) - \tilde{\rho}^*\|_1 < +\infty$. Finally, since $\frac{d}{dt}\|\tilde{\rho}(t) - \tilde{\rho}^*\|_1 < 0$ whenever $\tilde{\rho}(t) \neq \tilde{\rho}^*$, one can apply LaSalle's theorem to obtain $\lim_{t\to\infty} \tilde{\rho}(t) = \tilde{\rho}^*$, and thus $\lim_{t\to\infty} \rho(t) = \rho^*$. Hence, ρ^* is a globally asymptotically stable equilibrium.

V. Simulations

In this section, we present simulations to illustrate the theoretical results of this paper, and to compare the stability conditions under static and a dynamic route choice behavior. We consider the simple local system depicted in Figure 3. Notice that $e(1) = e(2)$ and that $e(3) = e(4)$. In the first set of simulations, the capacity of lanes are given by $C_1 = 1.5$, $C_2 = C_4 = 3$ and $C_3 = 2$. The components of the static route choice vectors are $G_1^{u_1} = 0.4$, $G_2^{u_1} = 0.6$, $G_3^{u_2} = G_4^{u_2} = 0.5$, the external inflow to the network is given by $\lambda_{e(1)} = \lambda_{e(3)} = 1$, and the departure rates are $\theta_{e(1)} = \theta_{e(2)} = 0$. One can check that the corresponding induced flows are $f_1 = 0.4$, $f_2 = 0.6$, $f_3 = f_4 = 0.5$, and that these flows are feasible. We use the monotonic distributed green light policy from Example 1 with $\kappa = 0.1$.

Figure 4 shows the time-evolution of occupancies on the lanes starting from initial condition $\rho(0) = \mathbf{0}$ and from initial condition $\rho_i(0) = 0.5$, for all $i = 1, \ldots, 4$. The necessary

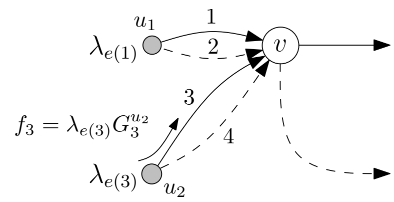

Fig. 3. The simple network used for simulations.

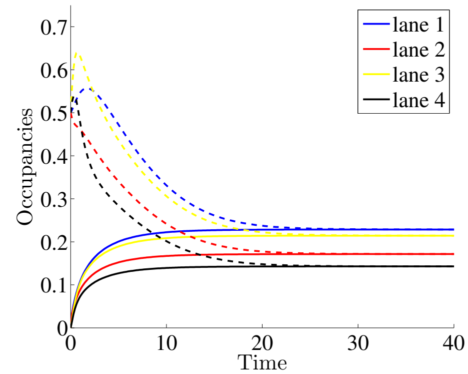

Fig. 4. Time evolution of lane occupancies with different initial conditions under static route choice. The solid lines correspond to $\rho(0) = \mathbf{0}$, whereas the dashed lines correspond to $\rho_i(0) = 0.5$, for all $i = 1, \ldots, 4$.

and sufficient condition for stability, as given in Theorem 1, is satisfied, since

$$\frac{f_1}{C_1} + \frac{f_2}{C_2} + \frac{f_3}{C_3} + \frac{f_4}{C_4} = 0.8833 < 1\,,$$

and indeed the system admits a unique globally asymptotically stable equilibrium.

A second set of simulations is performed with the the lane capacities $C_1 = 0.5$, $C_2 = C_4 = 4$, $C_3 = 5$, and the external arrival rates λ and departure ratios θ same as the first set of simulations. We compare the time-evolution of lane occupancies starting with initial condition $\rho(0) = \mathbf{0}$ under static and dynamic route choice, as shown in Figure 5. The static route choice G is the same as in the first set of

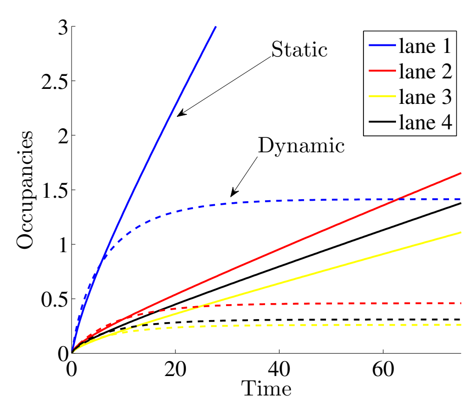

Fig. 5. Comparison of time evolution of lane occupancies with initial condition $\rho(0) = \mathbf{0}$ under static and dynamic route choice behavior. The solid lines correspond to static route choice, and the dashed lines correspond to dynamic route choice. This example suggests that the system has better stability properties under dynamic route choice.

simulations, whereas the dynamic route choice is given by

$$G_1^{u_1}(\rho_1,\rho_2) = \frac{e^{-\rho_1}}{e^{-\rho_1}+e^{-\rho_2}} = 1 - G_2^{u_1}(\rho_1,\rho_2),$$

$$G_3^{u_2}(\rho_3,\rho_4) = \frac{e^{-\rho_3}}{e^{-\rho_3}+e^{-\rho_4}} = 1 - G_4^{u_2}(\rho_3,\rho_4).$$

The choice of this dynamic route choice function is inspired by our previous work [15], where we analyzed the stability of dynamical transportation networks under such dynamic route choice behavior. In this case, since $\frac{f_1}{C_1}+\frac{f_2}{C_2}+\frac{f_3}{C_3}+\frac{f_4}{C_4} = 1.175 > 1$, Theorem 1 implies that the network is unstable under static route choice. This also corroborates with Figure 5, where the lane occupancies are growing unbounded. On the other hand, as Figure 5 shows, the lane occupancies remain bounded and in fact converge to an equilibrium under dynamic route choice. This suggests that, for these problem parameters, the stability condition for dynamic route choice is relaxed in comparison to the one for static route choice.

A third set of simulations shows that the observations from the second set simulations may not hold in general. We set the flow capacities to be $C_1 = C_3 = 0.5$, $C_2 = 2.1$, and $C_4 = 2.5$, route choice vectors be given by $G_1^{u_1} = G_3^{u_2} = 0$ and $G_2^{u_1} = G_4^{u_2} = 1$, and the λ and θ same as in the first set of simulations. The induced flows under the static route choice are $f_1 = f_3 = 0$ and $f_2 = f_4 = 1$. Therefore, $\frac{f_1}{C_1}+\frac{f_2}{C_2}+\frac{f_3}{C_3}+\frac{f_4}{C_4} = 0.8762 < 1$, and hence Theorem 1 implies that the system is stable. This is also corroborated by Figure 6. However, as shown in Figure 6, the lane occupancies grow unbounded under the dynamic route choice functions used in the second set of simulations.

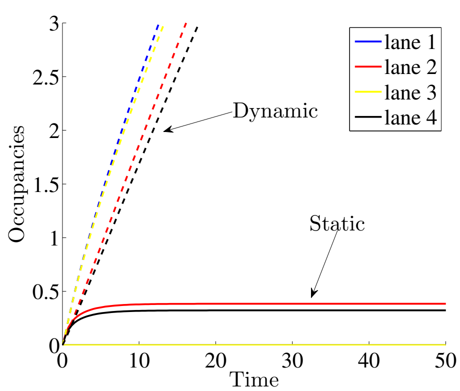

Fig. 6. Comparison of time evolution of lane occupancies with initial condition $\rho(0) = \mathbf{0}$ under static and dynamic route choice behavior. The solid lines correspond to static route choice, and the dashed lines correspond to dynamic route choice. This example suggests that the system has better stability properties under static route choice.

Finally, we report simulation results to support our conjecture that Theorem 1 extends to cyclic network topologies. We considering the cyclic network shown in Figure 7. The flow capacities are $C_i = 1$ for all $i = 1,\dots,8$, external arrival rates are $\lambda_{e(1)} = 0.4$ and $\lambda_{e(3)} = 0.2$, and the departure ratios are $\theta_e = 1$ for $e \in \{d_1, d_2\}$, and $\theta_e = 0$ otherwise, i.e., d_1 and d_2 are destination links. The static route choice vectors are such that $G_i^{\sigma_i} = 0.5$ for all $i = 1,\dots,8$.

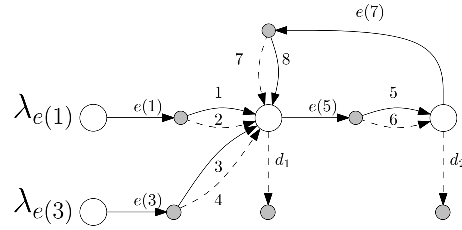

Fig. 7. The cyclic network used for last set of simulations.

We run two simulations: one with initial condition $\rho(0) = \mathbf{0}$ and the other with $\rho_i(0) = 0.5$ for all $i = 1,\dots,8$. Figure 4 shows the time-evolution of occupancies on the lanes, and one can see that the trajectories remain bounded and seem to converge to a unique globally asymptotically stable equilibrium. Even though we have no formal proof, we conjecture that, also for cyclic networks, a sufficient condition for the existence of a globally asymptotically stable equilibrium is monotonicity of the green light policy and

$$\sum_{i\in\mathcal{E}_v^-} \frac{f_i(G,\lambda,\theta)}{C_i} < 1, \quad \forall v \in \mathcal{N}$$

where $f(G,\lambda,\theta)$ is the (unique, under connectedness assumptions) vector of flows induced in the network by G, λ and θ. Indeed, for the present case, direct computation gives the induced flows to be $f_1 = f_2 = 0.2$, $f_3 = f_4 = 0.1$, $f_5 = f_6 = 0.2$ and $f_7 = f_8 = 0.1$. Therefore,

$$\frac{f_1}{C_1}+\frac{f_2}{C_2}+\frac{f_3}{C_3}+\frac{f_4}{C_4}+\frac{f_7}{C_7}+\frac{f_8}{C_8} = 0.8 < 1$$

and

$$\frac{f_5}{C_5}+\frac{f_6}{C_6} = 0.4 < 1\,.$$

Conversely, and not reported here for brevity, one can pick the parameters of the network in such a way that the previous conditions are *not* satisfied and observe that the resulting system is unstable. This numerical study supports our conjecture that Theorem 1 extends to cyclic network topologies too.

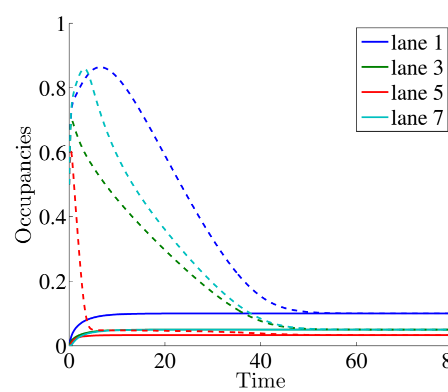

Fig. 8. Time evolution of lane occupancies with different initial conditions under static route choice. The solid lines correspond to $\rho(0) = \mathbf{0}$, whereas the dashed lines correspond to $\rho_i(0) = 0.5$ for all $i = 1,\dots,8$.

VI. Conclusions

In this paper, we analyzed stability of traffic networks under signal control policies that, at every intersection, give green light to at most one incoming lane at any time. We show that distributed green light policies that exhibits monotonicity in the green light durations with respect to the occupancy levels is maximally stabilizing for acyclic network topologies, and admits a globally asymptotically stable equilibrium. The attractive feature of the policies in comparison to the ones existing in literature is their minimalism in the usage of information, and hence being conducive to being adaptive in dynamic and uncertain environments.

There are several interesting directions for future research. We plan to formally establish Theorem 1 for cyclic network topologies. We also plan to extend our formalism and stability results to bounded occupancy capacities on the lanes, and to dynamic route choice behaviors. These extensions will likely require green light policies at intersections that use information about occupancies on incoming as well as outgoing lanes at that intersection, similar to the routing policies developed in [9]. We also plan to consider general traffic signal control architectures that allow activation of multiple (non-conflicting) phases. It is natural to expect that the stability regime under such a general architecture would be more than the one obtained under the setup of this paper. Therefore, an another interesting line of research is to optimize over phase combinations that maximize stability regime. Finally, we plan to consider other metrics of performance such as delay and resilience to disruptions, possibly using tools from [16], [17].

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