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A theory of haste *

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Abstract

We consider a hyperbolic discounting agent. At each period, he can undertake an irreversible consumption decision that yields an uncertain current benefit and a delayed cost. If he decides to defer it for the future, some information exogenously flows in. We show that the agent may rationally decide to consume with negative expected net present value (NPV), only to prevent himself from consuming in the future which could be profitable from a future perspective but highly detrimental from the current viewpoint. Comparative statics reveal that the value of information is U-shaped.

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1 Introduction

Irreversible choices are quite common in Economics. As the consequences of such decisions have usually some uncertainty component, looking for additional information before undertaking an action can be a sensible strategy. This simple observation is at the heart of the well-known "investment under uncertainty" literature, pioneered by Henry (1974) and furthered developed by Dixit and Pindyck (1994) among others. The literature shows that the ability to delay an irreversible investment and obtain some information in the meantime affects the decision to invest. Basically, if an individual embarks on a project in the current period, he gives up the possibility of acquiring new information about its profitability, i.e. he looses his *information value of waiting*. As a consequence, a project will be undertaken if and only if its expected benefit exceeds its cost by an amount at least equal to the value of keeping the option of deciding at a future date. As demonstrated in these papers, the information value of waiting is always positive and increases with the number of periods in which it can be exerted.

Starting from this observation, the goal of the paper is to explore whether there exist situations in which the information value of waiting can be negative. More precisely, our objective is (i) to present a rationale for *haste*, defined as the decision of an individual to embark on an irreversible activity anticipating a net expected loss, and (ii) to provide a systematic analysis of the type of stochastic environments in which this behavior is likely to occur.

Our theory relies on two building blocks. The first one, irreversible investment under uncertainty and exogenous flow of information, has already been introduced. Importantly, we assume that the irreversible action yields an uncertain current benefit and a delayed cost. This contrasts with the standard literature, where the timing of costs and benefits is irrelevant and only net expected payoffs matter.[1] The importance of this specific order will become clear when we present our results.

The second building block is a hyperbolic discounting of the flow of returns. Clearly, this is a non-standard ingredient in utility theory. It states that the decision maker has a taste for immediate gratification or, in other words, that he discounts short term events relatively more heavily than long term events. This type of non-exponential discounting is often accepted in the Psychology literature as an accurate way of capturing the intertemporal rate of substitution of individuals. Given the substantial amount of

[1]From now on, we will not refer to an 'investment' but rather to a 'consumption' decision because the former term is often associated to corporate investment choices where costs come earlier than benefits while, in our setting, it is crucial that benefits come earlier than costs.

experimental evidence (see e.g. the survey by Frederick et al. (2002)), it is also becoming increasingly accepted in Economics.[2] From a theoretical perspective, Strotz (1956) is the first study which accounts for the tendency of individuals to satiate instant gratification. Phelps and Pollak (1968) analyze the intertemporal coherence of decisions taken by governments in a model where 'implicit' social preferences are dynamically inconsistent. More recently, Laibson (1996) and Barro (1999) have analyzed standard models of consumption and growth under hyperbolic discounting. In both works, decisions of households are, in the absence of commitment devices, observationally equivalent to those obtained under exponential discounting. Hence, time-inconsistency induces inefficiencies but discrimination between hyperbolic and exponential discounting is possible only if agents possess some commitment technology.

The paper combines the two ingredients previously mentioned to provide a model of irreversible consumption under uncertainty and hyperbolic discounting. It shows that the consumption decision of a hyperbolic discounting agent will or will not be observationally equivalent to that of his exponential discounting peer depending exclusively on the magnitude of the (exogenous) flow of information transmitted between periods. More precisely, if the amount of uncertainty resolution is "sufficiently high", the information value of waiting will be positive and decreasing over time (i.e. as the horizon where it can be exerted diminishes), like in the standard literature. By contrast, if the amount of uncertainty resolution is "positive but sufficiently small", the information value of waiting will be *negative and increasing over time*.

The intuition is as follows. Under hyperbolic discounting and given that consumption has immediate benefits and delayed costs, there is what we call an "inconsistency region". For beliefs in that region, the project has an expected positive Net Present Value (NPV) from the current perspective and, at the same time, an expected negative NPV from a past perspective. This means that consumption is profitable for the "incarnation" of the agent who undertakes it but implies net expected losses for a previous incarnation of that same agent. As a result, the decision of the agent whether to consume today must be contingent not on what he would optimally do at future dates in case of postponing it, but rather on the anticipation of what future incarnations plan to do. Naturally, the behavior of those future incarnations will be a function of the signals they receive. Therefore, the amount of information transmission between two periods determines the incentives of the current incarnation to exert his information value of waiting.

[2]However, it is still generating a fair amount of controversy, as witnessed by the criticisms of Rubinstein (2000) and Read (2001) among others.

The paper shows that the value of information is U-shaped. First, under no information transmission, the agent consumes if and only if the expected NPV is positive. Second, under 'substantial' information transmission, the expected profitability of the project my vary strongly from one period to the next. The probability of avoiding the inconsistency region in the next period conditional on not consuming in the current one is high relative to the probability of falling in that region. As a result, waiting for new information is a relatively safe strategy and, once and again, consumption only occurs under positive (and large) NPV. Last, under 'coarse' information transmission, the chances that extra evidence plunges the agent into the inconsistency region are relatively important. Then, an incarnation with a project of expected NPV negative but close to zero may find it optimal to undertake the project. Hasty consumption takes place not because of its good prospects but only as a commitment device against future consumption with positive NPV from a future perspective but large negative NPV from a current one.

The practical implications of this result are immediate. The overuse of credit facilities by many consumers in the US and the insufficient protection of endangered species and the tropical forest may often be imputed to the irrationality of individuals and governments. Although it is certainly possible to find a number of plausible reasons for each particular behavior, our paper provides a unified theory that may help explain why and when rational decision-makers will undertake actions anticipating net losses. Naturally, its empirical relevance will depend on the specific issue at stake (a brief review of some potential applications is provided in the concluding section).

Before presenting the formal analysis, we would like to mention some papers in the behavioral economics literature that are related to ours. O'Donoghue and Rabin (1999) is the first paper to show that a (sophisticated) hyperbolic discounting agent may undertake actions with negative NPV. The crucial difference is that, in their work, costs and benefits vary deterministically over time. Our stochastic set up with dynamic resolution of uncertainty allows us to characterize the value of information, that is the evolution of the agent's expected payoff as a function of the accuracy of news. Its U-shaped form is, in our view, both interesting and surprising. Carrillo and Mariotti (2000) was the first paper to point out that a hyperbolic discounting agent may optimally avoid free information (see also Brocas and Carrillo (1999) and Benabou and Tirole (2002)).[3] Caplin and Leahy (2000) argue that restricting the access to information can be desirable if a standard exponential discounting agent derives

[3]Note however that in Carrillo and Mariotti (2000) learning and consumption are possible at every period. Therefore, unlike in our setting, current consumption cannot be used as a commitment device.

some utility from anticipatory feelings about uncertain future events. None of these papers characterizes the value of information as a function of its amount transmitted per-period.

The paper is organized as follows. We first describe a model of consumption under uncertainty in a finite but arbitrarily large horizon and a binary signal structure. Section 2 restates in this particular setting the standard results under exponential discounting, while section 3 characterizes the optimal decision under hyperbolic discounting. We then study in section 4 the same problem in a (simpler) two-period model but with a richer structure of signals. This enables us to provide an in-depth analysis of the relationship between the flow of information transmission and the incentives to take hasty actions. Finally, in section 5 we provide a brief informal discussion of possible applications and some concluding remarks. Proofs are relegated to the Appendix.

2 Benchmark model: consumption under uncertainty and exponential discounting

We consider the decision of an agent to undertake an activity which yields an uncertain intertemporal payoff. If, at some date, the agent decides not to undertake the activity (hereafter "waits"), he receives a signal about the net value of the activity and faces the possibility to undertake it (hereafter "consume") in the following period. The consumption decision is irreversible in the sense that once the agent decides to consume, uncertainty is resolved, payoffs are realized, and the game ends (considering partial irreversibility would not change the analysis). We are therefore in the standard case of irreversible consumption under uncertainty in the tradition of Dixit and Pindyck (1994). Unlike in the learning by doing literature, information flows exogenously in our model. Moreover, given the complete irreversibility of consumption, the timing of the uncertainty resolution after consumption is irrelevant. We also make the following important assumption.

Assumption 1 *Benefits of consumption are immediate while costs are delayed.*

This assumption contrasts with previous works in the field, where the expected NPV of costs and benefits are relevant but not the order of their realization. For analytical simplicity, we will assume that benefits are realized the period of consumption while costs are delayed one period. However, what matters for our theory is that benefits are concentrated more in the short term than costs.

2.1 Preliminaries

We analyze the simplest case of consumption under uncertainty.

• States: There are two possible states of the world $s \in \{H, L\}$. The benefit of consumption is G (> 0) if the true state is $s = H$ and 0 if the true state is $s = L$. There is uncertainty on the state. At the beginning of the game, nature chooses state H with probability $P(H)$ and state L with probability $P(L) = 1 - P(H)$. These prior probabilities are common knowledge, but the realization of the choice by nature is not.

• Consumption: We consider an irreversible consumption decision possible during a finite (but possibly arbitrarily large) number of periods. Formally, the agent can consume in period $t \in \{0, 1, ..., T\}$. As long as he does not consume, his payoff is normalized to zero. If the agent has not consumed before T and decides not to consume at T, his payoff is also zero.[4]

• Information: If the agent decides not to consume in period t ($< T$), he receives a signal $\sigma_t \in \{h, l\}$ about the true state of the world that can be used to update his beliefs. Naturally, this signal influences his consumption decision at $t + 1$. Signals h and l are imperfectly correlated with states H and L. More precisely,

$$\Pr(h \mid H) = \Pr(l \mid L) = \theta \;\; (> 1/2) \quad \text{and} \quad \Pr(h \mid L) = \Pr(l \mid H) = 1 - \theta.$$

Denote by n_h and n_l the number of signals h and l received respectively. From standard probability theory, it is easy to check that if draws of σ_t are independent then:

$$\Pr(H \mid n_h, n_l) = \frac{\theta^{n_h - n_l} P(H)}{\theta^{n_h - n_l} P(H) + (1-\theta)^{n_h - n_l} P(L)}$$

Note that two different signals cancel each other. Therefore, the relevant variable that will be used from now on is n ($= n_h - n_l$), that is the difference between the number of h signals and the number of l signals. As the agent receives one signal per period, in the T-horizon model $n \in \{-T, \dots, T\} \subset \mathbb{Z}$. We can define the following function:[5]

$$p(n) \equiv \Pr(H \mid n_h, n_l) = \frac{\theta^n P(H)}{\theta^n P(H) + (1-\theta)^n P(L)} \tag{1}$$

[4]Finite horizon allows us to focus on a model with a unique equilibrium (even under hyperbolic discounting) that can be computed by backward induction. Normalizing to zero the payoff under no consumption and the benefit of consumption when $s = L$ is also taken without loss of generality.

[5]One can immediately check the following properties of this function: (i) $p(n+1) > p(n)$; (ii) $p(0) = P(H)$; (iii) $\lim_{T \to +\infty} p(-T) = 0$; and (iv) $\lim_{T \to +\infty} p(T) = 1$.

• Benefits: Given the previous information structure, the expected benefit of consumption for a difference of signals n, is:

$$\pi(n) = p(n)\, G$$

Denote by $\gamma(n_t)$ the probability that, if at some period t the difference of signals is n_t, then at $t+1$ the difference is $n_t + 1$. Formally, $\gamma(n_t) \equiv \Pr(n_{t+1} = n_t + 1)$. Given the independence of signals, we can suppress time subscripts. We have:

$$\begin{aligned} \gamma(n) &= \Pr(h \,|\, H)p(n) + \Pr(h \,|\, L)(1 - p(n)) \\ &= (1-\theta) + (2\theta - 1)p(n) \end{aligned}$$

Note that $\gamma(n+1) > \gamma(n)$. Due to our binary signal structure, $1 - \gamma(n_t) \equiv \Pr(n_{t+1} = n_t - 1)$. Also, for any n, $p(n) = \gamma(n)\, p(n+1) + (1 - \gamma(n))\, p(n-1)$.

• Costs: For simplicity, we assume that if the agent consumes at date t, he incurs a deterministic cost at $t+1$ equal to C/δ, where δ is the exponential discount factor.[6]

• Net Present Value of Consumption: Given the costs, benefits and uncertainty described above, the expected net profit of consuming in the current period given a difference of signals n is simply:

$$\pi(n) - C \tag{2}$$

In order to focus on the interesting case, we assume the existence of an interior solution. Formally, consuming in the last period is profitable if the agent receives h-signals in all periods ($n_T = T$), and not profitable if he receives l-signals in all periods ($n_T = -T$).

Assumption 2 $\pi(-T) < C < \pi(T)$.[7]

Given this assumption and the fact that $\pi(n)$ is increasing in n, there exists one and only one value $\tilde{n} \in [-T, T]$ defined by:

$$\pi(\tilde{n}) = C \tag{3}$$

[6]All the results of the paper hold if the uncertainty is on the cost of consumption (rather than on the benefit) or on both the cost and the benefit. Obviously, our model remains formally equivalent if we assume that consumption at t entails a benefit g_i during all the periods $t + i \geq t$ if $s = H$ and a cost c_i during all the periods $t + i > t$, where $G \equiv g_0\ (> 0)$ and $C \equiv \sum_{i=1}^{+\infty} \delta^i\, (c_i - g_i)\ (> 0)$.

[7]Using (1), the assumption can be rewritten in terms of the primitives of the model as:

$$C\left[1 + \left(\frac{1-\theta}{\theta}\right)^T \frac{P(L)}{P(H)}\right] < G < C\left[1 + \left(\frac{\theta}{1-\theta}\right)^T \frac{P(L)}{P(H)}\right].$$

As $T \to +\infty$, the assumption simply becomes $C < G$.

such that the current payoff of consuming is nil. For all n greater (resp. smaller or equal) than the integer part of $\tilde{n}$, consuming yields expected benefits (resp. losses).[8] By abuse of notation and in order to avoid working with integer parts, we will from now treat n and $\tilde{n}$ as real numbers.

2.2 Consumption and the information value of waiting

We can now restate the well-known results of consumption under uncertainty in our particular framework with discrete time, finite horizon and a binary signal structure.

Lemma 1 **[Henry-Dixit-Pindyck]** *For all t, there exists one and only one cutoff value $\tilde{n}_t$ above which the agent consumes at date t. Furthermore, $\tilde{n}_T = \tilde{n}$ and $\tilde{n}_t > \tilde{n}_{t+1}$.*

Proof. See Appendix A1. □

This result is standard. At date T, the agent knows that the current period is his last chance for consuming so, given (2), consumption occurs if and only if $n_T \geq \tilde{n}$. At $T-1$, the agent can delay his consumption decision for at most one period. This may be valuable because in the meantime he will receive a signal σ_{T-1} about its profitability. Naturally, waiting is not always desirable because the future is discounted at a rate δ. In any case, at $T-1$ the agent has an information value of waiting one more period. Consumption will then take place only if its expected benefit exceeds its cost by an amount at least equal to the value of this information. Formally, the cutoff above which it is optimal to consume at $T-1$ is $\tilde{n}_{T-1} > \tilde{n}$. Last, for any given difference of signals, the information value of waiting is higher at date t than at date $t+1$ because it can be exerted during one more period so, for all $t \in \{0, \dots, T-1\}$, $\tilde{n}_t > \tilde{n}_{t+1}$.

3 Consumption under hyperbolic discounting

The analysis becomes more surprising when we consider a hyperbolic discounting agent.[9] Formally, this is to say that the discount rate between two consecutive periods t and $t+1$ increases as date t approaches. As stated in the introduction, this characteristic in the preferences of individuals has received support in Psychology and Economics. In recent years, several works have applied these types of preferences to a

[8] Using (1), $\tilde{n}$ can be rewritten in terms of the primitives of the model as: $\tilde{n} = \dfrac{\log\left(\frac{C}{G-C}\frac{P(L)}{P(H)}\right)}{\log(\theta) - \log(1-\theta)}$.

[9] For the rest of the paper we will assume that the agent overweighs current payoffs.

wide set of problems.[10] Besides, we also assume that the individual cannot commit at any time to his future behavior.

For analytical tractability we will use the elegant quasi-hyperbolic discount function introduced by Phelps and Pollak (1968). In their work on imperfect intergenerational altruism, period $t + s$ ($s \geq 1$) is, from the perspective of the agent at date t (also called "self-t"), discounted at a rate $\beta\delta^s$ with $0 < \beta < 1$.[11] Given the structure of payoffs described in section 2.1 and the intrapersonal conflict of interests, the net expected value of consuming depends on the date of reference. More precisely, from the perspective of self-t, the net payoff of consumption at t is:

$$\pi(n) - \beta C \tag{4}$$

while the net payoff of consumption at $t + s$ ($s \geq 1$) is:

$$\beta\delta^s \left[\pi(n) - C\right] \tag{5}$$

Note the existence of an *inconsistency region* $(\beta C, C)$. If $\pi(n) \in (\beta C, C)$ then consuming yields expected net profits from the perspective of the current self (equation (4)) but expected net losses from the perspective of past selves (equation (5)). This crucial difference between time-consistent and time-inconsistent preferences is the key of our analysis. We now introduce the analogue of Assumption 2. We require that if the agent receives only h-signals then consumption is profitable from the perspective of both past and present selves, and if he receives only l-signals then consumption is unprofitable from the perspective of both past and present selves.

Assumption 3 $\pi(-T) < \beta C < C < \pi(T)$.

Given this assumption, there exists one and only one value n^* defined by:

$$\pi(n^*) = \beta C \tag{6}$$

[10]Some of these issues (the list is clearly not exhaustive) are: procrastination (Akerlof (1991), O'Donoghue and Rabin (1999)), private side-bets and personal rules (Caillaud et al. (1996) and Benabou and Tirole (2001)), consumption (Laibson (1996), Harris and Laibson (2001)), portfolio choice (Palacios-Huerta, 2001), learning (Carrillo and Mariotti (2000), Brocas and Carrillo (1999)), and memory management (Benabou and Tirole (2002)). See also Loewenstein and Prelec (1992) for an accurate modelling of hyperbolic discounting, Caillaud and Jullien (2000) for other possible ways to model time-inconsistent preferences and Frederick et al. (2002) for a survey on time discounting.

[11]Note that $\beta = 1$ is the "standard" case of exponential discounting. As β decreases, the taste for immediate gratification becomes more acute; the agent is less able to internalize the effects of current decisions on future welfare, which increases his intrapersonal conflict of interests.

such that the current net payoff of consumption by a hyperbolic discounting individual is nil. For all n greater (resp. equal or smaller) than the integer part of n^*, consumption yields expected net benefits (resp. net losses) from the current perspective.[12] It is crucial to notice that a consumption yielding zero net utility from the current perspective necessarily implies net expected losses from the perspective of past selves. Formally, $\beta\delta^s[\pi(n^*) - C] = \beta\delta^s(\beta - 1)C < 0$. Now, consider the following function:

$$\mathcal{J}(\theta, \beta, G, C) \equiv \gamma(n^*)\, \beta\delta[\pi(n^* + 1) - C] \tag{7}$$

The function $\mathcal{J}(\cdot)$ represents self-$T-1$'s benefits of waiting when current consumption yields zero profit. To see this, suppose that $n_{T-1} = n^*$. By (6), we know that consumption implies zero current expected profits. If self-$T-1$ decides instead to wait, consumption will take place at date T (the last possible period) if and only if $\sigma_{T-1} = h$. This occurs with probability $\gamma(n^*)$, it implies a difference of signals $n^* + 1$, and therefore an anticipated NPV from self-$T-1$'s perspective equal to $\beta\delta[\pi(n^*+1) - C]$. From (1) and the definition of n^* in (6), it can be easily shown that:

$$\mathcal{J}(\theta, \beta, G, C) \;\propto\; \frac{\beta G - \beta C}{G - \beta C} - \frac{1-\theta}{\theta} \tag{8}$$

where $\propto$ stands for "proportional to". We are now in a position to state our first result.

Proposition 1 *Under hyperbolic discounting, there exists for all t one and only one cutoff n_t^* ($< \tilde{n}_t$) above which the agent consumes at t. Furthermore, $n_T^* = n^*$ and:*

(i) $n_t^* > n_{t+1}^*$ if $\mathcal{J}(\theta, \beta, G, C) > 0$;

(ii) $n_t^* \leqslant n_{t+1}^*$ if $\mathcal{J}(\theta, \beta, G, C) < 0$.

Proof. See Appendix A2. □

By definition (see equation (7) and its interpretation), if $\mathcal{J}(\cdot) > 0$ then the agent at date $T-1$ with a difference of signals $n_{T-1} = n^*$ has incentives to wait one more period. Therefore, we necessarily have that the cutoff for which self-$T-1$ is indifferent between consuming and not is $n_{T-1}^* > n_T^* = n^*$. Note that waiting is desirable for both selves when $n_{T-1} = n^*$, although it is especially attractive for self-T (consumption would imply net losses). To sum up, the case $\mathcal{J}(\cdot) > 0$ is similar to the time-consistent situation described in Lemma 1: the information value of waiting is always positive

[12] As in footnote 8 and using (1), n^* can be rewritten as: $n^* = \dfrac{\log\left(\frac{\beta C}{G-\beta C}\frac{P(L)}{P(H)}\right)}{\log(\theta) - \log(1-\theta)}$. Again by abuse of notation, we will treat n^* as a real number.

and greater at date t than at date $t+1$ because it can be exerted during one more period. Formally, cutoffs decrease as the consumption horizon shrinks ($n_t^* > n_{t+1}^*$).

We want to emphasize that the intrapersonal conflict of preferences causes an inefficiency. It is easy to show that each self would prefer to commit to a different future behavior. However, just like in Laibson (1996) or Barro (1999), the no-commitment behavior when $\mathcal{J}(\cdot) > 0$ is qualitatively similar to that of a time-consistent individual. Since the information value of waiting is positive and decreasing in the consumption horizon (see Figure 1), an agent who refrains from consuming at some date may decide to consume in a later period with the same expected benefit.

[INSERT FIGURE 1 HERE]

A more surprising situation arises when $\mathcal{J}(\cdot) < 0$. By (7) and given the inconsistency region $(\beta C, C)$, if $n_{T-1} = n^*$ then self-$T-1$ strictly prefers to consume in the current period and get a zero expected profit rather than wait and obtain some net expected losses. By continuity, the cutoff above which it is optimal to consume at $T-1$ is $n_{T-1}^* < n_T^* = n^*$. This implies that for all $n_{T-1} \in (n_{T-1}^*, n_T^*)$ self-$T-1$ consumes with negative NPV from his own perspective. Put it differently, in this region the agent has an expected negative information value of waiting. Consumption occurs not because of its intrinsic value but rather as a *commitment device* against an even more inefficient future consumption decision. Besides, the information value of waiting at date t is negative and smaller than at date $t+1$, precisely because there is one more period in which a future action undesirable from the current perspective can be undertaken. Formally, $n_t^* \leq n_{t+1}^*$ for all t, so that the likelihood of consumption with negative NPV decreases as date T approaches. This implies that a longer consumption horizon translates into a higher likelihood of a future inefficient action, and therefore into a higher willingness to accept a current consumption with negative NPV. Summing up, the agent says to himself: "it would be best if I did not consume now and then consumed optimally in the future; but since I am going to be too prone to consume in the future if I get any small encouragement in the meantime (and this is likely to occur), I might as well consume now so as to, at least, enjoy the benefits right away."

It is important to realize that the agent's knowledge of his intrapersonal conflict (and therefore of his future incentive to act inefficiently from the current perspective) is crucial for our result. His behavior is rational given his time-varying preferences. Naturally, in the absence of learning, an agent would never consume with an expected negative NPV. Therefore, in our setup it is precisely the unavoidability of information that "forces" the current incarnation to take non-desirable decisions. Last, but not least, this case illustrates the idea that hyperbolic discounting may change qualitatively the standard results in the literature. As depicted in Figure 2 and contrary to the previous and the exponential cases, when $\mathcal{J}(\cdot) < 0$ an agent who refrains from consuming at some date will never choose to consume in a later period given the same expected benefit.

[INSERT FIGURE 2 HERE]

A corollary which deserves special attention follows directly from Proposition 1.

Corollary 1 (i) *If $\mathcal{J}(\cdot) < 0$, there exists a non-empty set of expected profits $\pi(n)$ such that the agent consumes even though it is a strictly dominated strategy from the perspective of all incarnations.*

(ii) $\frac{\partial}{\partial \beta}\mathcal{J}(\cdot) > 0$, $\left[\frac{\partial}{\partial G}\mathcal{J}(\cdot) + \beta \frac{\partial}{\partial C}\mathcal{J}(\cdot)\right] < 0$, *and* $\frac{\partial}{\partial \theta}\mathcal{J}(\cdot) > 0$.

Part (i) is a direct consequence of Proposition 1(ii). For values of $\pi(n)$ below βC and above the indifference path in Figure 2, consumption has negative NPV from the viewpoint of all selves (including the current one) and therefore it is dominated by a strategy of never consuming. Yet, in the absence of commitment devices, consumption with expected losses for all selves cannot be avoided.

Recall that hasty actions can occur if $\mathcal{J}$ is negative and cannot if $\mathcal{J}$ is positive. Using (8), we can perform simple comparative statics. First, $\partial \mathcal{J}/\partial \beta > 0$: the stronger the taste for immediate gratification, the higher the agent's willingness to use current consumption as a commitment device, even if it implies a negative payoff.[13] Second, keeping constant the net current profit when the good state is realized $(G - \beta C)$, hasty decisions are more likely to occur when the stakes (or size of the projects) are important. Last, and most importantly, $\partial \mathcal{J}/\partial \theta > 0$. As the correlation θ between state and signal increases, the informational content of an observation also increases. Thus, agents are more prone to haste and consume with negative NPV when the flow of information transmitted between two periods is positive but small. Formally, for all (β, G, C), there exists a value $\theta^* \equiv \frac{G-\beta C}{(1+\beta)G - 2\beta C}$ such that $\mathcal{J}(\theta; \beta, G, C) \gtreqless 0$ if $\theta \gtreqless \theta^*$. Given Assumption 3, $\theta^* \in (1/2, 1)$. Hence, the information value of waiting at any date t is negative for all $\theta \in (1/2, \theta^*)$, positive for all $\theta \in (\theta^*, 1)$, and nil for $\theta \in \{1/2, \theta^*\}$. Figure 3 illustrates the relationship between the inter-period flow of information and the incentives to undertake hasty actions. As we think that this effect is key, we leave for the next section a more comprehensive analysis of this relationship.

[13] It is easy to check that $\mathcal{J}(\theta, 1, G, C) > 0$ for all θ, G and C.

[INSERT FIGURE 3 HERE]

Remarks. Although we have assumed a specific learning process, all we need for our theory is a non-deterministic change in the payoff of consumption. That is, our results would still hold under any stochastic fluctuation of benefits (e.g., a random walk). Also, note that the sign of $\mathcal{J}$ (and therefore the likelihood of a hasty action) is independent of the consumption horizon T.

We have highlighted the possibility of behaving suboptimally from the perspective of all selves due to hyperbolic discounting. Our next goal is to provide some prescriptions about a simple way to avoid such inefficient behavior.

Proposition 2 *When $\mathcal{J} < 0$, it might be optimal for the agent to spend resources that increase the value of not consuming even if the direct costs offset the direct benefits.*

Proof. See Appendix A3. □

It is well-known from the literature on hyperbolic discounting that partial commitment on future behavior reduces the problem of self-control and therefore mitigates the inefficiencies due to the dynamic inconsistency of preferences. In Proposition 2 we offer a different commitment mechanism. If the agent cannot affect the decision of future selves, he can at least distort the costs and benefits of actions. One extreme possibility would be to deter future consumption by decreasing its benefit. Still, there is a much better commitment device which consists of increasing the payoff of *not undertaking* the activity (which in our model is normalized to 0). Formally, it has the same effect of reducing the incentives to engage in the activity in the future. However, it has the advantage that the resources are not wasted. The interesting issue is that this strategy can be optimal even if it requires an investment that offsets the direct benefits, just because it avoids future consumption with negative NPV.

4 Flow of information as a determinant for haste

In this section, we investigate in more detail the relationship outlined in Corollary 1 between amount of information transmission and likelihood of haste. To this purpose, we consider a richer structure of signals than previously. At the same time, we restrict the attention to a consumption horizon $T = 2$. Admittedly, this a limitation, although not excessively critical since we just showed in section 3 that the sign of $\mathcal{J}$ was independent of the horizon T.

4.1 Preliminaries

The structure of payments is the same as in section 3: π denotes the (instantaneous) benefit of current consumption and βC the (delayed) cost. As before, if consumption takes place one period later, then $\beta\delta\,\pi$ is the benefit and $\beta\delta\,C$ the cost from the current self's perspective. The information structure is modified in the following way. The benefit π of consuming in period 1 is a random variable drawn from a Normal distribution with known mean m and precision (i.e. inverse of variance) h.

$$\pi \;\sim\; \mathcal{N}\,(m, 1/h) \tag{9}$$

If the agent decides to wait in period 1, he receives s exogenous and independent signals $\{x_i\}_{i=1}^{s}$ about the benefit of consumption, and faces the same decision problem in period 2. After that date, there are no further consumption opportunities. As previously, signals are correlated with the true (unknown) profitability. Formally,

$$x_i = \pi + \epsilon_i \qquad \text{where} \quad \epsilon_i \;\text{ i.i.d. }\; \mathcal{N}(0,1)$$

Signals x_i are used to update in a Bayesian way the beliefs about the benefit of consumption. Denote by $X_s = \sum_{i=1}^{s} x_i$. A standard result in statistics is that:

$$\pi\,|\,X_s \;\sim\; \mathcal{N}\left(\lambda_s\, m + \frac{1-\lambda_s}{s}\,X_s, \frac{\lambda_s}{h}\right) \qquad \text{where} \quad \lambda_s = \frac{h}{h+s} \tag{10}$$

Note that the mean of the posterior distribution of π given s signals is a weighted average of the prior m and the observations $\{x_i\}_{i=1}^{s}$. The weight of the prior λ_s decreases with the number of observations. The variance is deterministic and decreasing in s.

4.2 Time inconsistency and haste

As in the previous section (see (4)), the risk-neutral agent consumes in period 2 if:

$$E[\pi\,|\,X_s] \geqslant \beta C$$

Hence, from (10) and conditional on having decided to wait in period 1, the agent consumes in period 2 if:

$$\lambda_s\, m + \frac{1-\lambda_s}{s}\,X_s \geqslant \beta C \quad \Rightarrow \quad X_s \geqslant \frac{\beta C - \lambda_s\, m}{(1-\lambda_s)/s}$$

At date 1, the agent anticipates his future behavior and chooses to consume rather than wait for extra information if and only if:

$$E[\pi] - \beta C \geqslant N(m)$$

where $N(m)$, the expected net payoff of waiting from the perspective of date 1, is:

$$N(m) = \beta\delta \int_{\frac{\beta C - \lambda_s m}{(1-\lambda_s)/s}}^{+\infty} \Big(E[\pi \mid X_s] - C \Big) dF(X_s) \tag{11}$$

and we can state our next result.

Proposition 3 *There exists a unique cutoff value m^* such that the agent consumes at date 1 if and only if $m \geq m^*$. Furthermore, for all $\beta < 1$, there exists $h^*(\beta) \in (0, +\infty)$ and $s^*(\beta, h)$ (> 1) such that $m^* < \beta C$ if and only if $h > h^*(\beta)$ and $1 \leq s < s^*(\beta, h)$.*

Proof. See Appendix A4. □

As in the previous section, there might be a negative information value of waiting. For instance, suppose that the agent waits and the signals X_s are such that $E[\pi \mid X_s] \in (\beta C, C)$. In this case, he will consume in period 2, which is not desirable from self-1's perspective. To avoid this inefficiency, the agent may decide to incur another (less important) one: consume in period 1 with an NPV of $E[\pi] - \beta C$ (< 0).

However, our main concern is not to check the robustness of Proposition 1 to a more comprehensive signal structure, but rather to analyze under which conditions consumption with negative NPV is more likely to occur. Proposition 3 states that if the agent has very little prior knowledge about the profitability (h small) and a large number of signals are transmitted between periods 1 and 2 (s high), then there is a positive information value of waiting. The idea is that, in this case, (i) the mean of the posterior distribution is mostly determined by the signals X_s and (ii) the decrease in the variance between the prior and posterior distribution is substantial (see equations (9) and (10)). Both effects add up implying that the *quantity and accuracy of information transmitted* between periods 1 and 2 is going to be important. In other words, with high probability the mean of the posterior will be relatively far from the prior. So, even if m is initially close to βC, there are relatively small chances that the expected posterior falls in the inconsistency region $(\beta C, C)$. Waiting is quite 'safe', and therefore desirable. Conversely, when the initial estimate of profitability is very accurate and few signals are coming, there is little transmission of information between periods. In this case, the chances that the posterior falls in $(\beta C, C)$ are high provided that the prior is sufficiently close to βC. In order to avoid this inefficiency, the agent is willing to consume in period 1, even at the expense of a negative NPV.

To sum up, Proposition 3 generalizes the intuition presented in section 3, where we argued that consumption with expected net losses was more likely when θ, the informational content of a signal, was small. Here, we make the argument more precise. The likelihood of a hasty action increases with the precision of the prior assessment h and decreases with the amount of information transmission s.[14]

4.3 Value of information

Arguing that haste is more frequent under limited transmission of information is different from saying that extra pieces of news are always desirable. We now show whether and when an agent will find information to be harmful.

Proposition 4 *For any positive number of signals s, there always exists a value $\hat{m}$ ($< \beta C$) such that extra signals are harmful for all $m < \hat{m}$.*

Proof. See Appendix A5. □

A hyperbolic discounting agent will sometimes be reluctant to obtain information. Proposition 4 states that the willingness of an agent to get free information decreases as the agent becomes more pessimistic about the expected value of consumption. The idea is that if the expected benefit is sufficiently small, it is unlikely that the agent ever consumes. Therefore, $N(m)$ the NPV of not consuming at date 1 is close to zero. Still, there is some (small) probability that after receiving the signals, the expected posterior falls in $(\beta C, C)$ and some (even smaller) probability of the posterior being above C. In this case, small doses of information are detrimental because they increase more the chances of hitting the inconsistency region $(\beta C, C)$ than the chances of hitting the region above C. Two results follow from Propositions 3 and 4.

Corollary 2 *The value of information is U-shaped. If $s = 0$ consumption with negative NPV never occurs. Moreover, for any h, the access to information decreases welfare if $s \in \{1, \dots, s^*(h)\}$ and it increases welfare if $s > s^*(h)$.*

Corollary 3 *The capacity to acquire costly information reduces but does not eliminate the possibility of an investment with negative NPV.*

According to Corollary 2, a positive NPV is a necessary condition for consumption at date 1 only if $s > s^*$ or $s = 0$ (in that case, it is also sufficient). The main inefficiency occurs for intermediate amounts of information transmission ($s \in \{1, ..., s^*\}$) where

[14] Again, $\partial h^* / \partial \beta > 0$ and $\lim_{\beta \to 1} h^*(\beta) = +\infty$: as the intrapersonal conflict decreases, the likelihood of a future behavior inefficient from the current perspective also decreases. When the conflict vanishes, positive expected NPV is a necessary condition to observe consumption at date 1.

consumption with negative NPV is possible. In other words, the value of information is U-shaped: starting from no information, each extra piece of news has a negative effect on welfare up to a certain point and then a positive effect. After s^* pieces of news, we know that having access to information is strictly better than not receiving any news at all.

How does the capacity to buy costly pieces of news modifies this conclusion? The non-monotonicity of the value of information has interesting implications. In some circumstances, an agent is willing to pay to reduce the number of signals (e.g. from $s = 1$ to $s = 0$). In some others, the marginal value of extra information is positive and large (e.g. from s^* to $s^* + 1$). Overall, as Corollary 3 states, granting the agent the possibility to pay for extra information will necessarily decrease the likelihood of consuming with negative NPV. Our conjecture is that for initially small levels of information transmission, extra news are harmful and therefore avoided independently of their cost. For initially high levels of information, the marginal effect of an extra piece of news is positive but negligible and therefore not worth the cost. Last, for intermediate levels of information (around s^*) news are extremely valuable, so the agent will have incentives to pay for signals.[15] For a deeper analysis of the incentives to choose endogenously the amount of acquired information, we refer to Carrillo and Mariotti (2000), and the extensions by Brocas and Carrillo (1999) and Benabou and Tirole (2002).

5 Applications and concluding remarks

As we have thoroughly discussed, our theory is based on four key ingredients: (i) hyperbolic discounting of payoffs, (ii) irreversible consumption, with an uncertain net payoff, short term benefits and long term costs, (iii) possibility of delaying the decision to consume, and (iv) exogenous flow of information relative to its net value (learning or random evolution of payoffs). We have relied on some (still controversial) experimental evidence to justify the first ingredient. In this final section we argue that the other three ingredients are present in economic situations of very different nature. Naturally, our model only captures one specific aspect in decision making processes that are very complex. We do not pretend that, in the examples presented below, the effect highlighted by our simple model is the only, or even the main, driving force for the behavior of agents. Still, we believe that ours is a plausible theory to keep in mind when studying these issues, and that it may account for part of the problem. Its importance

[15] We thank an anonymous referee for pointing out this issue.

relative to other motives is an empirical question, interesting but out of the scope of this paper.

Credit purchase and impulse buying. The recent trends in consumer spending and use of credit opportunities are usually associated with changes in the buying habits, namely with an increase in present-oriented, unplanned and impulse buying. Impulse buying is recognized as being more emotional than rational (see e.g. Rook, 1987). Our simple model suggests an alternative (rational) explanation for this phenomenon. Credit facilities have modified consumers' habits by turning the 'buying' decision into a 'consumption' decision with current uncertain benefits and delayed costs. In this context, the lack of confidence in their future self-control pushes consumers to become currently indebted in order to buy goods, even under the anticipation that, on average, they will not necessarily enjoy them too much.

Preservation of endangered species. The benefits of preserving wildlife are not only limited to basic issues of animal rights and legacy to future generations. Even from a selfish viewpoint, many endangered species serve us as pharmaceuticals, food sources, and industrial products. Yet, the threat of extinction affects an increasing number of species. Naturally, the opportunity cost to preserve some species is sometimes extremely high, so sustained economic development has often been invoked as the reason for limited conservation policies. Yet, in our view, this is a good example where the forces of our model are at play, at least to a certain extent. Weak policies have uncertain and partly irreversible effects. Some of this uncertainty is resolved over time. Most costs are delayed whereas most benefits (including in some cases political support from powerful corporations) are immediate. The anticipation of future inefficient conservation policies may have induced governments to take insufficient care of species right away, even under the knowledge that the costs offset the benefits.

Destruction of the tropical rainforest. Tropical deforestation is responsible for the extinction of species and the liberation of carbon monoxide (the greenhouse effect). Yet, the tropical forest area destroyed every year keeps increasing dramatically. This is probably the best example of myopia and the commons problems since studies show that traditional agriculture lasts, on average, only two years due to soil erosion. However, we argue that our haste theory may also account for part of the problem. Note all the ingredients are present: deforestation is partially irreversible, benefits of wood cutting and slash and burn agriculture are immediate while costs are delayed, and there is uncertainty (resolved over time) on the costs of deforestation for the natural environment. In that context, our model claims that a lack of confidence in the preservation policy of future governments pushes current decision-makers to sacrifice the land and reap at least some of the benefits of its exploitation.

To sum up, we have studied the relation between self-control problems and the tendency to take decisions with pernicious long run consequences. We have provided a characterization of the value of information as a function of its (exogenous) amount transmitted between periods and pointed out its U-shaped form. We have highlighted the individual willingness to undertake activities with negative NPV only to avoid detrimental incoming information. Last, we have also argued that the cost-benefit analysis of investment policies should incorporate the shadow benefit of decreasing the incentives to take hasty actions.

We would like to conclude by pointing out two natural extensions. First, in the traditional theory, stochastic changes in the environment decrease the (positive) value of information, because signals about the current state of the world may quickly become obsolete. Given that, in our setting, the information value of waiting is not always positive, the effects of continual shocks on the likelihood of consumption should be ambiguous. Second, in standard analyses, there is no loss of generality in assuming a deterministic consumption horizon. On the contrary, under hyperbolic discounting, multiple equilibria may coexist if the horizon is stochastic or infinite. The coexistence of equilibria with and without haste has an interesting interpretation; individuals choose whether or not to sacrifice long run wealth depending on the degree of 'trust' in future generations (i.e., on the anticipation of the equilibrium that will be played).[16] These and other extensions are left for future work.

[16]Note that the interpretation of hyperbolic discounting as an overlapping generation model with imperfect altruism goes back to Phelps and Pollak (1968). For recent recursive utility models where self-t integrates in his utility function the utility of self-$t+1$ and/or that of self-$t-1$, we refer to Caplin and Leahy (2001) and Palacios-Huerta (2004).

Appendix

A1. Proof of Lemma 1.

Denote by $F_i(n)$ the value function of the agent at date $i \in \{0, ..., T\}$ given $n = n_h - n_l$. Similarly, pose $H_i(n) = F_i(n) - [\pi(n) - C]$. At date T, $F_T(n) = \max\{\pi(n) - C; 0\}$ and $H_T(n) = F_T(n) - [\pi(n) - C]$. Given that $\pi(n) - C$ is strictly increasing in n, $H_T(n)$ is non increasing in n. Under Assumption 2 and given (3), there is a cutoff $\tilde{n}_T \equiv \tilde{n}$ such that for all $n_T > \tilde{n}_T$, $\pi(n_T) - C > 0$ so that the agent consumes in the last period T.

At date $T-1$ and given the definition of $\gamma(n)$, we have $F_{T-1}(n) = \max\{\pi(n) - C;\ \delta[\gamma(n)F_T(n+1) + (1-\gamma(n))F_T(n-1)]\}$ which is equivalent to:

$$H_{T-1}(n) = \max\{0; -(1-\delta)[\pi(n) - C] + \delta[\gamma(n)H_T(n+1) + (1-\gamma(n))H_T(n-1)]\}$$

Note that:

$$\frac{\partial}{\partial n}\Big[\gamma(n)H_T(n+1) + (1-\gamma(n))H_T(n-1)\Big] < 0 \qquad (12)$$

since $\gamma'(n) > 0$ and, again, $H_T(n)$ is non increasing in n. In addition $\pi'(n) > 0$, so the right hand side (r.h.s.) of $H_{T-1}(n)$ is strictly decreasing in n and the value function $H_{T-1}(n)$ is non increasing in n. As a consequence, there exists $\tilde{n}_{T-1}$ that satisfies:

$$\pi(\tilde{n}_{T-1}) - C = \delta\Big[\gamma(\tilde{n}_{T-1})F_T(\tilde{n}_{T-1}+1) + (1-\gamma(\tilde{n}_{T-1}))F_T(\tilde{n}_{T-1}-1)\Big] > 0$$

Therefore, $\tilde{n}_{T-1} > \tilde{n}_T$.

The proof is completed using a recursive method. Suppose that the following properties hold at date t:

(A1) $H_t(n)$ is non increasing in n;
(A2) $H_t(n) \geq H_{t+1}(n)$.

$$\begin{aligned} H_t(n) &= \max\Big\{0; -(1-\delta)[\pi(n) - C] + \delta[\gamma(n)H_{t+1}(n+1) + (1-\gamma(n))H_{t+1}(n-1)]\Big\} \\ H_{t-1}(n) &= \max\Big\{0; -(1-\delta)[\pi(n) - C] + \delta[\gamma(n)H_t(n+1) + (1-\gamma(n))H_t(n-1)]\Big\} \end{aligned}$$

Combining **(A1)** and the fact that $\gamma'(n) > 0$, we get that the r.h.s. of $H_{t-1}(n)$ is decreasing in n. Therefore, if $H_t(n)$ is non increasing in n, $H_{t-1}(n)$ is also non increasing in n. Moreover, there exists a unique cutoff $\tilde{n}_{t-1}$ above which the agent consumes at date $t-1$. In addition, given **(A2)**, it is clear that:

$$\gamma(n)\Big[H_t(n+1) - H_{t+1}(n+1)\Big] + (1-\gamma(n))\Big[H_t(n-1) - H_{t+1}(n-1)\Big] \geq 0$$

and therefore, the r.h.s. of $H_{t-1}(n)$ is greater than the r.h.s. of $H_t(n)$ which is sufficient to prove that $H_{t-1}(n) \geq H_t(n)$. Overall, we have shown that if **(A1)** and **(A2)** hold at date t, then they also hold at date $t-1$ which completes the recursion. This in turn implies that $\tilde{n}_{t-1} > \tilde{n}_t$ for all t.

A2. Proof of Proposition 1.

Step 1: consumption behavior at date T. Denote by $F_i^*(n)$ the value function of self-i ($i \in \{0, ..., T\}$) under time inconsistency, and pose $H_i^*(n) = F_i^*(n) - [\pi(n) - \beta C]$. At date T, self-T's value function is $F_T^*(n) = \max\{\pi(n) - \beta C; 0\}$ and $H_T^*(n) = \max\{0; -[\pi(n) - \beta C]\}$. As for $H_T(n)$ in Lemma 1, $H_T^*(n)$ is non increasing in n. Therefore, under Assumption 3 and given (6), there exists $n_T^* \equiv n^*$ such that $\pi(n_T^*) = \beta C$ and the agent consumes if and only if $n_T \geq n_T^*$.

Step 2: consumption behavior at date $T-1$. Determining the behavior at $T-1$ is more complicated than in Lemma 1 because, under time inconsistency, we cannot apply the Bellman principle directly. At date $T-1$, the value function of self-$T-1$ is:

$$F_{T-1}^*(n) = \max \left\{\pi(n) - \beta C; K_{T-1}^*(n)\right\}$$

where:

$$K_{T-1}^*(n) = \beta\delta\Big[\gamma(n)[\pi(n+1) - C]\mathbb{1}_{n+1 \geq n_T^*} + (1-\gamma(n))[\pi(n-1) - C]\mathbb{1}_{n-1 \geq n_T^*}\Big]$$

This can be rewritten as:

$$K_{T-1}^*(n) = \delta E_{T-1}\left[[\pi(n') - \beta C]\mathbb{1}_{n' \geq n_T^*} \,\Big|\, n\right] - \delta(1-\beta)E_{T-1}\left[\pi(n')\mathbb{1}_{n' \geq n_T^*} \,\Big|\, n\right]$$

where, given our binary signal structure, n' is equal to $n+1$ with probability $\gamma(n)$ and to $n-1$ with probability $1-\gamma(n)$. Hence:

$$K_{T-1}^*(n) = \delta E_{T-1}\left[F_T^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_T^*} \,\Big|\, n\right]$$

Overall, we get:

$$F_{T-1}^*(n) = \max\left\{\pi(n) - \beta C; \delta E_{T-1}\left[F_T^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_T^*} \,\Big|\, n\right]\right\}$$

In terms of $H_{T-1}^*(n)$ and $H_T^*(n)$, this is equivalent to:

$$H_{T-1}^*(n) = \max\left\{0; -(1-\delta)[\pi(n) - \beta C] + \delta E_{T-1}\Big[H_T^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_T^*} \,\Big|\, n\Big]\right\}$$

Given $\frac{\partial}{\partial n} E_{T-1}[\pi(n') \mid n] \geq 0$, $\frac{\partial}{\partial n} \Pr(n' > n_T^* \mid n) \geq 0$ and (12), the r.h.s. of $H_{T-1}^*(n)$ is strictly decreasing in n. Therefore, there exists a unique cutoff n_{T-1}^*. Call $L_{T-1}^*(n)$ the r.h.s. of $H_{T-1}^*(n)$, so that $H_{T-1}^*(n) = \max\{0, L_{T-1}^*(n)\}$. Given that $\pi(n_T^*) = \beta C$,

$$L_{T-1}^*(n_T^*) = \delta[\gamma(n_T^*)F_T^*(n_T^*+1) - (1-\beta)\gamma(n_T^*)\pi(n_T^*+1)]$$

Hence, $L_{T-1}^*(n_T^*) > 0$ if and only if $F_T^*(n_T^*+1) > (1-\beta)\pi(n_T^*+1)$. Thus,

$$n_{T-1}^* > n_T^* \;\Leftrightarrow\; \pi(n_T^*+1) - \beta C > (1-\beta)\pi(n_T^*+1) \;\Leftrightarrow\; \pi(n_T^*+1) - C > 0$$

Given the definition of $\mathcal{J}(\cdot)$ in (7) we get that $n_{T-1}^* > n_T^*$ if $\mathcal{J}(\theta, \beta, G, C) > 0$ and $n_{T-1}^* < n_T^*$ if $\mathcal{J}(\theta, \beta, G, C) < 0$.

Step 3: recursion. The last step consists of proving that if $n_t > n_{t+1}$ then $n_{t-1} > n_t$ and if $n_t \leq n_{t+1}$ then $n_{t-1} \leq n_t$. Suppose that self-t's value function is:

$$F_t^*(n) = \max\left\{\pi(n) - \beta C; \delta E_t\left[F_{t+1}^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' > n_{t+1}^*} \;\middle|\; n\right]\right\}$$

At date $t-1$:

$$F_{t-1}^*(n) = \max\left\{\pi(n) - \beta C; K_{t-1}^*(n)\right\}$$

where:

$$K_{t-1}^*(n) = \beta\delta\left\{E_{t-1}\left[[\pi(n') - C]\mathbb{1}_{n' > n_t^*} \;\middle|\; n\right]\right.$$
$$\left. + \delta E_{t-1}\left[\mathbb{1}_{n' < n_t^*} E_t\left[[\pi(n'') - C]\mathbb{1}_{n'' > n_{t+1}^*} \;\middle|\; n'\right] \;\middle|\; n\right] + \ldots\right\}$$

This can be rewritten as:

$$K_{t-1}^*(n) = \delta\left\{E_{t-1}\left[[\pi(n') - \beta C]\mathbb{1}_{n' \geq n_t^*} + \mathbb{1}_{n' < n_t^*}\left[\delta E_t\left[[\pi(n'') - \beta C]\mathbb{1}_{n'' \geq n_{t+1}^*} \mid n'\right] + \ldots\right.\right.\right.$$
$$\left.\left.\left. - (1-\beta)\delta E_t\left[\pi(n'')\mathbb{1}_{n'' \geq n_{t+1}^*} \mid n'\right] - \ldots\right] \;\middle|\; n\right] - (1-\beta)E_{t-1}\left[\pi(n')\mathbb{1}_{n' \geq n_t^*} \;\middle|\; n\right]\right\}$$

Therefore:

$$F_{t-1}^*(n) = \max\left\{\pi(n) - \beta C; \delta E_{t-1}\left[F_t^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' > n_t^*} \;\middle|\; n\right]\right\}$$

Given that we can rewrite the value function $F_t^*(n)$ (and therefore $H_t^*(n)$) for each self-t, we can proceed to the recursion. Suppose first that $n_{T-1}^* < n_T^*$ (i.e. $\mathcal{J}(\cdot) < 0$) and assume that the following properties hold at date t:

(A1) $H_t^*(n)$ is non increasing in n;
(A3) $H_t^*(n) \leq H_{t+1}^*(n)$.

Note that **(A1)** and **(A3)** imply that $n_t^* \leq n_{t+1}^*$. Here:

$$H_t^*(n) = \max\left\{0; -(1-\delta)[\pi(n) - \beta C] + \delta E_t\left[H_{t+1}^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_{t+1}^*} \;\middle|\; n\right]\right\}$$

$$H_{t-1}^*(n) = \max\left\{0; -(1-\delta)[\pi(n) - \beta C] + \delta E_{t-1}\left[H_t^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_t^*} \;\middle|\; n\right]\right\}$$

Note as before that $\pi'(n) > 0$ and that $\frac{\partial}{\partial n} E_{t-1}\left[H_t^*(n') \mid n\right] \leq 0$ since $H_t^*(n)$ is decreasing by **(A1)**. In addition, $\frac{\partial}{\partial n}\Pr(n' > n_t^* \mid n) \geq 0$ which is sufficient to prove that if **(A1)** holds at date t then it also holds at $t-1$.

Now, according to **(A3)**:

$$\delta E_t\left[[H_t^*(n) - H_{t+1}^*(n)] - (1-\beta)[\pi(n)\mathbb{1}_{n \geq n_t^*} - \pi(n)\mathbb{1}_{n > n_{t+1}^*}]\right] < 0$$

and therefore $L_{t-1}^*(n) < L_t^*(n)$ for all n which implies that if **(A3)** holds at date t then it also holds at $t-1$. Overall, by recursion, if $\mathcal{J}(\cdot) < 0$ then $n_t^* \leq n_{t+1}^*$.

If $n_{T-1}^* > n_T^*$ (i.e. if $\mathcal{J}(\cdot) > 0$) the recursion is the same as in the proof of Lemma 1 and we get that $n_t^* > n_{t+1}^*$.

Step 4: comparison between $\tilde{n}_t$ and n_t^*. Suppose that $H_{t+1}^*(n) < H_{t+1}(n)$. This is true at $t+1 = T$. Moreover:

$$H_t(n) = \max\left\{0; -(1-\delta)[\pi(n) - C] + \delta E_t[H_{t+1}(n') \mid n\right\}$$

$$\begin{aligned} H_t^*(n) &= \max\left\{0; -(1-\delta)[\pi(n) - \beta C] + \delta E_t\left[H_{t+1}^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_{t+1}^*} \;\middle|\; n\right]\right\} \\ &= \max\left\{0; -(1-\delta)[\pi(n) - C] + \delta E_t\left[H_{t+1}^*(n') - (1-\beta)\pi(n')\mathbb{1}_{n' \geq n_{t+1}^*} \;\middle|\; n\right]\right. \\ &\qquad \left. -(1-\delta)(1-\beta)C\right\} \end{aligned}$$

Trivially, if $H_{t+1}^*(n) < H_{t+1}(n)$ then the r.h.s. of $H_t^*(n)$ is smaller than the r.h.s. of $H_t(n)$. As a consequence $H_t^*(n) < H_t(n)$, which proves that $n_t^* < \tilde{n}_t$ for all t.

A3. Proof of Proposition 2.

Recall that the value of not consuming is normalized to 0. Suppose now that increasing this value by Δ units has a cost for the agent smaller than $C - \pi(n_{T-1}^*)$. If $\pi(n_{T-1}^* + 1) - \beta C < \Delta$, then when $n_{T-1} = n_{T-1}^*$, self-$T-1$ strictly prefers to spend resources in increasing the value of not consuming by Δ rather than consuming himself.

A4. Proof of Proposition 3.

Note that $X_s \sim \mathcal{N}\left(ms, \frac{s^2}{\lambda_1}\right)$. Call $M(m) = \frac{(\beta C - m)\sqrt{\lambda_1}}{1-\lambda_s}$ and denote $\phi(\cdot)$ the density and $\Phi(\cdot)$ the c.d.f. of the standard normal distribution. From (11) and using the properties of the truncated normal distribution, we can rewrite the value of waiting $N(m)$ as:

$$N(m) = \beta\delta\,(m - C)\,[1 - \Phi(M(m))] + \beta\delta\,\frac{1-\lambda_s}{\sqrt{\lambda_1}}\,\phi(M(m)) \tag{13}$$

Given that $\phi'(x) = -x\,\phi(x)$, we have that:

$N'(m) = \beta\delta\left[1 - \Phi(M(m)) - \phi(M(m))\frac{\sqrt{\lambda_1}\,C(1-\beta)}{1-\lambda_s}\right]$ and $N''(m) \propto 1 - \frac{\lambda_1\,C(1-\beta)}{(1-\lambda_s)^2}(\beta C - m)$

Therefore: (i) $\lim_{m\to-\infty} N(m) = 0$; (ii) $\lim_{m\to-\infty} N'(m) = 0$; (iii) there exists a value $\tilde{m}$ ($< \beta C$) such that if $m < \tilde{m}$ then $N''(m) < 0$ and if $m > \tilde{m}$ then $N''(m) > 0$; (iv) for all m, $N'(m) < 1$. Results (i) to (iv) imply that the cutoff m^* is unique and given by:

$$m^* = \beta C - \beta C\,\frac{\delta(1-\beta)[1 - \Phi(M(m^*))]}{1 - \beta\delta[1 - \Phi(M(m^*))]} + \beta\,\frac{1-\lambda_s}{\sqrt{\lambda_1}}\,\frac{\delta\,\phi(M(m^*))}{1 - \beta\delta[1 - \Phi(M(m^*))]} \tag{14}$$

The last part of the proof is done by contradiction. Fix β and denote by (h^*, s^*) the pairs such that $m^*(h^*, s^*) = \beta C$. Note that, by construction, this implies $M(m^*(h^*, s^*)) = 0$. Hence, from (14):

$$C\,(1-\beta) = \frac{\phi(0)}{1 - \Phi(0)}\,\frac{1 - \lambda_s(h^*, s^*)}{\sqrt{\lambda_1(h^*, s^*)}} \tag{15}$$

Now take $h > h^*$. Naturally, $\lambda_s(h, s^*) > \lambda_s(h^*, s^*)$. Suppose that $m^*(h, s^*) > \beta C$ (so that $M(m^*(h, s^*)) < 0$). This would imply that:

$$\frac{\phi(0)}{1 - \Phi(0)}\,\frac{1 - \lambda_s(h^*, s^*)}{\sqrt{\lambda_1(h^*, s^*)}} < \frac{\phi(M(m^*(h, s^*)))}{1 - \Phi(M(m^*(h, s^*)))}\,\frac{1 - \lambda_s(h, s^*)}{\sqrt{\lambda_1(h, s^*)}}$$

However, this is impossible because; (i) $\frac{\phi(0)}{1-\Phi(0)} > \frac{\phi(M(m^*(h,s^*)))}{1-\Phi(M(m^*(h,s^*)))}$ (since $\frac{\phi(x)}{1-\Phi(x)}$ is increasing in x) and (ii) $\frac{1-\lambda_s(h^*,s^*)}{\sqrt{\lambda_1(h^*,s^*)}} > \frac{1-\lambda_s(h,s^*)}{\sqrt{\lambda_1(h,s^*)}}$. So, for $s = s^*$ if $h >$ (resp. $<$) $h^*(\beta)$ then $m^* <$ (resp. $>$) βC. The same argument shows that for $h = h^*$ if $s <$ (resp. $>$) s^* then $m^* <$ (resp. $>$) βC. Last, when $\beta = 1$, equation (14) becomes:

$$m^* = C + \frac{1-\lambda_s}{\sqrt{\lambda_1}}\,\frac{\delta\,\phi(M(m^*))}{1 - \delta[1 - \Phi(M(m^*))]} > C.$$

A5. Proof of Proposition 4.

We have to show that for all $m < \hat{m}$, $N(m,s) > m - \beta C$ and $\frac{\partial}{\partial s} N(m,s) < 0$. From the definition of $N(m,s)$ in (13) it is easy to see that:

$$\frac{\partial N(m,s)}{\partial s} \propto 1 - C\,(1-\beta)(\beta C - m)\frac{\lambda_1}{(1-\lambda_s)^2} \tag{16}$$

As $\lim_{m \to -\infty} N(m,s) = 0$, then for all m smaller than a cutoff $\hat{m}$, if s increases the cutoff above which the agent consumes decreases. Besides, $\frac{\partial N}{\partial s} < 0$, so the agent consumes more often and, even when he waits, he gets on average a smaller payoff.

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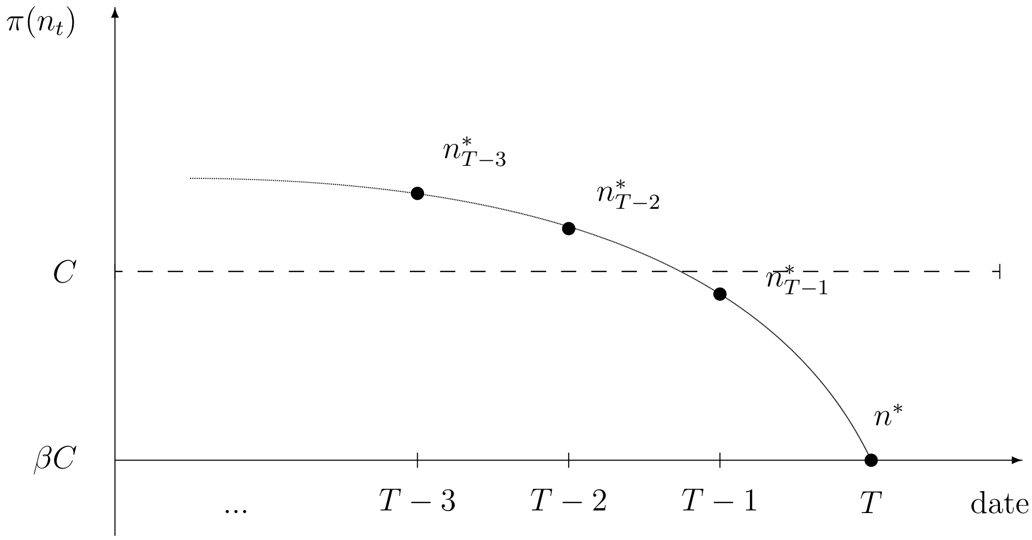

Figure 1. Consumption when $\mathcal{J}(\theta, \beta, G, C) > 0$.

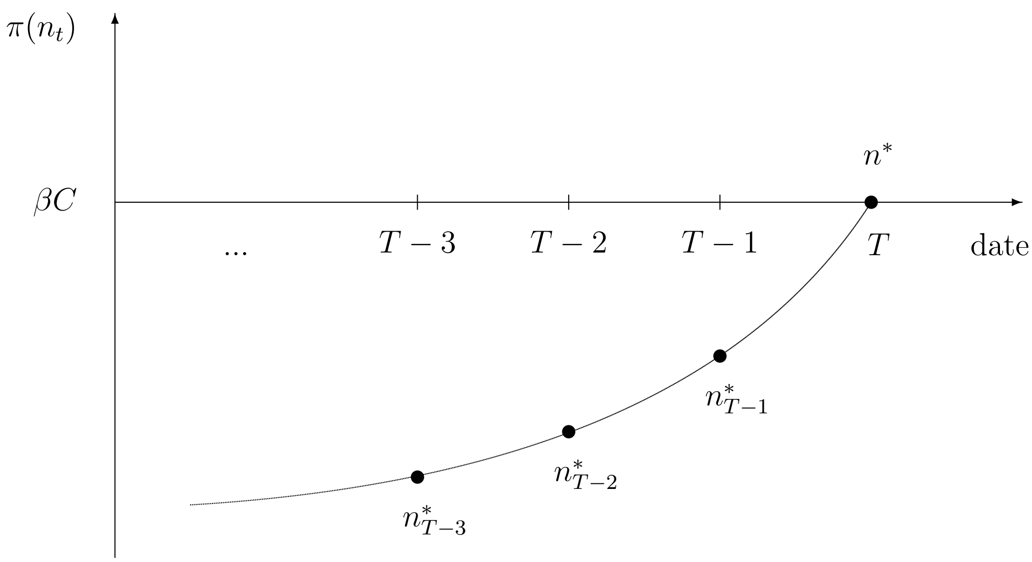

Figure 2. Consumption when $\mathcal{J}(\theta, \beta, G, C) < 0$.

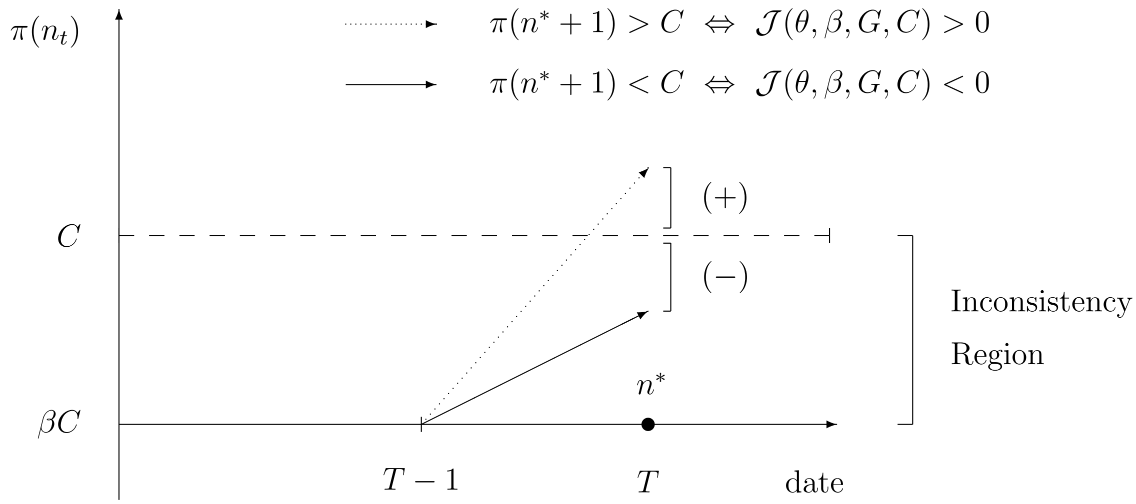

Figure 3. Transmission of information and incentives to consume.