

All Roads Lead to Violations of Countable Additivity

In an important recent paper, Brian Weatherson (2010) claims to solve a problem I have raised elsewhere,¹ namely the following. On the one hand, there appears to be strong reason to favor the one-third solution to the Sleeping Beauty problem. This problem has been tackled from a variety of perspectives, but, with a few notable exceptions, they have converged on the one-third solution.² On the other hand, there are a variety of considerations supporting the principle of countable additivity (hereafter, CA).³ However, there appears to be a deep tension between the one-third solution and CA. For the main arguments for the one-third solution all share a problematic feature: they rest on premises that entail that, under certain possible circumstances, an agent is rationally required to violate CA. The challenge, therefore, for anyone who wants to accept both the one-third solution and CA, and who wants to deny that the requirements of rationality can conflict, is to provide an argument for the one-third solution that avoids this problematic feature.

Weatherson argues that we should reject traditional arguments for the one-third solution, since they require an indifference principle that conflicts with CA. He then takes up the challenge of providing a CA-friendly argument for the one-third solution. I aim to show, however, that Weatherson's argument fails to meet this challenge, since, like the arguments he rejects, his argument is based on premises that conflict with CA. To show this, I begin, in the first section, by setting out Weatherson's argument informally. I then formalize the argument in the second section, indicating the general assumptions on which it rests. Then I show, in the third section, that these assumptions entail the very kind of indifference principle that Weatherson sought to avoid, a principle that conflicts with CA. In the fourth section, I respond to an objection to my argument, and in the final section, I reflect briefly on its broader significance.

¹ Ross (2010).

² Philosophers who have argued for the one-third solution include Frank Arntzenius, Dennis Dieks, Cian Dorr, Kai Draper, Adam Elga, Christopher Hitchcock, Terrance Horgan, Bradley Monton, Robert Stalnaker, Michael Titelbaum, Ruth Weintraub, as well as the sixteen members of the OSCAR Seminar (Seminar, 2008). Philosophers who have argued for the one-half solution include Joseph Halpern, David Lewis and Christopher Meacham. For references to the relevant literature, see Ross 2010 and Pust forthcoming.

³ One reason to accept CA is that it is supported by Dutch book arguments—see Williamson 1999 and Ross 2010. Another is that it follows from the very plausible principle of conglomerability—see Schervish *et al.*, 1984. And a third is that it plays an important role in scientific reasoning and statistical inference—see Earman 1992 and Kelly 1996.

1. Weatherson's Argument: Informal Version

Consider a scenario in which a fully rational agent named Beauty falls asleep on Sunday evening after being told the rules of the game, and then a fair coin is tossed. She is awoken on Monday morning and then put back to sleep. If *Tails*, her memories of Monday are erased on Monday night and she undergoes a second, indistinguishable awakening on Tuesday morning. But if *Heads*, no memories are erased and she is not awoken on Tuesday. Either way, on Wednesday morning the experiment ends, and Beauty is awoken under circumstances that cannot be confused with those of any other awakening (call this Wednesday awakening the *post-experimental awakening*). The problem is to determine what Beauty's credence in *Heads* should be on Monday morning.

To solve this problem, Weatherson borrows an idea from Robert Stalnaker (2008), namely, that we can think of self-locating belief as a species of *de re* belief. For example, when Beauty awakens on Monday morning, her belief that she would express by saying "it is now Monday or Tuesday" can be thought of as the *de re* belief, concerning her present awakening experience, that it occurs on Monday or Tuesday. The advantage to thinking about self-locating belief in this manner is that it allows us to understand Beauty's beliefs on Monday morning as having the same content as thoughts she can entertain at other times. And this commonality of content may allow us to determine what her credences should be on Monday by applying diachronic principles, such as conditionalization.

Weatherson's argument involves applying diachronic principles both to the transition between Sunday and Monday, and to the transition between Monday and Wednesday. Let's first consider the latter transition. Weatherson argues as follows. On Wednesday, Beauty doesn't have any information she could use to discriminate between *Heads* and *Tails*, and so her credence in *Heads* should be one-half. And on Wednesday, Beauty will remember exactly one awakening, though which awakening she will remember depends on the outcome of the coin toss. Let's assume that the coin came up heads, and hence that the awakening she remembers on Wednesday is the one that occurred on Monday—call this awakening α . We can assume this without loss of generality, since our aim is to figure out her credences on Monday, and these should not be affected by the outcome of the coin toss.

On this assumption, on Wednesday Beauty will not only remember α , but she will know strictly more about α than she did on Monday. For on Wednesday, Beauty will continue to know that α occurred on Monday if *Heads* and on Monday or Tuesday if *Tails*. But she will also know one further fact about α , namely, that it is an awakening event that she can still remember on Wednesday. All along, however, she knows that she will remember α on Wednesday just in case (either α occurs on Monday and *Heads* or else α occurs on Tuesday and *Tails*). Thus, since what she learns on Wednesday is equivalent to this disjunction, her Wednesday credences should equal her Monday credences conditional on this disjunction. Hence, since, on Wednesday, her credence in *Heads* is one-half, on Monday her credence in *Heads* conditional on this disjunction must be one-half. And, given what Beauty knows about the set-up, this implies:

$$\text{P1. } Cr_1(\alpha \text{ is on Monday and } \textit{Heads}) = Cr_1(\alpha \text{ is on Tuesday and } \textit{Tails})$$

where Cr_1 represents Beauty's credences on Monday morning in the scenario we are now considering (call this scenario *SB*).

This takes us half-way to the one-third solution. To take us the rest of the way, Weatherson connects Beauty's credences on Monday with her credences on Sunday. An obstacle to drawing this connection is that, in *SB*, on Sunday Beauty is not yet acquainted with α , and so it's unclear that she can have *de re* thoughts about it. Weatherson gets around this obstacle by introducing a variant of *SB*, which we may call *SB'*, in which a time traveler films Beauty's awakening on Monday, and then travels back to Sunday and shows her the film. Let's call the time traveler 'Tom', and let α' denote Beauty's first experimental awakening in *SB'*, the one Tom films on Monday. When Tom shows Beauty the film, he points to α' and tells her that it occurs Monday. Beauty knows that Tom is telling the truth, and she understands that she would have experienced just such a visit from Tom regardless of the outcome of the coin toss. Then, when Beauty falls asleep on Sunday night, Tom erases all her memories from his visit.

It seems that in, *SB'*, Beauty's credence in *Heads* on Sunday should be unaffected by Tom's visit, and so it should be one-half. And in this scenario, Beauty will know strictly more about α' on Sunday than on Monday: for on Sunday she will know that α' occurs on Monday. Since Beauty loses information between Sunday and Monday, we can't apply conditionalization, but we can apply *reverse conditionalization*, according to which her prior credences must equal

her posterior credences conditional on the information she loses. Applying this principle, we may infer that Beauty's Sunday credence in *Heads*, namely one-half, must equal her Monday credence in *Heads* conditional on (α' is on Monday). And, given what Beauty knows about the set-up, this implies that when Beauty awakens on Monday in *SB'*, she must have the same credence in (α' is on Monday and *Heads*) as she has in (α' is on Monday and *Tails*).

But if this is true in the time traveler variant, *SB'*, then the corresponding claim should also be true of the original case, *SB*. For, given the memory erasure that occurs in *SB'*, it doesn't seem there should be any difference between these cases in Beauty's credences on Monday. And so we may infer the following:

$$\mathbf{P2.} \quad Cr_1(\alpha \text{ is on Monday and } \textit{Heads}) = Cr_1(\alpha \text{ is on Monday and } \textit{Tails})$$

And P1 and P2 together imply that when Beauty awakens on Monday in *SB*, her credence in *Heads* must be one-third.

2. Weatherson's Argument: Formal Version

In order to show that this argument rests on assumptions that conflict with CA, it will be useful to formalize the argument. Let us begin by defining two kinds of Sleeping Beauty scenario.

Standard Sleeping Beauty Scenario (SSBS): Any scenario in which a fully rational agent, Beauty, will undergo one or more mutually indistinguishable awakenings, and in which the number of awakenings she will undergo is determined by the outcome of a random process. Where *H* is the partition of alternative hypotheses concerning the outcome of this process, prior to her first experimental awakening, for each hypothesis in *H*, Beauty knows both its objective chance and how many times she will awaken if it is true. But she has no other relevant information.

Time traveler Variant: For any SSBS defined by a partition *H* of hypotheses and a specification of the prior objective chance, and of the number of awakenings, for each hypothesis in *H*, a time traveler variant of this scenario has the same structure, and differs from it only in the following respect. In the time traveler variant, Beauty is visited by a time traveler (named Tom) on the day before her first experimental awakening. For some property *A*, Tom carries out a protocol that ensures that, regardless of which hypothesis in *H* is true, he films one of Beauty's experimental awakenings, and then returns to the day before her first experimental awakening, shows

her the film, and truthfully tells her, of the awakening shown in the film, that it has property A . Beauty understands that Tom would show her a film of one of her experimental awakenings, and tell her the same thing about this awakening, regardless of which hypothesis in H were true. Then, before Beauty's first experimental awakening, all her memories from the visit are erased.

Given these definitions, we can now state general principles on the basis of which Weatherson's argument for the one-third solution can be reconstructed.

Posterior Chance-Credence Principle. In any SSBS defined by a partition H , if, at the end of the experiment, Beauty is awoken under circumstances that cannot be confused with those of any other awakening, and if those circumstances are not affected, in any discernible way, by the truth values of the hypotheses in H , then her credences in the hypotheses in H must equal their prior objective chances.

Information Gain. In any SSBS, if a given awakening, α , is Beauty's last experimental awakening, then the only epistemically relevant change (if any) to occur between α and her post-experimental awakening is that she learns, of α , that it was her last experimental awakening.

Conditionalization. If an agent is fully rational, and if the only epistemically relevant change (if any) to occur between two times is that she learns some proposition p in which she antecedently had positive credence, then her later credences must equal her earlier credences conditional on p .

Prior Chance-Credence Principle. In any time traveler variant of an SSBS, defined by a partition H , right after Tom's visit on the eve of the first experimental awakening, Beauty's credences in the hypotheses in H must equal their prior objective chances.

Information Loss. In any time traveler variant of an SSBS, where p is the *de re* proposition that Tom tells Beauty, the only epistemically relevant change (if any) to occur between the time she is told that p and the time of her first experimental awakening is that she ceases to know that p .

Reverse Conditionalization. If an agent is fully rational, and if the only epistemically relevant change (if any) to occur between two times is that she ceases to know some proposition p in which she subsequently has positive credence, then her earlier credences must equal her later credences conditional on p .

Outcome Independence. For any SSBS defined by a partition H , the truth values of the hypotheses in H have no bearing on what Beauty's credences should be in these hypotheses at the time of her first experimental awakening.

Time Traveler Independence. The difference between an SSBS and a time traveler variant thereof has no bearing on Beauty's credences at the time of her first experimental awakening. In particular, for any property A , these two scenarios do not differ with respect to Beauty's credence, concerning her first experimental awakening, that it has property A .

We can now show that these assumptions, together with the axioms of probability, entail the one-third solution. As before, let α and α' represent Beauty's first awakenings in SB and in SB' , respectively. Let Cr_0 , Cr_1 and Cr_2 represent her credences, in SB , on Sunday evening, Monday morning and Wednesday morning, respectively, and let Cr'_0 , Cr'_1 and Cr'_2 represent her credences at these respective times in SB' . Let Ch represent prior objective chance, so that $Ch(Heads)$ represents what the objective chance of *Heads* was on Sunday evening, before the coin was tossed. Let M represent the proposition that α occurs on Monday, and let M' represent the proposition that α' occurs on Monday.

Now assume that, in SB , the coin comes up heads. On this assumption, the Posterior Chance-Credence Principle implies:

$$Cr_2(Heads) = Ch(Heads) = \frac{1}{2}$$

And by Conditionalization, together with Information Gain,

$$Cr_2(Heads) = Cr_1(Heads \mid ((M \& Heads) \vee (\neg M \& \neg Heads)))$$

The last two lines entail

$$Cr_1(Heads \mid ((M \& Heads) \vee (\neg M \& \neg Heads))) = \frac{1}{2}$$

Since, in SB , Beauty knows that *Heads* iff $(M \& Heads)$, and since $M \& Heads$ is incompatible with $\neg M \& \neg Heads$, from the above line we can infer Weatherson's P1, namely

$$Cr_1(M \& Heads) = Cr_1(\neg M \& \neg Heads)$$

We can draw this conclusion with full generality, since Outcome Independence allows us to discharge the assumption that the coin came up heads.

Now consider SB' . By the Prior Chance-Credence Principle,

$$Cr'_0(Heads) = Ch(Heads) = \frac{1}{2}$$

And by Reverse Conditionalization, together with Information Loss,

$$Cr'_0(Heads) = Cr'_1(Heads | M)$$

The last two lines entail

$$Cr'_1(Heads | M) = \frac{1}{2}$$

Since Beauty knows, on Monday morning, that if *Heads* then her present awakening is on Monday, we may infer

$$Cr'_1(M \& Heads) = Cr'_1(M \& \neg Heads)$$

Hence, by Time Traveler Independence, we may infer Weatherson's P2, namely

$$Cr_1(M \& Heads) = Cr_1(M \& \neg Heads)$$

P1 and P2 together imply that, upon first awakening, Beauty must have equal credence in each of the following possibilities: $(M \& Heads)$, $(M \& \neg Heads)$ and $(\neg M \& \neg Heads)$. And since these possibilities form a partition, her credence in each must be one-third. Hence, since she knows that $M \& Heads$ is true iff *Heads*, it follows that, when Beauty first awakens in *SB*, her credence in *Heads* must be one-third.

3. How Weatherson's Assumptions Conflict with CA

In order to show that Weatherson's assumptions conflict with CA, I will first consider an arbitrary SSBS, as well as a time traveler variant of this scenario. I will show that, by applying a proper subset⁴ of Weatherson's assumptions to these scenarios, we can derive the very kind of indifference principle that Weatherson sought to avoid. I will then illustrate the conflict between this indifference principle and CA.

It will be useful to introduce the following conventions. In any Sleeping Beauty scenario defined by a partition H , for any h in H , let $N(h)$ be the number of times Beauty awakens if h is true. And for any experimental awakening, α , and any integer, n , let $W(\alpha, n)$ represent the *de re* proposition that α is Beauty's n th experimental awakening.

⁴ Weatherson's first three assumptions are not required, since the argument I will give makes no use of the post-experimental awakening.

We now introduce two scenarios. Let \mathbf{SB} be any arbitrary SSBS defined by a partition H . Let h be an arbitrary hypothesis in H , and let n be an arbitrary positive integer no greater than $N(h)$. Let \mathbf{SB}' be a time traveler variant of \mathbf{SB} in which the protocol followed by Tom is this. If h is true, he films Beauty's n th experimental awakening, and otherwise he films her first experimental awakening. He then returns to the day before her first experimental awakening and shows her the film. Then, specifying the actual values of n and h , he tells her, of the awakening shown in the film, that either it is her n th experimental awakening and h is true, or it is her first experimental awakening and h is false. Then, on the night before her first experimental awakening, he erases her memories from his visit.

Let α and α' represent Beauty's first experimental awakenings in \mathbf{SB} and in \mathbf{SB}' , respectively. Let Cr_1 and Cr_1' represent her credences, at the time of her first experimental awakening, in \mathbf{SB} and in \mathbf{SB}' , respectively. And let Cr_0' represent her credences in \mathbf{SB}' on the preceding day, right after Tom's visit.

Let us assume that, as a matter of fact, h is false, so that, in \mathbf{SB}' , the awakening that Tom films and shows to Beauty on Sunday is her first experimental awakening, α . Thus, the *de re* proposition he tells to Beauty is the disjunction $(h \& W(\alpha', n)) \vee (\neg h \& W(\alpha', 1))$. And so on Sunday evening, Beauty knows that this disjunction is true, and so she knows that h has the same truth value as $h \& W(\alpha', n)$. Consequently,

$$Cr_0'(h) = Cr_0'(h \& W(\alpha', n))$$

And by Reverse Conditionalization, together with Information Loss,

$$Cr_0'(h \& W(\alpha', n)) = Cr_1'(h \& W(\alpha', n) | (h \& W(\alpha', n)) \vee (\neg h \& W(\alpha', 1)))$$

But by the Prior Chance-Credence Principle,

$$Cr_0'(h) = Ch(h)$$

The last three lines clearly entail

$$Cr_1'(h \& W(\alpha', n) | (h \& W(\alpha', n)) \vee (\neg h \& W(\alpha', 1))) = Ch(h)$$

Hence, by Time Traveler Independence,

$$Cr_1(h \& W(\alpha, n) | ((h \& W(\alpha, n)) \vee (\neg h \& W(\alpha, 1)))) = Ch(h)$$

And, by the axioms of probability, this entails⁵

$$Cr_1(h \& W(\alpha, n)) = \frac{Ch(h)Cr_1(\neg h \& W(\alpha, 1))}{1 - Ch(h)}$$

And since h is an arbitrary hypothesis in H , and n is an arbitrary positive integer no greater than $N(h)$, we can generalize this result as follows:

$$\text{For all } h \text{ in } H, \text{ and all } n \leq N(h), \quad Cr_1(h \& W(\alpha, n)) = \frac{Ch(h)Cr_1(\neg h \& W(\alpha, 1))}{1 - Ch(h)}$$

Note that n does not occur on the right hand side of this last equation. Hence, the value of $Cr_1(h \& W(\alpha, n))$ is independent of n . Consequently,

$$\text{For all } h \text{ in } H, \text{ and all } m, n \leq N(h), \quad Cr_1(h \& W(\alpha, m)) = Cr_1(h \& W(\alpha, n))$$

We can draw this conclusion with full generality, since Outcome Independence allows us to discharge the assumption that h is false. Let us call this conclusion the principle of Sleeping Beauty Indifference, which can be stated, in natural language, as follows:

Sleeping Beauty Indifference: In any SSBS defined by a partition H , for any hypothesis h in H , at the time of Beauty's first experimental awakening (call it α), Beauty must have equal credence in each of the possibilities of the form (h is true, and α is my n th experimental awakening) for values of n between 1 and the maximum number of times she would awaken if h is true.

Note that Sleeping Beauty Indifference is very similar to the indifference principle that figures in Elga's (2000) original argument for the one-third solution, and that Weatherson(2005) has so strenuously criticized. Moreover, the main reasons Weatherson gives for rejecting Elga's principle apply equally to Sleeping Beauty Indifference. In particular, the latter principle, like Elga's principle, conflicts with countable additivity. This conflict can be seen if we consider an

⁵ By the ratio analysis, the previous line entails $Cr_1(h \& W(\alpha, n)) / Cr_1(((h \& W(\alpha, n)) \vee (\neg h \& W(\alpha, 1)))) = Ch(h)$.

And by finite additivity, the latter entails $Cr_1(h \& W(\alpha, n)) / (Cr_1(h \& W(\alpha, n)) + Cr_1(\neg h \& W(\alpha, 1))) = Ch(h)$.

Solving for $Cr_1(h \& W(\alpha, n))$, we obtain $Cr_1(h \& W(\alpha, n)) = Ch(h)Cr_1(\neg h \& W(\alpha, 1)) / (1 - Ch(h))$.

SSBS in which H consists of only a single hypothesis, h , where h entails that Beauty will undergo an infinite sequence of awakenings. In this scenario, Sleeping Beauty Indifference implies that Beauty must have equal credence in each of the infinitely many possibilities of the form (h is true and the present awakening is my n th experimental awakening) for positive integer values of n . And so her credences in these possibilities cannot possibly sum to her credence in their disjunction, which is one.⁶

4. How the Defender of Weatherson's Argument Might Respond

In order to resist the conclusion that Weatherson is committed to Sleeping Beauty Indifference, the defender of Weatherson's argument would need to maintain that I have misdescribed the argument's underlying premises. Most plausibly, my opponent could maintain that Weatherson's argument does not require anything so general as the Prior Chance-Credence Principle, as the following, restricted principle would suffice:

Restricted Prior Chance-Credence Principle. In any time traveler variant of an SSBS defined by a partition H , if Tom follows a protocol that ensures that the awakening he shows to Beauty is numerically the same regardless of which hypothesis in H is true, then, right after Tom's visit, Beauty's credences in the hypotheses in H must equal their prior objective chances.

Such a restriction would block my argument, since I require the more general Chance-Credence Principle. But in order to avoid the charge of ad hocery, my opponent would need to justify this restriction, which she might attempt to do as follows. If the truth values of the hypotheses in H can affect which awakening Tom shows to Beauty, then they will also affect which awakening Beauty has *de re* information about. And if Beauty's *de re* information depends on which hypothesis in H is true, then Beauty will have information bearing on H . Hence, to borrow a phrase from Lewis 1980, she will have *inadmissible information*, and so her credences in the hypotheses in H needn't equal their prior objective chances.

But while this move may be *prima facie* plausible, it can't plausibly be made by the defender of Weatherson's argument. For this argument requires that when Beauty awakens on Wednesday in the original Sleeping Beauty scenario, her credence in *Heads* is one-half. And

⁶ This argument has the same structure as Weatherson's argument against Elga's indifference principle in section 5 of Weatherson 2005.

this argument also requires that, on Wednesday, Beauty has *de re* information about the awakening she remembers. However, which awakening Beauty remembers, and hence which awakening she has *de re* information about, will depend on the outcome of the coin toss: if *Heads*, she'll remember her Monday awakening, whereas if *Tails*, she'll remember her Tuesday awakening. Thus, if my opponent maintains that Beauty's credence in hypotheses needn't equal their prior objective chances when their truth values affects her *de re* information, then she cannot maintain that Beauty's credence in *Heads* must equal the objective chance of *Heads* when she awakens on Wednesday in the original Sleeping Beauty problem. Such a move, therefore, would undermine Weatherson's argument.

More generally, Beauty's epistemic situation, in the original Sleeping Beauty scenario on Wednesday morning, is relevantly similar to her situation in a time traveler variant of an SSBS on the eve of her first experimental awakening. In both cases, she knows her temporal location, and she knows that the outcome of the random process under consideration has no internalistically discernible effect on her mental state, but that it may affect her *de re* information. Given this similarity, any ground for denying that Beauty's credence should be aligned with objective chance in the latter case is likely to underline the claim that they should be so aligned in the former.

5. Conclusion

There are two lessons that we may draw from Weatherson's commitment to Sleeping Beauty Indifference. First, it indicates that conflict with countable additivity isn't simply an accidental feature of the traditional arguments for the one-third solution; for such conflict emerges even when a concerted effort is made to avoid it. This provides strong evidence that there is a deep tension between CA and the one-third solution, and hence that there is a deep tension between standard views about probabilistic coherence and standard views about self-locating belief.

Second, Weatherson's commitment to Sleeping Beauty Indifference helps to shed light on the status of indifference principles. Some critics of such principles, including Weatherson, have maintained that they rest on a confusion between uncertainty and risk, or between cases in which one lacks any reason to regard any possibility as more probable than any other, and cases in

which one has positive reason to regard the possibilities as equally probable.⁷ It now appears, however, that the acceptance of indifference principles needn't be based on any such confusion. It may instead be based on a reasoned argument, such as the argument for Sleeping Beauty Indifference given above on the basis of Weatherson's assumptions.

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⁷ See Keynes 1921 and Weatherson 2005 and 2010.