Abstract

A robust adaptive controller is designed for a flexible structure which is modeled as a single rigid body mode. The controller design consists of a first order fixed compensator in an inner loop and an adaptive controller in the outer loop. The purpose of the fixed compensator is to reduce the effect of the unmodelled flexible modes and therefore ease the effort of the adaptive controller. The robustness of the proposed control design with respect to unmodelled flexible modes and input disturbances is established using a rigorous mathematical analysis. Our analysis shows that the closed loop adaptive control scheme is globally stable despite the presence of plant uncertainties, such as unmodelled flexible modes, parameter changes and disturbances.

A simple example consisting of one rigid mode and four flexible modes is used to demonstrate our results. We show using simulations that the proposed robust adaptive controller performs much better than a fixed proportional-derivative (PD) or proportional-integral-derivative (PID) controller. The learning capability of the adaptive control scheme produces a more appropriate input signal, which gives a better tracking and disturbance-rejection performance.

1. Introduction

The availability of the space shuttle to transport large payloads into orbit created considerable interest [1-5] in large flexible space structures (LFSS). Typical LFSS under consideration include the space station, large optical systems for earth surveillance, large antenna reflectors, etc. [7]. One of the important control problems in LFSS such as in the case of the space station is adequate attitude control during normal and transition operations involving docking, berthing, assembling, etc. During the operations, significant configuration changes may take place which may not be accommodated by a fixed controller whose design is based on the original configuration of the structure. A more sophisticated control scheme with the ability of learning about changes within the system on line, and adjusting the controller gains appropriately is therefore needed for stability and acceptable performance. An important class of such learning schemes is called adaptive control.

In this paper we investigate the design, analysis and performance of a robust adaptive controller for attitude control of a LFSS. For clarity of presentation we assume that the LFSS can be modeled as single-input, single-output (SISO) system with one rigid body mode and an arbitrary number of flexible modes. Our methodology, however, can be easily extended to LFSS with multiple inputs and outputs [8]. The robust adaptive control design is divided into two parts. The first part involves the design of a proportional derivative (PD) controller based on the rigid-body mode and the second part involves the design of a robust adaptive controller based on the rigid body mode together with the PD controller. The purpose of the PD controller is to give a crude control action and ease the effort of the adaptive controller. The proposed design is analyzed in the presence of the unmodelled flexible modes and input disturbances and shown to be robust.

A simple example consisting of one rigid body mode and four flexible modes is used to demonstrate our results. Our simulations show that even though stability can be maintained by the PD controller alone, i.e., without the adaptive controller, the performance in the presence of large inertia changes and input disturbances deteriorates considerably. When the adaptive controller is used, however, the performance improves considerably. The learning capability of the adaptive control scheme produces a more appropriate input signal which gives a better tracking and disturbance-rejection performance.

2. Model of LFSS

Let us assume that the dynamics of the LFSS along a single axis can be approximated by a SISO system

\[ y = \frac{1}{\omega_0^2 + \omega_n^2 + 2\omega_n\zeta\omega_0 s} (u + T_d) \]

where \( y \) is the attitude; \( u \) the input torque; \( T_d \) the torque disturbance; \( \omega_0 \) is the modal frequency and damping ratio respectively and \( \zeta, \omega_n > 0 \).

The inertia \( I \) as well as the disturbance torque \( T_d \) are considered to be unknown. The control objective is to find a bounded control input \( u \) so that the closed-loop system is bounded and \( y(t) \) follows a constant command \( r \). Since the number of flexible modes in (1) may be large or even infinity a reduced-order model has to be assumed for practical design purposes. In our analysis we express (1) as

\[ y = \frac{1}{\omega_0^2 + \omega_n^2 s} (u + T_d) \]

where \( \omega_0(s) = \omega_0(s) + \omega_n(s) \) is treated as a plant uncertainty, and make the following assumptions:

(A1) A lower bound \( a \) for \( \omega_0(s) \) such that \( \omega_0(s) \geq a \) is known.

(A2) The real function \( \zeta(s) \) satisfying

\[ ||s\omega_0(s)\omega_0(s) + \omega_n(s)s\omega_n(s)|| \leq \zeta(s) \]

is known.

(A3) The bounds \( l_1, l_2 \) in \( l_1 \leq s \leq l_2 \) are known.

Remark 2.1: It should be noted that assumptions (A2), (A3) are used only for the design and stability of the fixed PD controller in the inner loop. The adaptive control design in the outer loop allows (A2), (A3) to be violated without serious effects on stability.

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3. PD Control Design

Since the flexible modes $G(s)$ have stable poles, we can stabilize (5) by a PD controller, i.e., with the control law

$$ u = K_F r - K_G y $$

(4)

where $r$ is the input command, $K_F$, $K_G$ are controller parameters and $y$, $\dot{y}$ are available for measurement. The stability and robustness properties of the closed-loop system (2)-(4) is given by the following theorem:

**Theorem 1.** Let $K_F$, $K_G$ be chosen to satisfy

$$
\begin{align*}
& (1) \quad K_F(x_G(s))^2 + \frac{1}{s} + K \geq K_G(s) \\
& (2) \quad K(s) = K_F(s)^2 + \frac{1}{s} + K \leq \frac{1}{|s|}
\end{align*}
$$

then the closed loop system can be expressed as

$$ y = [H(s) + \alpha H(s)] r + [uH(s) + \alpha uH(s)] \tilde{y}$$

(5)

where $\alpha = K_F/\alpha_0$ is a second order transfer function and $H(s)$, $\alpha H(s)$ have stable poles. Furthermore, the tracking error $e = r - y$ satisfies

$$ \limsup_{t \to \infty} |e(t)| \leq \epsilon$$

(6)

where $\epsilon_0$ is the upper bound for $|\tilde{y}|$.

**Proof:** The closed-loop system can be expressed as

$$ y = H(s) r + \frac{1}{s} H(s) \tilde{y}$$

(7)

where $H(s) = K_F(s)^2 + \frac{1}{s} + K$, $\alpha H(s) = K_F(s)^2 + \frac{1}{s} + K$ and $\alpha H(s) = \frac{1}{s} H(s)$.

Thus, condition (1) implies that $H(s)$ has stable poles $\alpha H(s)$ also has stable poles, i.e.,

$$ \alpha = K_F(s)^2 + \frac{1}{s} + K \geq 1$$

(8)

Using assumption (2) in section 2, and condition (1) it follows that (8) is true and therefore $\alpha H(s)$ has stable poles.

The tracking error $e = r - y$ can be written as

$$ e = H(s) r + \frac{1}{s} H(s) \tilde{y}$$

(9)

Since $H(s)\tilde{y}$ and $H(s)\frac{1}{s}$ have stable poles and $\lim H(s) \to 1$, the first term in the right hand side of (9) is zero at steady state and hence (6) follows.

**Remark 3.1:** From Theorem 1, we can see that if $K$ is large, then $\epsilon \to 0$, therefore the modeling error $\alpha H(s)$ and the disturbance response due to $\tilde{y}$ are reduced considerably in the closed loop.

**Remark 3.2:** The sensitivity function of $H(s)$ with respect to the variation of the inertia $I$ is

$$ \frac{\partial H(s)}{\partial I} = \frac{1}{2s^2 + K_G H(s) + K} $$

(10)

where $\delta I$ is the change of inertia, that is, for larger $K$, $H(s)$ is less sensitive to $I$ changes and therefore the parameter variation $\delta I$ will have less effect on the output response $y$.

**Remark 3.3:** If the PID controller is used instead of PD controller, i.e.,

$$ u = K_F r - \frac{K_G}{\alpha_0} \left[ \left( r - \frac{1}{2} K_G y \right) - \frac{1}{2} K_G y \right] $$

then zero steady state tracking error can be achieved for constant $r$ and constant disturbance $\tilde{y}$. The analysis of the PID controller can be carried out in a similar way as in the case of the PD controller.

4. Adaptive Control Scheme

The PD and PID controllers presented in section 3 may be insufficient to meet the stability and performance specifications for the following reasons: (1) Larger $K$ results in a larger bandwidth and therefore it may amplify the input high frequency noise and may introduce oscillations in the transient response. (2) Larger $K$ may give a larger input torque $u$ which may exceed the saturation limits of the actuators and (3) the conditions (1), (2) in Theorem 1 for choosing $K$ and $K_G$ may not be satisfied if there is large change in $I$. In this case, the PD controller may even fail to stabilize the system.

To compensate for these limitations of the PD, PID controllers an outer loop consisting of a robust adaptive controller is introduced.

The design of the adaptive controller is based on the model (5) with $u = 0$. The PD controller is kept in the inner loop as a part of the system in order to achieve a crude control action and ease the effort of the adaptive controller. The adaptive control law is chosen as

$$ u' = \theta u' + \epsilon r $$

(12)

where $u'$ is the control signal applied to the system (5) with the PD controller, $\theta = \theta_0, \theta_1, \theta_2, \theta_3, \theta_4$, $u = [u'_1, u'_2, u'_3, u'_4, u'_5]$, and $f(s) = y, y'$ and $f(s)$ is a first order stable transfer function with relative degree 1. The robust adaptive law used to update the parameters $\theta$, $\epsilon$ is proposed by Ioannou and Savakis [10], and is summarized below:

$$
\begin{align*}
\dot{\theta}_1(t) &= \gamma_1 (c_1(t) - c_2) \\
\dot{\theta}_2(t) &= \gamma_2 (c_1(t) y(t) - c_3) \\
\dot{\theta}_3(t) &= \gamma_3 (c_1(t) y(t) - c_4) \\
\dot{\theta}_4(t) &= \gamma_4 (c_1(t) y(t) - c_5) \\
\dot{\theta}_5(t) &= \gamma_5 (c_1(t) y(t) - c_6)
\end{align*}
$$

(13)

where $\gamma_i$ are positive constants with $\gamma_i \in [0, 1]$, $c_i$ is the reference signal and it is given by $c_i = \bar{c}_i \cdot \bar{c}_i$, $c_i$ is a stable transfer function with relative degree 2 and it describes a desired input-output relationship of the desired system (for the tracking purpose, we have $c_i(t) = 0$). For the sake of simplicity, we choose $c_i(t) = c_i$ with $c_i > 0$ and $c_i$ is a Hurwitz polynomial. $\alpha$ in (13), (14) and (15) is given by

$$
\alpha(t) = \begin{cases}
0 & \text{if } \alpha(t-1) \\
\alpha(t-1) + \alpha_0 & \text{if } \alpha(t-1) < \alpha_0
\end{cases}
$$

(14)

where $\alpha = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5]$ and $\alpha_0$ is fixed constants and $\alpha_0$ is the desired controller parameter vector which can be calculated provided that the inertia $I$ is known exactly.

If the adaptive algorithm (13)-(18) is applied to (5), i.e., the ISS with the PD controller, then the following theorem gives the global stability properties of the adaptive control system:

$$ u = K_F r - \frac{K_G}{\alpha_0} \left[ \left( r - \frac{1}{2} K_G y \right) - \frac{1}{2} K_G y \right] $$

(11)
Theorem 2: Choose $f(s)$, $W(s)$ such that $f(s)W(s) \geq 0$ have stable poles. Then there exists a $\mu > 0$ such that for $u \in C_0^2$, all the signals in the closed-loop system are bounded for any bounded initial conditions. Furthermore, the tracking error $\varepsilon(t) = y(t) - y_d(t)$ is driven to the following residual set:

$$\begin{align*}
\mathcal{D}_\mu &= \left\{ \gamma \leq 0, \quad \gamma(t) \text{ is bounded on } [T, T+\tau] \right\}
\end{align*}$$

(19)

where $\gamma$ is a positive constant, and $\epsilon_d$ is the upper bound on the bounded disturbance $D_T$.

The proof of Theorem 2 is given in Appendix A.

Remark 2: The choice of $W(s)$ is arbitrary as long as the conditions are satisfied.

5. Simulation Results

The control design procedure proposed in section 4, 5 and 6 has been applied to a LFSW which has these transfer function

$$\begin{align*}
g(s) &= \frac{\frac{1}{2} \omega_d \omega_s}{s^2 + \omega_d^2 s + \omega_s^2} 
\end{align*}$$

(20)

The parameters take the values $\omega_d = 0.5$, $\omega_s = 1.2$ rad/sec, $\omega_{id} = 0.05$, $c_1 = 0.02$, $c_2 = 0.008$, $c_3 = 0.004$, $c_4 = 0.007$, $c_5 = 0.002$, $c_6 = 10^{-2}$, $c_7 = 10^{-2}$.

A PD, PID and an adaptive controller are designed for (20) and simulated on the computer. The performance of these different control strategies is evaluated and compared in the presence of input torque disturbance, disturbances and unmodeled flexible modes. Figure 1 gives the disturbance change and input torque disturbance as functions of time. Figure 2 shows the output response for the plant (20) with PID, PID and adaptive controller when the system is subjected to the perturbation given by Fig. 1. It is clear from the figure that the adaptive controller produces a much better performance and it is able to accommodate the disturbance and inertia changes. The adaptive controller is a much smaller model and smoother. The learning capability of the adaptive scheme provides a more appropriate input signal which results in a better performance.

For these simulations, we chose $K = 8100$. We observed that reducing the value of $K$ worsened the output response in the presence of disturbance torque and inertia changes, especially for the PID and PD controller. When we increase $K$, some undesired oscillations can be observed in the case of the PD and PID controller. The performance of the adaptive controller when $K = 10011.2$ is much affected. The simulation results demonstrate the effectiveness of the gain $K$ in the inner loop are not included due to lack of space.

6. Conclusion

The application of robust adaptive control to LFSW, LTFSW is studied by using analysis and computer simulation. We first show that the proposed robust control design is globally stable in the presence of inertia changes, input disturbances and unmodeled flexible modes. We then use simulation results to demonstrate that the adaptive control design performs much better than the fixed PD or PID control design in terms of the output response and input torque.
where $T_1, T_2$ are bounded signals and they are proportional to $t^a, \tau_1, 1 = \tau_2$ by $m$. System (A.9), (A.10) together with (13) can be realized by the following minimal state-space representation [14]:

$$
\dot{x} = Ax + f(s, t) + e(t)
$$

$$
u = C_1 x + T_1 + y = C_2 u + T_2
$$

where $A = \text{diag}(A_1, A_2)$, $C = \text{diag}(c_1, c_2)$, $\dot{e}_m = b_1 e_1 + b_2 e_2$, for some constant vectors $b_1, b_2, c_1, c_2$ and

$$
I(s, t) = \left[ e_1(s, t) + \frac{\dot{e}_m(s, t) + \dot{e}_m(0)}{2} \right] T
$$

where $\sigma = \left[ \left( 1 + s^2 \right) \sigma_1^2, \left( 1 + s^2 \right)^2 \sigma_2^2 \right]$, $Z = \text{diag}(Z_1, Z_2, Z_3)$, $s = s_1, s_2, s_3$ and $e_1, e_2, e_3$ are constants proportional to $e_0$.

Let us assume that $m = 1$ is bounded, then for any $1 < c < 0$, there will be a time $t_0$ such that $m(t_0) > 1/c$.

Since $\sigma = \sigma_0(t), m(t_0) > 1/c$ implies that $m(t) > 1/c_0$ for $c_0 = 0$.

Therefore, $m(t) > 1/c_0$. Then $m(t_0) > 1/c_0$ for $c_0 = 0$.

By definition, for $c_1, c_2, c_3, 

$$
\int_{t_0}^{T} e_1^2(s, t) \leq c_1 \int_{t_0}^{T} e_2^2(s, t) \leq c_2 \int_{t_0}^{T} e_3^2(s, t)
$$

for some constants $c_1$, $c_2$, $c_3$. And any $T > t_0$.

The expression for the tracking error (A.19) follows.

$$
\int_{t_0}^{T} e_1^2(s, t) \leq c_1 \int_{t_0}^{T} e_2^2(s, t) \leq c_2 \int_{t_0}^{T} e_3^2(s, t)
$$

for some constant $c_1, c_2, c_3$.

References