ROBUST REDESIGN OF ADAPTIVE CONTROL IN THE PRESENCE OF DISTURBANCES AND UNMODELED DYNAMICS

Petros Ioannou
University of Southern California
Department of Electrical Engineering-Systems
Los Angeles, CA 90089-0781

Abstract
The effects of unmodeled weakly observable stable dynamics and bounded disturbances on the stability and performance of adaptive control schemes are analyzed. A second order example is used to illustrate the non-robust behavior of the present adaptive controllers when disturbances and/or unmodeled dynamics are present. An earlier modified adaptive law [1,2] is shown to guarantee the existence of a region of attraction for boundedness. A new adaptive controller has been introduced which guarantees boundedness for bounded initial conditions.

Introduction
Recently several attempts [1-14] have been made to analyze the stability properties of several adaptive control schemes in the presence of modeling errors and/or bounded disturbances. It has been shown that bounded disturbances [3,4,14] or unmodeled stable dynamics [1,2,7,10-13] can make the closed-loop system unstable. The conclusion of these studies is that the present adaptive control schemes need to be redesigned or modified for robustness.

Several modified adaptive laws have been introduced to counteract the effects of disturbances. In [3,4,6] a dead zone is introduced in the adaptive laws, that is, adaptation is stopped when the output error becomes smaller than a computed bound. This modification guarantees boundedness of all the signals in the closed loop but can lead to large output errors if the size of the dead zone, which depends on the unknown disturbances and plant parameters, is overestimated. Alternative approaches which retain the potential of obtaining small output errors in the limit when the disturbances are small are taken in [3,5]. These approaches, however, require the knowledge of an upper bound for the desired constant controller parameter vector.

A modified adaptive law which guarantees robustness with respect to unmodeled fast dynamics and/or disturbances is introduced in [1,2]. This is a linear modification and guarantees the existence of a region of attraction from which all signals converge to a small residual set. In [14] a boundedness result is obtained for an indirect adaptive control scheme under the assumption that the difference between the actual plant and the ideal plant is linearly dominated by the information vector. The modification consists of a dead zone, by which the adaptation is switched off whenever some signals become smaller than a given number. Local stability has also been proved for a reduced-order indirect adaptive regulator in [12].

In this paper we examine the effects of unmodeled weakly observable stable dynamics and disturbances on the stability properties of a continuous time direct adaptive control scheme. We first use a second order example to demonstrate the instability phenomena which can arise when disturbances and/or unmodeled weakly observable dynamics are present. We then use the p-modification, first introduced in [1,2,15,16] to handle unmodeled fast dynamics and unmodeled interconnections, to obtain sufficient conditions for boundedness in the presence of disturbances and/or unmodeled weakly observable dynamics. This modified adaptive law guarantees the existence of a region of attraction from which all signals converge to a small residual set. Furthermore, we introduce a new adaptive controller which guarantees boundedness for any bounded initial conditions.

1. A Scalar Adaptive Control Problem
We start with a simple second order plant with a uniformly bounded input disturbance d(t).

\[ u(t) \rightarrow \begin{bmatrix} \sigma \xi_1 \\ \sigma_2^2 \end{bmatrix} \rightarrow y(t) \]

where \( \sigma^2, \sigma_1 \geq 0 \) and \( \sigma_2^2 \) are unknown constants. We assume that \( u^2, \sigma_1 \) is small, that is, the stable mode is weakly observable. The output of the plant is required to track the state \( y_0 \) of a first order system.

\[ y_0 = -\alpha \xi_1 + r(t) \quad \alpha > 0 \quad (1.1) \]

where \( r(t) \) is a reference input, a uniformly bounded function of time. This example illustrates some of the stability problems arising in adaptive control when disturbances or unmodeled dynamics are present and serves as a motivation for an introduction to the general methodology to be developed in the next section.
The state representation for the second order plant is
\begin{align}
\dot{x} &= c_2 x + y z + d \\
\dot{z} &= c_2 x + y z + d \\
y &= x
\end{align}
(1.2) (1.3) (1.4)

A simplified model for the plant (1.2) to (1.4) is obtained by assuming zero disturbances (d=0) and exact zero-pole cancellation (c=0), i.e.,
\begin{align}
x &= c_2 x + y \\
y &= x
\end{align}
(1.5) (1.6)

For the simplified model (1.5), (1.6) the adaptive controller
\begin{align}
u &= K(t)y + v(t)
\end{align}
(1.7)

\begin{align}
\dot{K} &= \lambda \delta y_x \quad \lambda > 0
\end{align}
(1.8)

guarantees the following properties.

Lemma 1: For any bounded initial condition \((x_0, y_0, z_0)\), all the signals of the closed loop system (1.5) to (1.8) are bounded and \(\lim t \to \infty e(t) = 0\).

The equations to be answered in this paper are the following: Now will the adaptive controller (1.7), (1.8) designed for the simplified plant (1.5), (1.6) behave when applied to the actual plant (1.2) to (1.4) with disturbances and/or weakly observable unmodeled dynamics? Will the properties of Lemma 1 be preserved for small disturbances and small \(c_2\)? Which modification of the adaptive law would help to preserve some of the desirable properties?

Let us consider the effect of disturbance \(d(t)\) on the adaptive controller (1.7), (1.8) when applied to the plant (1.2) to (1.4), with \(c = 0\). The error equations for (1.5) to (1.8) are
\begin{align}
\dot{e} &= -x - y (k - c_2) y x \\
\dot{K} &= \lambda \delta y_x
\end{align}
(1.9) (1.10)

where \(K = c_2 + y_x\). In the disturbance-free case, \(d = 0\), we can use the Lyapunov function
\begin{align}
V(x, e) = \frac{1}{2} e^T (K - K_0) e
\end{align}
(1.11)

that the equilibrium \(e = 0, K = K_0\) is stable and \(\lim t \to \infty e(t) = 0\) for any uniformly bounded bounded reference input signal \(r(t)\). If the disturbance \(d(t)\) is not zero, the derivative of \(V\) satisfies
\begin{align}
\dot{V}(x, e) = \delta e - \delta e (K - c_2 z) y + d
\end{align}
(1.12)

Thus \(e(t)\) is bounded and there exist positive constants \(c\) and \(\eta\) such that
\begin{align}
|e(t)| \leq \sup_{t \geq T} |e(t)|, \quad \forall t \geq T
\end{align}
(1.13)

However, this does not guarantee that \(K(t)\) is bounded. For example, take
\begin{align}
\dot{e} &= \frac{1}{\gamma} (z + y)^2 \quad \text{and} \\
\dot{y} &= \gamma (y - e)^2
\end{align}
(1.14)

where \(\gamma > 0\) and \(b\) are some constants. When this bounded disturbance, which decays to zero, is present in the regulation case \((x(t) = 0, y(t) = 0)\) the output is still regulated
\begin{align}
y(t) = \frac{1}{\gamma b} (e(t) + e)^2 = 0, \quad \text{as} \quad t \to \infty
\end{align}
(1.15)

but the adaptive controller is nonrobust because \(K(t) = (\gamma b)^{1/2} x\) as \(t \to \infty\). (1.16)

Similar instability phenomena can be observed for \(r(t) \neq 0, y \neq 0\) as well as for the case of output disturbances.

In the presence of disturbances as well as weakly observable unmodeled dynamics, the equations describing the stability properties of the adaptive controller (1.7), (1.8) are
\begin{align}
\dot{e} &= -x - y (K - c_2) y x + d \\
\dot{z} &= -c_2 x (y x) + d \\
\dot{K} &= \lambda \delta y_x
\end{align}
(1.17) (1.18) (1.19)

when both signal \(r\) and disturbance \(d\) are constant and \(y x = r\) the equilibrium
\begin{align}
\dot{e} = 0, K = c_2 y + c_2 y x (r - d)
\end{align}
(1.20)

\begin{align}
\dot{x} = \frac{2 c_2 d + 2 c_2 z}{c_2} - z c_2 z
\end{align}
(1.21)

is unstable if \(r > \frac{2 c_2 d + 2 c_2 z}{c_2} - c_2 z\). For local asymptotic stability, it is sufficient that the disturbance-to-signal ratio and the perturbation parameter \(c_2\) be small. Because of the disturbances and the unmodeled dynamics, the adaptive system (1.17) to (1.19) may not converge to or may not even possess an equilibrium for general bounded reference input signals. A practical goal is then to guarantee some boundedness properties. We first show that the \(r\)-modification introduced in (1.12) guarantees the existence of a region of attraction for boundedness. We then introduced a new adaptive controller which guarantees boundedness for any bounded initial conditions provided that \(c_2\) is small.

Theorem 1: Let the reference input \(r(t)\), state \(y_x(t)\) and disturbance \(d(t)\) satisfy
\[ |r(t)| < r_1, \quad |y(t)| < r_2, \quad |d(t)| < d_0 \quad (1.24) \]

where \( r_1, r_2 \) and \( d_0 \) are finite positive constants. Then there exist positive constants \( t_1, \gamma_1, \sigma, \alpha, \beta, d_1, d_2 \) such that for all \( \|e(t, x, z)\| \leq 0 \) every solution of (1.21) to (1.23) starting at \( t = 0 \) from the set

\[ D = \{ e(x, z) : |e|_c \leq d_2, |z| \leq d_2 \} \]

enters the residual set

\[ D_0 = \{ e(x, z) : \frac{\gamma_1}{d_0} < |z| < \frac{1}{d_1} \} \]

\[ \leq \frac{\gamma_1}{d_0} < |z| < \frac{1}{d_1} \quad (1.25) \]

at \( t = t_1 \) and remains in \( D_0 \) for all \( t > t_1 \).

Proof. Choosing the function

\[ V(e, z, x) = \frac{1}{2} \sum_{i=1}^{n} (x_i)^2 + |z|^2 \quad (1.26) \]

we can see that for each \( e \), \( z \geq 0 \), \( x \neq 0 \) the equality

\[ V(e, z, x) = 0 \quad \text{if} \quad e = 0 \quad (1.27) \]

defines a closed surface \( S(e, z, c) \in \mathbb{R}^2 \) space. The derivative of \( V(e, z, x) \) along the solution of (1.21) to (1.23) is

\[ \dot{V}(e, z, x) = -\alpha |e|^2 - \sigma |z|^2 - |x|^2 \quad (1.28) \]

and can be rewritten as

\[ \dot{V}(e, z, x) = -\alpha |e|^2 - \sigma |z|^2 - \frac{1}{2} |x|^2 \quad (1.29) \]

\[ -\frac{1}{2} |x|^2 + \frac{1}{2} |e|^2 + \frac{1}{2} |z|^2 \quad (1.30) \]

Inside \( S(e, z, c) \) quantities \( |e|, |z| \) can grow up to \( O(|e|^2) \), whereas \( |x| \) can grow up to \( O(|e|^2) \). Therefore there exists constants \( b_1, b_2, b_3 \) such that

\[ |e| \leq b_1, \quad |z| \leq b_2, \quad |x| \leq b_3 \quad (1.31) \]

for all \( e, z, x \) inside \( S(e, z, c) \). Hence (1.30) becomes

\[ \dot{V}(e, z, x) = -\alpha |e|^2 - \sigma |z|^2 - \frac{1}{2} |x|^2 \]

\[ -\frac{1}{2} |x|^2 + \frac{1}{2} |e|^2 + \frac{1}{2} |z|^2 \quad (1.32) \]

for all \( e, z, x \) inside \( S(e, z, c) \). Choosing \( a_1, \alpha, \gamma, \delta \), it can be shown that there exists constants \( \varepsilon > 0 \) and \( \gamma > 0 \) such that for

\[ \sigma > \gamma_1 \]

and each \( e \) \( \in \{e \in D_1 : \gamma_1 \} \quad (1.33) \) can be written as

\[ \dot{V}(e, z, x) = -\frac{\alpha}{\varepsilon} |e|^2 - \frac{\sigma}{\varepsilon} |z|^2 \quad (1.34) \]

\[ -\frac{1}{2} |x|^2 + \frac{1}{2} |e|^2 + \frac{1}{2} |z|^2 \quad (1.35) \]

It is clear that outside \( D \) and inside \( S(e, z, c) \) \( V \) is strictly decreasing. Hence there exists positive constants \( c_1, c_2, c_3 \) to \( e \) and \( z \) such that

\[ \text{for each} \ e \in \{e \in D_1 : \gamma_1 \} \]

\[ \text{inside} \ S(e, z, c), \quad D \text{is enclosed by} \ D_0 \text{and any solution starting from} D_0 \text{remains inside} S(e, z, c). \]

Since inside \( D/D_0 \), every solution starting at \( t = 0 \) from \( D_0 \), will enter \( D_0 \) at some finite time \( t \leq \tau_1 \) and remain in \( D_0 \) for all \( t > \tau_1 \). Since \( D_0 \) is uniformly bounded, the solution \( e(t), z(t), x(t) \) is bounded for any initial condition in \( D \).

Remark 1. As \( e = 0 \), domain \( D \) becomes the whole space, that is, the adaptive control problem (1.21) to (1.23) is well posed with respect to the unmodeled dynamics.

Remark 2. From (1.26) it is clear that the size of \( D_0 \) depends on the disturbance \( d(t) \), reference input \( \psi(t) \) and the design parameter \( \gamma \). Given \( e, z \), a sufficient increase in \( r_1 \) and \( r_2 \) can no longer guarantee the property that \( \varepsilon > 0 \) everywhere in \( D_0 \). For this reason, our formulation excludes high amplitude reference input signals or disturbances.

Remark 3. For small \( \varepsilon \), the design parameter \( \sigma \) can be small and therefore its contribution to the size of \( D_0 \) is small. However, in the disturbance free case and when \( e = 0, \gamma = 0 \) causes an output error of \( 0(\varepsilon) \). This is a trade-off between boundedness in the presence of disturbances and/or unmodeled dynamics and the loss of exact convergence of the output error to zero in the absence of uncertainties.

b. New Adaptive Controller

Instead of the adaptive controller (1.7), (1.8), we propose the controller

\[ u(t) = K(t) - x(t) \rightarrow \text{controller} (1.35) \]

\[ K = \text{vector} \quad (1.36) \]

\[ \zeta = a \varepsilon + c(t) = 0 \quad (1.37) \]

where \( a, c \) are design parameters to be selected. The stability properties of (1.25) to (1.27) when applied to the actual plant (1.2) to (1.4) are described by

\[ \dot{z} = -a_2 z + (\alpha + \gamma) \zeta e + \sigma z \quad (1.38) \]

\[ \dot{\delta} = -a_2 (\alpha + \gamma) e + \sigma z \quad (1.39) \]

where \( \zeta = \frac{\partial V}{\partial e} \) is the differential operator, \( a_2 = (\alpha + \gamma) e \) and \( f(t) = -K(t) \text{gradient} \).
Theorem 2. There exists an $e > 0$ such that for each $\varepsilon > 0$, the solution $e(t), x(t), y(t)$ of (1.38) to (1.40) is bounded for any bounded initial condition. Furthermore, the solution enters the set

$$
D_y = \left( e, z \right) := \left\{ \left( e, z \right) \in \mathbb{R}^n \times \mathbb{R}^n \mid \sum_{i=1}^{n} \left| z_i \right|^2 \leq \frac{\varepsilon}{\alpha} \left( \frac{e}{\left| e \right|} \right)^2 \right\}
$$

in finite time.

Proof. Choose the positive definite function

$$
V(e, z) = \frac{1}{2} \left( e, x \right)^2 + \frac{1}{2} \left( z, \overline{x} \right)^2 + \frac{1}{2} \left( e, z \right)^2
$$

Then, along the solution of (1.38) to (1.40) we have

$$
\dot{V}(e, z) = -\frac{1}{2} \left( e, x \right)^2 - \left( e, z \right)^2 - \frac{1}{2} \left( z, \overline{x} \right)^2 + \frac{1}{2} \left( e, z \right)^2
$$

Completing the squares we can write (1.43) as

$$
\dot{V}(e, z) = -\frac{1}{2} \left( e, x \right)^2 - \frac{1}{2} \left( z, \overline{x} \right)^2 + \frac{1}{2} \left( e, z \right)^2
$$

Then there exists a $c^*$, where

$$
c^* = \min \left\{ c \left| \frac{1}{2} \left( e, x \right)^2, \frac{1}{2} \left( z, \overline{x} \right)^2 \right| \right\}
$$

such that for each $\varepsilon > 0$, we have

$$
\dot{V}(e, z) \leq c^* \min \left\{ e^2, |e|, |z|^2 \right\}
$$

Clearly outside $D_y$, $V < 0$ and therefore for any bounded initial condition, the solution $e(t), x(t), y(t)$ is bounded and enters $D_y$ in finite time.

Remark 4. The new adaptive controller guarantees global boundedness at the expense of increasing slightly the complexity of the reduced-order controller. The value of $c^*$ in this case can be arbitrarily small in contrast to the case of the linearization with $\alpha > 0$.

Remark 5. We note that the condition of disturbance boundedness and unmodeled dynamics is the new adaptive controller guarantees global stability and $\lim e(t) = 0$ provided $\alpha > 0$.

II. Adaptive Control with Unmodeled Disturbances

We now consider the general problem of adaptive control of a SISO time-invariant plant of order $n$ in the order of the weakly observable stable dynamics and $m$ is the order of the plant to be controlled.

The plant is assumed to have the following state representation

$$
\begin{align*}
\dot{x} &= Ax + Bu + c_d \delta(t) \\
\dot{y} &= Fx + w(t) + D_2 \\
y &= Cx + y(t)
\end{align*}
$$

where $x(t) \in \mathbb{R}^n$, $\delta(t) \in \mathbb{R}^m$, $F$ is a stable matrix, $u$ is a small positive scalar and $D_1, D_2$ are bounded vector disturbances. Such a representation can be obtained from the transfer function of the plant in a similar manner as in Section I.

In (2.3) we assume that the output $y$ does not depend on disturbances explicitly...
where $\mathbf{A}$ is an $(n-1)\times(n-1)$ stable matrix and $(x,y)$ is a controllable pair.

a. The \textit{a}-Modification for the General Problem

The control input is given by

$$u = -y_k(t)u(t)$$

(2.12)

where $v(t) = \left[v_1(t), v_2(t), (v_1(t), v_2(t))\right]$ and

$\xi(t) = \left[x(t), C(t), d(t), \mu(t)\right]$. The parameter vector $\theta(t)$ is updated using the adaptive law

$$\dot{\theta} = -\mu_\pi \eta(t) \theta(t)$$

(2.13)

It can be shown [17] that a constant vector $\theta$ exists such that for $\theta = \theta^*$ the transfer function of the simplified plant (2.4) together with the controller (2.5) to (2.12) matches that of the reference model given by (2.27).

Defining $\mathbf{Y} = \left[y_1(t), y_2(t)\right]$ and using $\theta^*$, the closed-loop system becomes

$$\dot{y} = \mathbf{A}_{y}\mathbf{y} + \mathbf{B}_y u + \mathbf{D}_y$$

(2.14)

For $\theta = \theta^*$, $\theta = 0$ and $\mathbf{D}_y = 0$, (2.14) is a non-minimal representation of the reference model

$$\mathbf{A}_{y} = \mathbf{A}_{y} \theta^* + \mathbf{B}_y$$

(2.16)

The equations for the error $e = y - y^*$ can be expressed as

$$\dot{e} = \mathbf{A}_{e} e + \mathbf{B}_e u + \mathbf{D}_e$$

(2.17)

$$\mathbf{A}_{e} = \mathbf{A}_{y} - \mathbf{A}_{y} \theta^*$$

(2.18)

$$\mathbf{B}_e = \mathbf{B}_y$$

(2.19)

where $\mathbf{B}_y = \left[1, \ldots, 0\right]$ and $f(t)$ is a vector.

The equations (2.13) and (2.17) to (2.19) describe the stability properties of the adaptive control scheme in the presence of disturbances and unmodelled dynamics. For $e = 0, \mathbf{D}_e = 0$, we can show that the solution $e(t)$ is bounded for any bounded initial condition and $\lim_{t \to \infty} e(t) = 0$. When $\mathbf{D}_e \neq 0$, the following theorem gives sufficient conditions for boundedness.

Theorem 3. There exist positive constants $P_1, P_2, \ldots, P_n$ and $P_{n+1}$ to $P_{2n}$ such that for each $e(t)$ every solution of (2.13), (2.17) to (2.19) which starts from $\mathbf{D}_e = (e(t), x(t), y(t))$ enters the residual set

$$\mathbf{D}_e = (e(t), x(t), y(t)) \subseteq \mathbf{D}_e$$

(2.20)

at $t = T$, and remains in $\mathbf{D}_e$ for all $T \geq t$. In (2.21), $P_{n+1}$ to $P_{2n}$ are positive finite constants and are such that $\mathbf{D}_e$ is enclosed by $\sup_{t \geq 0} \mathbf{D}_e$ and every solution $e(t), x(t), y(t)$ which starts from $\mathbf{D}_e$ remains inside $\mathbf{D}_e$. Since $\mathbf{D}_e$ is not empty, there exists an $e^*$ and constants $P_1$ to $P_2$ such that for each $e(t)$ and $e^*$

$$V(e(t)) \leq \frac{1}{2} \mathbf{D}_e^2 - \left[\frac{1}{2} \mathbf{D}_e^2 - \frac{1}{2} \mathbf{D}_e^2 \right]$$

(2.22)

where $\mathbf{D}_e^2$ is bounded for all $e \geq 0$.
D_0 every solution which starts from D_0 will enter D_0 in finite time T>0. Once in D_0, it cannot escape and remain there for all T>0.

b. New Adaptive Controller for the General Case

The control input is chosen as

\[ u(t) = \beta(t)w(t) + \beta(t)\sigma(t) \]  

(2.30)

where

\[ \dot{\beta} = -\tau_1 D_0 \beta \]  

(2.31)

and \( \sigma \) is a design parameter to be chosen. The parameters are updated as

\[ \dot{\beta} = -\gamma_1 \beta - \gamma_2 \beta \]  

(2.32)

using the same procedure as in (a) of this section. We can show that the equations for the error can be expressed as

\[ \dot{e} = A_2 e + b_2 (p_1) e' + \eta_2 e + \xi_1 \]  

(2.33)

and

\[ \dot{e_1} = -\gamma_2 e_1 \]  

(2.34)

where \( \eta_2 = \eta_2(\eta) \) and \( p \) is the differential operator \( \frac{d}{dt} \).

Theorem 4. There exists an \( c > 0 \) such that for each \( \beta(0), \sigma(0) \) and

\[ \|E_1(\beta(t)) \| \leq c \|E_1(\beta(0)) \| \]  

(2.36)

the solution \( e(t), \sigma(t), \beta(t) \) of (2.32) to (2.35) is bounded for any bounded initial condition. Furthermore, there exists a finite time \( T > 0 \) such that for all \( t > T \) the solution \( e(t), \sigma(t), \beta(t) \) is inside the set \( D_0 \) given by

\[ D_0 = \{ e, \sigma : \frac{1}{\alpha_1} \leq e_1 \leq \frac{1}{\alpha_2}, \sigma \leq 2 \} \]  

(2.38)

where \( \alpha_1, \alpha_2 \) are positive finite constants.

Proof. Choose the positive definite function

\[ V(e, \sigma) = \frac{1}{2} \sigma \sigma^T + \frac{1}{2} e_1 e_1 \]  

(2.39)

where \( \sigma \) and \( e_1 \) are the same matrices as in (2.23) to (2.25).

Along the solution of (2.32) to (2.35)

\[ \dot{V}(e, \sigma) = \frac{1}{2} \sigma \sigma^T + \frac{1}{2} e_1 e_1 \]  

(2.39)

choosing

\[ c = \min \left\{ \frac{\sigma_1}{\alpha_1}, \frac{\sigma_2}{\alpha_2}, \frac{\sigma_3}{\alpha_3}, \frac{\sigma_4}{\alpha_4}, \frac{\sigma_5}{\alpha_5}, \frac{\sigma_6}{\alpha_6} \right\} \]  

(2.40)

where \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 \) are the eigenvalues of the matrices \( D_0 \).

Since \( D_0 \) is uniformly bounded and \( \forall B \) outside \( D_0 \), then every solution of (2.32) to (2.36) with a bounded initial condition will be bounded and will enter \( D_0 \) in some finite time \( T > 0 \). Once in \( D_0 \), it cannot escape but will remain there for all \( t > T \).

Remark 5. Theorem 4 requires that the design parameter \( \gamma \) has to satisfy (2.36) for boundedness. An overestimated large value of \( \gamma \) results in a smaller \( c \), i.e., to a smaller set of allowable unmodelled dynamics.

Remark 6. A similar analysis can be used to find bounds for the parameter \( \alpha \) for boundedness.

Conclusion

In this paper, we analyzed the stability properties of adaptive control schemes with respect to bounded disturbances and model-plant mismatch caused by unmodelled weakly observable dynamics. We showed that the adaptive controller with the \( \eta \)-modification guarantees the existence of a region of attraction for boundedness. We introduced a new adaptive controller which guarantees boundedness for any bounded initial condition provided some design parameters are chosen properly. A further investigation of this new controller and the extension of these results to more general adaptive schemes is a topic for future research.

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References


