Control Techniques for a Large Segmented Reflector

K. Li, E. B. Kosmatopoulos, P. A. Ioannou

Dept. of Electrical Engineering-Systems, University of Southern California, Los Angeles, CA 90089-2563

H. Boussalis, M. Mirmirani

School of Engineering, California State University, Los Angeles, CA 90032

A. Chassiakos

School of Engineering, California State University, Long Beach, CA 90840-5602

I Introduction

The performance of astronomical systems is directly related to the size of their reflectors. Since it is very difficult to cast mirrors larger than 7 meters in diameter from a single piece of glass, to reach unprecedented performance levels, it is planned to construct future optical systems with diameters in the 7 – 12 meters range from arrays of mirror segments. The problem with segmented optics is to make them behave like the conventional ones made from a single piece of glass; optical performance requires positioning the mirror segments to a fraction of the wavelength of light. No support structure can provide the mechanical rigidity needed to maintain the position of the mirror segments to such an accuracy. To compensate for the mechanical imperfections, the deformations due to the gravity and thermal loads and to attenuate the seismic or maneuver induced vibrations, the mirror segments must therefore be actively controlled.

To study the complex dynamic behavior of large segmented optical systems, NASA has founded a five year project to design and construct a test-bed in the Controls and Structures Laboratory (CSRL) at the California State University, Los Angeles. The CSRL test-bed will serve as a generic experimental facility capable of performing experiments that simulate the complex dynamic behavior of a large segmented optical system. It will be used as an experimental facility for addressing in an integrated way, problems associated with structural dynamics, control of Multi-Input Multi-Output (MIMO) systems, optics, electronics, actuators, and sensor design.

In this paper we describe our latest results regarding robust control, decentralized control, adaptive control and neural network control of the CSRL structure.

II Description of the Structure

The CSRL structure emulates a f/2.66 Cassegrain telescope. It consists of a primary mirror, a secondary mirror, a supporting light-weight flexible truss structure, and an isolation platform. The active optical elements are the primary mirror segments and the secondary mirror which interact dynamically with the actuators, sensors and the supporting truss structure in an integrated way. The primary mirror is a 2.66 meters diameter dish. The major components of the CSRL test-bed are discussed below:

Truss Structure. One of the most fundamental design goals has been a strong, light-weight truss structure whose structural-dynamic characteristics are representative of a large, flexible space borne systems. These include low frequency modes, high-modal density and global mode shapes that properly reflect the coupling of the sub-elements of the structure. A careful trade-off between the need for the structure to support itself in the 1 – g environment versus the need to keep the frequency of the first mode as low as possible was achieved using multi-criteria optimization techniques and Pareto optimality concept. The truss is made of thinwall stainless steel tubing in a unique geometric configuration to achieve highest strength with lowest mass.

Primary Mirror. The CSRL primary mirror is designed to emulate the critical properties of a real segmented mirror. These properties include segmentation geometry, inter-segment spacing, segment mass, inertia, and stiffness, and optical focal ratio. The primary mirror consists of seven hexagonal segments; the six peripheral segments are actively controller by 18 linear electromagnetic precision actuators (3 per segment). Collocated to each actuator is a position sensor which measures the displacement of the actuator. 24 edge position sensors are used to measure the relative displacements between the segments. The total of 18 + 24 = 42 sensors generate commands for the actuators to keep the segments optically aligned, the central segment acting as a reference.

Secondary Mirror. The CSRL test-bed secondary reflector consists of an actively controlled mirror whose housing is supported by a tripod that is attached to the...
primary truss at three points. The mirror is suspended from its housing by means of isolation springs. The mirror is designed to provide two-axis, active beam-stem control. It is equipped with 3 reluctance actuators and 3 position sensors, in order to make the control system which is currently being designed capable of aligning the secondary mirror to the focal plane and removing all relative angular motion between the secondary and the reference central segment of the primary structure.

II.1 FEM and Reduced-Order Modelling of the Structure

A 1870 mode (i.e. 3740 states) Finite Element Model was constructed using the NASTRAN software. This FEM model was carefully reduced to a 40 modes (i.e., 80 states) model, in such a way that the reduced-order model retains most of the original model's physical characteristics. In particular we employed an iterative technique as follows: (i) the FEM model was reduced by neglecting most of its highest frequency modes. (ii) The resulted model was further reduced by keeping the most controllable and observable modes using the combined observability/controllability criterion of \cite{1}. (iii) Finally, we reduce further the model by employing the Hankel norm model reduction method \cite{7} for reducing linear systems.

By neglecting a different number of modes at each of the steps (i), (ii), and (iii) we end up with a different reduced order model. The best reduced order model is chosen based on time-domain and frequency-domain comparisons between the original FEM model and the reduced-one. Figure 1 plots the first 40 modes of the FEM model; those are the modes of the reduced order system as well. Figure 2 plots the frequency response of the reduced order model with respect to external disturbances. The frequency response of the original FEM model is almost the same with the one of the reduced order system.

III Control Requirements & Constraints

The segment alignment control system must achieve the optical quality of a single mirror in the segmented primary mirror. In particular, the RMS distortion with respect to the reference single-mirror-surface must be within 1 micron \cite{12}. In order to achieve the above performance, the control design must take into account the following:

Damping of Resonance Peaks. Figure 1 shows the open-loop singular value plot of the disturbance response of the CSRL structure. The singular value plot of the plant transfer function (from the control actuators to the sensor outputs) is exactly the same. A similar behavior must be expected from the CSRL structure (in fact, the plots of Figure 1 are typical for flexible segmented reflectors). Clearly, the first modes of the system induce large resonance peaks in the plant transfer function from the control actuators to the sensor outputs, as well as in the transfer matrix from the disturbances to the outputs. There is no need to convince the reader that these open-loop responses are highly unsatisfactory. The need for damping the resonance peaks in the 20 – 30 Hz range is obvious.

Excitation of Resonant Modes of the Truss. As the segment alignment bandwidth increases, the control starts exciting the resonant modes of the light-weight support structure. This control structure interaction may result in closed-loop instability. For satisfactory performance, the segment alignment requirements must therefore be coupled with modal vibration suppression requirements. In this respect, the objective is not to worsen the natural response of the system to disturbances outside the control bandwidth and if possible to damp out the natural vibration mode of the system.

Model Reduction, Decoupling, and Decentralization. Usually, robust controllers designed for flexible structures have about the same order as the order of the mathematical model of the structure. Since, the controller order is limited by hardware limitations, the order of the mathe-
mathematical model of the structure must be kept as small as possible, or - in the case where the mathematical model of the structure is of large order - we must use model reduction techniques in order to "minimize" the order of the controller. On the other hand, the computational effort of the controller can be reduced dramatically by using either modal decoupling or decentralized control. Both methods reduce the difficulties of the controller synthesis in several ways: subcontrollers can be computed faster (the computation goes down like the inverse of the cube of the number of subsystems, when the subsystems have equal size \(^2\)) and more reliably than a controller for the global system and they are easier to test. Furthermore, optimality is not lost if the decoupling is nearly exact. Model decoupling consists of partitioning the modes of a system into subsets of modes that can each be controlled and sensed by different combinations of the physical actuators and sensors. These actuators and sensor combinations define new fictitious sensors and actuators. The transformation from the fictitious to the physical actuators must be well conditioned so that the physical actuator and control specifications such as maximum actuator authority are applicable without any modification to the fictitious actuators. Decentralized control can be thought as an extreme case of the modal decoupling, where the physical actuators' and sensors' subsets coincide with the subsets of fictitious actuators and sensors. In this case, each subcontroller requires measurements only from a subset of sensors and controls only a subset of actuators. In particular, we are interested in a decentralized control technique where each subcontroller controls the three actuators of each segment and uses measurements from the sensors of this particular segment. The symmetrical nature of the CSRI structure as well as the large order of the mathematical model describing such a structure make necessary the use of model reduction, decoupling and decentralization.

**Modeling Errors.** The control design must take into account the various modeling errors between the mathematical model and the actual behavior of the structure. Finite Element Methods (FEM) or system identification techniques are known to provide accurate models only in low frequencies, while in high frequencies the modeling errors may be up to 100% of the actual system's behavior. Finally, the various nonlinearities as well as the model reduction, decoupling, and decentralization are also sources of modeling errors that must be taken into account.

**Limited Control Authority, Bandwidth and Imperfect Measurements.** The control design must account for limited control authority and imperfect measurements. The maximum available actuator force is 15 lbf. The actuators have a 120 Hz current loop bandwidth. The sensors have a resolution of 1um and an operating range of 100nm-1mm.

Based on the above, the controller must satisfy the following design specifications:

- 10 – 20% stability margin with respect to modal frequency errors.
- 100% stability margin with respect to modal damping errors.
- 30 – 60 dB/decade of roll-off in the loop gain starting at the projected bandwidth (which must be expected to be about 45 – 50 Hz) in order to avoid spillover.
- 100 : 1 disturbance attenuation over the frequency range 0 – 15 Hz.
- The controller bandwidth must be limited to be less than 50Hz.

**IV Robust H\(_\infty\) Control**

Classical robust H\(_\infty\) methods \(^1\,^2\) were used in order to develop robust centralized controllers for shape control of the structure. The robust H\(_\infty\) control problem is formulated as an optimization problem where the objective is to construct a dynamic compensator that minimizes the H\(_\infty\)-norm of the transfer function T\(_{zd}\) from the disturbances d to a set of fictitious outputs z which is given by

\[
z = W_1(s)y + W_2(s)u
\]

where W\(_1\), W\(_2\) are stable design filters. Different choices for the filters W\(_1\), W\(_2\) give different solutions to the robust H\(_\infty\) control problem. In our case W\(_1\), W\(_2\) were chosen to be static matrices, which were carefully designed (using a trial and error iterative technique) in such a way that the resulting controller meets the control objectives and constraints of the previous section.

The robust H\(_\infty\) controller achieved a disturbance attenuation of up to 29 dB, and it also provides with a stability margin of up to 20% parametric uncertainty with respect to modal frequencies and up to 100% parametric uncertainty with respect to the modal damped. Finally, the controller provides with a stability margin of 80% with respect to output multiplicative uncertainties (unmodelled dynamics). The response of the closed-loop system is shown in figure 3.

We finally mention that the order of the H\(_\infty\) controller is 80 (same as the order of the model we used for the structure). We reduced the controller down to 50 states using the Hankel-norm model reduction method \(^7\), and we observed that the results were almost identical with the ones obtained for the original controller.
V Robust Decentralized Control

Decentralized control of large telescopes is extremely important. This is because the future generation telescopes will be consisted of hundreds of panels, which makes the application of decentralized control techniques necessary. One of the main scopes of our project is to apply decentralized control techniques to the structure and examine whether the control objectives can be meet by these techniques.

Three different decentralized control techniques have been tested and we are currently testing a fourth one. Next we briefly explain those four techniques and the results obtained by applying those techniques to the structure.

DC1 In the first technique, we rewrite the structure dynamics as follows

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_6
\end{bmatrix} =
\begin{bmatrix}
G_{11}(s) & G_{12}(s) & \cdots & G_{16}(s) \\
G_{21}(s) & G_{22}(s) & \cdots & G_{26}(s) \\
\vdots & \vdots & \ddots & \vdots \\
G_{61}(s) & G_{62}(s) & \cdots & G_{66}(s)
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_6
\end{bmatrix}
\]

where \(Y\) denotes the three-dimensional vector of sensor measurements that correspond to the \(i\)-th panel, \(U_i\) denotes the three-dimensional vector of the actuators displacements in the \(i\)-th panel and \(G_{ij}(s)\) represent transfer function matrices.

In this method, we use an iterative technique where at each step six robust \(\mathcal{H}_\infty\) controllers \(U_i = K_i(s)Y_i\) are designed for the subsystems

\[Y_i = G_{ii}(s)U_i\]

Then, we examine whether the overall closed-loop system meets the control objectives and constraints, and if not, we redesign the controllers \(U_i = K_i(s)Y_i\) using the same approach as we did in the previous section (i.e., we redesign the filters \(W_1(s), W_2(s)\)).

DC2 In the second technique, we rewrite the structure dynamics as follows

\[
\dot{x} = Ax + \sum_{i=1}^6 B_i U_i \\
Y_i = C_i x
\]

where \(Y_i, U_i\) are defined as in the previous technique. Six robust \(\mathcal{H}_\infty\) controllers are developed for the “subsystems”

\[
\dot{x} = Ax + B_i U_i \\
Y_i = C_i x
\]

Similar to the previous method, the filters \(W_1(s), W_2(s)\) of the six decentralized controllers are carefully designed in such a way that the overall closed-loop system meets the control objectives and constraints.

DC3 In this technique, we first design six robust \(\mathcal{H}_\infty\) decentralized controllers for every pair of two adjacent panels, i.e., we design controllers for the subsystems

\[
\begin{bmatrix}
Y_i \\
Y_{i+1}
\end{bmatrix} =
\begin{bmatrix}
G_{ii}(s) & G_{i,i+1}(s) \\
G_{i+1,i}(s) & G_{i+1,i+1}(s)
\end{bmatrix}
\begin{bmatrix}
U_i \\
U_{i+1}
\end{bmatrix}
\]

where (for notational convenience we assume that if \(i = 6\) then \(i + 1 = 1\)). The inclusion principle and overlapping decomposition method of \(14\) are then used in order to decompose those decentralized controllers into a set of six decentralized controllers each corresponding to a single panel.

DC4 In this technique, the six decentralized controllers are obtained by solving the following robust \(\mathcal{H}_\infty\) control problem: Find a block-diagonal controller of the from

\[
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_6
\end{bmatrix} =
\begin{bmatrix}
K_{11}(s) & 0 & \cdots & 0 \\
0 & K_{22}(s) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & K_{66}(s)
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_6
\end{bmatrix}
\]

that minimizes the \(\mathcal{H}_\infty\) norm \(\|T_{xd}\|_\infty\). In other words, this technique finds the optimal (wrt to an \(\mathcal{H}_\infty\) performance criterion) set of decentralized controllers. The paper of \(15\) has solved the above optimization problem, and an algorithm for calculating the \(\mathcal{H}_\infty\) (near)-optimal controllers has been developed.

In Table I, we summarize the results obtained by applying the three decentralized methods to the structure. For comparison reasons, we include in the table the results obtained using the centralized robust controller of the previous section (referred to as CC).
Figure 4: Closed-loop System Response - Decentralized Controller, Method 1

Figure 5: Closed-loop System Response - Decentralized Controller, Method 2

Figure 6: Closed-loop System Response - Decentralized Controller, Method 4

- Adaptive and neural control can deal with time-varying parameters.

- In many cases, adaptive and neural control have the ability to “cancel out” the effect of many disturbances.

- Adaptive and neural control can deal with general types of nonlinearities, while conventional linear controllers can deal only with nonlinearities that are either bounded, conic, or globally Lipschitz.

- Adaptive and control have very good steady state behavior (in the case of external disturbances and unmodelled dynamics).

Unfortunately, most of the existing techniques for adaptive and neural control are either not applicable to flexible structures or result in controllers with poor performance. For instance, most of the existing approaches require the structure to satisfy the ASPR (almost strictly positive real) assumption \(^{11}\), which assumes that the structure dynamics are stabilizable by means of static feedback. The ASPR assumption is met only by flexible structures equipped with both position and rate sensors, which is not the case for the CSRL structure.

Recently, we introduced a family of new adaptive and neural controllers \(^{8,9,10}\) that do not require the structure to satisfy any strict assumptions like the ASPR one.

In order to test the applicability of our adaptive and neural controllers we performed the following experiment:

- The Decentralized \(H_{\infty}\) controllers obtained using method DC4 were connected with decentralized adaptive (neural) controllers.

- 9 order adaptive (neural) controllers used.

- After “training” the adaptive controller, we update the parameters every 100 time-cycles.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Method} & \text{DA} & \text{MF} & \text{MD} & \text{MU} \\
\hline
\text{CC} & 29 \text{ dB} & 20\% & 100\% & 80\% \\
\text{DC1} & 21 \text{ dB} & 10\% & 100\% & 40\% \\
\text{DC2} & 15.5 \text{ dB} & 16\% & 100\% & 60\% \\
\text{DC3} & 25 \text{ dB} & 20\% & 100\% & 80\% \\
\text{DC4} & 24.5 \text{ dB} & 20\% & 100\% & 80\% \\
\hline
\end{array}
\]

where

DA: Disturbance Attenuation

MF: Stability margin wrt modal frequency errors

MD: Stability margin wrt modal damping errors

MU: Stability margin wrt output multiplicative uncertainties

Figures 4-6 plot the frequency response of the closed-loop system for the case the controllers DC1, DC2, DC4 are used.

As it can be seen from Table I and figures 4-6, the decentralized controllers meet the control objectives.

VI Adaptive and Neural Decentralized Control

One of the scopes of the project is to examine the applicability of adaptive and neural control techniques for the structure. Adaptive and neural control have certain advantages over the existing conventional methods:

- Adaptive and neural control can deal with large parametric uncertainties.
The nominal system parameters were "randomly perturbed" (10% modal frequencies variations, 100% modal damping variations).

"Small" unmodelled dynamics were added.

The control law is constrained to satisfy the control authority constraints.

In figure 7 we plot the system response in the case where no controller is applied, the case where the decentralized controller obtained using method DC4 is applied, and the case where the decentralized controller DC4 is accompanied with the adaptive controller. The system starts from a random initial state and is concatenated by bandlimited random disturbances.

The open-loop system dynamics achieve a steady-state error (RMS distortion) of $9 \cdot 10^{-3}$; the DC4 controller achieves a steady-state error of $8 \cdot 10^{-4}$ while the combined DC4/adaptive controller achieves a steady state error of $10^{-4}$. In figure 8 we have replaced the adaptive controller with a neural one. The neural controller achieves the same steady-state error as the adaptive one, but the neural controller has better transient behavior than the adaptive one.


