ROBUST ADAPTIVE CONTROL ALGORITHMS WITH AND WITHOUT PERSISTENCE OF EXCITATION

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KEYWORDS
Model Reference Control, Plant Uncertainties, Robust Adaptive Control, General Adaptive Law, Persistence of Excitation

ABSTRACT
In this paper, we consider the model reference control problem of a plant with unmodeled dynamics. We first show that the model reference control structure is robust with respect to plant uncertainties and obtained bounds for the tracking error by assuming that the parameters of the modeled part of the plant are known. We then combine the control structure with a general robust adaptive law and apply it to the actual plant with unmodeled dynamics and unknown parameters. We then show that the general adaptive law guarantees global stability and residual tracking errors which are small in the mean without any persistence excitation condition. Since within the residual set for the tracking error, burst phenomena cannot be eliminated, a persistence excitation condition is given which guarantees exponential convergence for the parameter estimates and tracking error to bounded residual sets. Within these sets no burst phenomena can occur and the bound for the tracking error in the limit as $t \to \infty$ depends only on the modeling error and a design parameter. In the case of the switching-modified adaptive law this design parameter becomes zero and therefore the parameter case becomes known in the obtained parameter case.

1. INTRODUCTION

Recently, several global robustness results have been obtained for adaptive control algorithms in the presence of unmodeled dynamics and bounded disturbances (Praly (1982), 1984); Ioannou and Tsakalis (1988); Goodwin et al. (1989); Cluett et al. (1986); Narendran and Annaswamy (1988). A common feature of these algorithms is that the controller structure is the same as the one which could be used if the parameters of the modeled part of the plant were known. The main contribution of this work is the design of suitable adaptive laws for estimating the unknown plant or controller parameters and proving stability in the presence of unmodeled dynamics, disturbances and time-varying parameters (Tsakalis and Ioannou (1988)). A common feature of these algorithms is the use of a normalizing signal (Narendra (1979); Praly (1983)), which bounds the modeling error and guarantees bounded speed of adaptation. In addition to the normalizing signal, several different types of modifications are used in the adaptive laws which need somewhat different assumptions and a priori knowledge about the unknown plant. These assumptions lead to different stability proofs and performance of the adaptive control algorithms.

In this paper we present a general adaptive law which is robust with respect to unmodeled dynamics, bounded disturbances, time-varying parameters, etc. We show that the dead-zone (Kreisselmeier and Anderson (1985); Goodwin et al. (1985); Cluett et al. (1986), fixed-time varying (Ioannou (1984); Ioannou and Tsakalis (1989)) and \text{-}\text{modification (Narendra and Annaswamy (1986))} are special cases of this general adaptive law and their stability properties and robustness can be established in a unified manner. The general robust adaptive law and its byproducts guarantee global stability and residual tracking error, which are small in the mean without requiring any persistence of excitation condition. However, within the residual set for the tracking error $e_1(t)$, burst phenomena (Narendra and Costa (1987)) cannot be excluded since $e_1(t)$ is only small (i.e., order of the modeling error and of a small design parameter) most of the time but it can be large for some short intervals of time. In order to avoid burst phenomena and improve the bound for the tracking error a persistence of excitation condition is imposed on a signal within the adaptive loop. If such a condition is satisfied by properly choosing the reference input signal then the parameter and tracking error converge exponentially to bounded residual sets. Within these sets no burst phenomena can occur and the bound for the tracking error in the limit as $t \to \infty$ depends only on the modeling error and a design parameter. We show that if the switching-modification is used this design parameter becomes zero and therefore the bound for the tracking error is qualitatively the same as the one which could be obtained in the known parameter case.

2. PLANT AND CONTROLLER STRUCTURE

Consider the SISO plant

$$y = G(s)u = G_0(s)(1 + \delta s)$$

where $G(s)$ is a strictly proper transfer function; $G_0(s)$ represents the modeled part of the plant; $\delta s$ is a multiplicative plant perturbation given by the positive scalar $\delta$.

For the plant perturbation $\delta(s)$, we assume that

(A1) A lower bound $\delta > 0$ on the stability margin $\rho > 0$ for which the poles of $\delta(s)$ are stable is known.

It should be noted that $G(s)$ can be improper when the relative degree of $G_0(s)$ is greater than 1 so
that
\[ u(s) \rightarrow - \infty \quad \text{as} \quad s \rightarrow -\infty \]  
for any \( u > 0 \). The assumptions about \( G(s) \), the modeled part of the plant transfer function, differ for different control objectives. These assumptions will be given in the following subsection, together with the control objectives.

### 2.1 Model Reference Control (MRC)

In MRC the plant output \( y(t) \) is required to track the output \( y_r(t) \) of the reference model given by
\[ y_r(t) = \frac{W(s)}{1 + W(s)G(s)} \]
where \( r(t) \) is any bounded piecewise continuous reference input signal, \( b \) is a constant, and \( W(s) \) is a monic Hurwitz polynomial. To achieve such an objective, the following assumptions are needed for the modeled part of the plant transfer function
\[ G(s) = \frac{Z(s)}{bP(s)} \]
where \( Z(s) \) is a monic Hurwitz polynomial of degree \( n \) and \( P(s) \) is a monic polynomial of degree \( n \). The sign of \( b \) is known. Without loss of generality we assume that \( b > 0 \).

#### (51) The relative degree \( n^+ = n - m \) is known.

#### (52) An upper bound \( n \) for the order \( n \) of \( G(s) \) is known.

Choosing degree \( n_g(s) = n^+ \) the following controller structure may be used:
\[ u = q_g(s) + \frac{1}{A(s)} \nu + q_g(s) \nu + q_g(s) \nu \]
where \( A(s) \) is an arbitrary monic Hurwitz polynomial of degree \( n^+ \):
\[ g_1(s) = g_{10} + g_{11}s + \cdots + g_{1(n-2)}s^{n-2} \]
\[ g_2(s) = g_{20} + g_{21}s + \cdots + g_{2(n-2)}s^{n-2} \]
\[ g_3 = [g_{30}, g_{31}, \ldots, g_{3(n-2)}] \]
and \( q_g(s), c_g(s) \in \mathbb{R}^n \).

It is shown in Narendra and Valavani (1978) that a \( q_g(s), c_g(s) \) exists such that the closed-loop transfer function of the modeled part of the plant satisfies the matching condition
\[ \frac{Z(s)}{B(s)} = \frac{G(s)}{1 + G(s)W(s)} \]
\[ \frac{Z(s)}{B(s)} = \frac{G(s)}{1 + G(s)W(s)} \]
If the coefficients of the polynomials \( G(s), Z(s) \) are known, then \( c_g(s), g_1(s), g_2(s), g_3(s) \) may be obtained from (2.2) and used in (2.5) to form the desired control input for MRC
\[ u = \frac{q_g(s)}{A(s)} \]
\[ u = \frac{q_g(s)}{A(s)} \]  
(2.3)

If the coefficients of \( G(s), Z(s) \) are unknown, (2.3) cannot be implemented. In this case, instead of (2.3), (2.5) is used together with an appropriate adaptive law to adjust the time-varying parameters \( b, \theta_1, \theta_2, \theta_3 \) and \( c_g(s) \). A class of such adaptive laws will be presented and analyzed in Section 3.

From the robustness point of view, it is important to analyze the stability properties and performance of the MRC law (2.8) when applied to the actual plant (2.1) with unmodeled dynamics.

#### Theorem 2.1

There exists a \( u_1 \geq 0 \) such that for each \( p \in \{0, 1, 2, \ldots \} \) all the signals of the closed loop plant (2.1), (2.8) are bounded for any bounded initial conditions. Furthermore, the tracking error \( \xi = y - y_r \) converges exponentially to the residual set
\[ \Omega_{\theta} = \{ \xi | \| \xi \| \leq \theta \} \]
where \( \theta \in \mathbb{R}^n \) and \( \theta \) is the upper bound for \( |r(t)| \).

**Proof.** From (2.8), (2.1) we have
\[ y = \frac{1}{a(s)} \left[ 1 - g_1(s) \frac{\xi}{A(s)} \right] \left( \frac{\xi}{A(s)} + \gamma \right) \]
\[ y = \frac{1}{a(s)} \left[ 1 - g_1(s) \frac{\xi}{A(s)} \right] \left( \frac{\xi}{A(s)} + \gamma \right) \]
(2.10)
Using the matching condition (2.7) in (2.10) we have
\[ y = k(s) \bar{r} + \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
\[ y = k(s) \bar{r} + \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
(2.11)
where
\[ \bar{H}_{ij} (s) \]
\[ \bar{H}_{ij} (s) \]
are strictly proper transfer functions and
\[ \bar{H}_{ij} (s) = \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
\[ \bar{H}_{ij} (s) = \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
(2.12)
Using the small gain theorem, \( \bar{H}_{ij} \) has stable poles if
\[ \frac{\bar{H}_{ij} (s)A(s)B(s)}{1 - \bar{H}_{ij} (s)A(s)B(s)} < 1, \quad \forall s \in (-\infty, 0) \]
\[ \frac{\bar{H}_{ij} (s)A(s)B(s)}{1 - \bar{H}_{ij} (s)A(s)B(s)} < 1, \quad \forall s \in (-\infty, 0) \]
(2.13)
Choosing
\[ \nu = \frac{1}{2} \inf \left( \frac{\bar{H}_{ij} (s)A(s)B(s)}{1 - \bar{H}_{ij} (s)A(s)B(s)} \right) \]
\[ \nu = \frac{1}{2} \inf \left( \frac{\bar{H}_{ij} (s)A(s)B(s)}{1 - \bar{H}_{ij} (s)A(s)B(s)} \right) \]
then (2.14) holds for any \( \nu \in (0, \infty) \).

From (2.11) we have
\[ \xi = \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
\[ \xi = \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \]
(2.16)
Since \( \sum_{i=1}^m \sum_{j=1}^{n_i} \bar{H}_{ij} (s) \leq k_0 \), for some \( k_0 \in \mathbb{R}^n \) it follows that
\[ \xi(t) \leq |\xi_0| + c_0 \]
\[ \xi(t) \leq |\xi_0| + c_0 \]
(2.17)
where \( c_0 \) is an exponentially decaying term and therefore (2.9) follows.

The chosen control law (2.8) guarantees that the MRC is robust with respect to unmodeled dynamics. Furthermore, it guarantees that the bound for the residual tracking error \( \xi(t) \) is small for small \( u \). When \( u > 0 \) (i.e., regulation case) the tracking error reduces to zero despite the
The presence of unmodeled dynamics. In order to implement (2.6) the parameters of $D_c(s)$ have to be known so that $H_1^*, H_2^*, H_3^*$ and $H_4^*$ can be calculated from (2.7). If these parameters are not known, the control law (2.5) has to be implemented together with a suitable adaptive law to update the controller parameters $\theta_1, \theta_2, \theta_3$ and $\theta_0$. The adaptation law makes the closed-loop system a non-linear one whose stability properties and robustness do not follow from the time-invariant case of the LFC. In fact, it was shown in Ioannou and Kokotović (1984) that several types of instability may arise, even if an adaptive law which guarantees stability in the linear case of no plant uncertainties is applied to the full-order plant with unmodeled dynamics and/or bounded disturbances.

In the following section, we present a general adaptive law and the sufficient conditions to be satisfied in order to guarantee robust adaptation of the controller parameters in (2.5).

### 3. ROBUST ADAPTIVE LAW

For clarity of presentation, we shall assume that $k_p = 1$, i.e. $c_0^*$. In order to derive the adaptive law, we form the estimation error $e(t)$ by using the following equation satisfied by $\dot{e} = \lbrack a_1^T a_2^T a_3^T \rbrack$ [Ioannou and Sun (1986)]

\[ \dot{e} = J^T \dot{e} = \psi(t) \]

where

\[ \psi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \end{bmatrix} = \begin{bmatrix} H_1^*(s) \phi(s) \\ H_2^*(s) \phi(s) \\ H_3^*(s) \phi(s) \end{bmatrix} \]

and

\[ \phi(s) = \begin{bmatrix} 1 \\ s \\ s^2 \end{bmatrix}. \]

In view of (3.1), the measured estimation error $e(t)$ is given by

\[ e = \psi - \dot{\eta} = \psi - \dot{\psi} + \dot{\gamma} \]

where $\psi(t)$ is the estimate of $H_1^*$ at time $t$. Defining the parameter error $e = \psi - \dot{\eta}$, (3.2) may be written as

\[ e = \dot{\gamma} + \dot{\psi}, \quad n = \dot{\psi} \]

where $\dot{\psi}(s)$ is a strictly proper transfer function with stable poles.

We derive an adaptive law for $\theta$ by considering the following cost

\[ J = \frac{1}{2n^2} \int \phi(t)^T \phi(t) \]

where $m(t) > 0$ and $\int_0^t \phi(t)^T \phi(t) dt$ is bounded. Such a signal can be generated from the following equation

\[ \dot{\eta} = \dot{\gamma} + \gamma + \phi^T(t) \phi(t) \]

where $\phi(t)$ is the design time-varying vector whose properties will be established below. These properties guarantee that $\int_0^\infty \phi(t)^T \phi(t) dt$ bounded $\eta(t)$ bounded.

From (3.4) we have

\[ \dot{J} = \frac{1}{2n^2} \dot{\phi}(t)^T \phi(t) + \phi(t)^T \dot{\phi}(t) \]

and therefore using the gradient method

\[ \dot{\eta} = -\frac{1}{2n^2} \dot{\phi}(t)^T \phi(t) + \phi(t)^T \dot{\phi}(t), \quad \gamma > 0 \]

The following theorem gives a set of sufficient conditions which have to be satisfied by $f(t)$ for stability and robustness.

**Theorem 3.1.** If $f(t)$ is chosen such that the following conditions are satisfied

\[ \frac{\dot{J}}{J} = \frac{\dot{\eta}}{\eta} + \frac{\dot{\phi}}{\phi} \frac{1}{n^2} < 0 \]

then there exists a $0 < n < \infty$ such that for $x_0(t)|\phi(t)|$ and $\int_0^\infty |\phi(t)| dt \leq \gamma |x_0|$ the plant (2.1) with the adaptive control law (2.5), (3.3) is globally stable. Furthermore, the tracking error $e(t)$

\[ e = e(t) = e(t) \int \begin{bmatrix} \gamma(t) \\ \phi(t) \end{bmatrix} dt \leq \gamma |x_0| + \int_0^\infty |\phi(t)| dt \]

**Corollary 3.1.** Let $f$ be designed such that condition (1) of Theorem 4.1 holds for some $u_0(t)$.

\[ f(t) = \begin{cases} f_1(t) & \text{if } c_1 \leq \gamma_1 \phi_1 \\ f_2(t) & \text{if } c_1 > \gamma_1 \phi_1 \end{cases} \]

where $f_1(t)$ is the upper bound for $f(t)$, $\gamma_1$ is a small parameter. Theorem 3.1 can be applied by setting $f = f_1$.

**Switching Law.**

\[ f = \begin{cases} f_1(t) & \text{if } c_1 \leq \gamma_1 \phi_1 \\ f_2(t) & \text{if } c_1 > \gamma_1 \phi_1 \end{cases} \]

where $f_1(t)$ is small. Theorem 3.1 can be applied by setting $f = f_1$. 
\[ f = \varphi_0 c - \delta \begin{cases} 0 & \text{if } \|c\| < \delta_0 \\ \frac{\|c\|}{\delta_0} - 1 & \text{if } \delta_0 \leq \|c\| < 2\delta_0 \\ 1 & \text{if } \|c\| \geq 2\delta_0 \end{cases} \]  

(3.12)

where \( \delta_0 > 0 \) is arbitrary and \( \delta_0 \geq 2\delta_0 \). In this case \( u = u_0 \), where \( p \in \mathbb{R}^n \) and therefore both Theorem 3.1 and Corollary 3.1 are applicable. A comparison of the properties of the time-varying or switching-0 modification versus those of the dead-zone, fixed-0 and \( \varphi_0 = 0 \) are presented in [Ioannou and Tskalakis 1986; Ioannou and Sun 1986].

The robust adaptive law (3.7) guarantees global stability and residual tracking error bounded by the bound on \( \Omega \) independent of any persistence excitation condition. This result, however, does not exclude the appearance of burst, or other bifurcation, phenomena [Hou and Costa 1987] within the residual set given by (3.8). In order to improve performance and obtain explicit bounds for the residual tracking error, a persistence of excitation condition needs to be satisfied by some signals in the adaptive loop.

Definition 3.1. The signal vector \( z \) is strongly persistently exciting (SPE) for the closed-loop plant (2.1), (2.5), (3.7) if there exists constants \( \alpha > 0 \) and \( \gamma > 0 \) such that

\[ \int_0^\infty e^\gamma dt \geq \alpha \]  

(3.12a)

Theorem 3.2. Let \( z \) be strongly persistently exciting. Then in addition to Theorem 3.1 the following result holds for the closed-loop plant (2.1), (2.5), (3.7): Assume that the parameter error \( \delta(t) \) and tracking error \( \epsilon(t) \) converge exponentially to the residual set

\[ \epsilon_0 = (\epsilon_0, \delta_0, \|\epsilon\| \leq \delta_0, \gamma \leq \gamma + 7) \]  

(3.13)

where \( \gamma = \sup \{ f(t) : t \in [0, \infty) \} \)

\[ \gamma \geq \gamma_0 \]

Corollary 3.2. If \( f(t) \) is chosen as in (3.12a) then there exists a \( 0 < c < c_0 \) such that for \( \epsilon(t) \in [0, \infty) \)

\[ \|\epsilon\| \leq \alpha \]  

(3.13)

Proof of Theorem 3.2. From Theorem 3.1 we have that \( z \) is SPE and \( \gamma > 0 \), therefore for any bounded initial conditions independent of whether \( z \) is persistently exciting (PE) or not. Since \( \gamma \) is SPE and it is bounded, \( \gamma \) is SPE and therefore

\[ \int_0^\infty e^\gamma dt \geq \alpha \]  

(3.14)

holds for some \( \alpha > 0 \) and for all \( \gamma > 0 \). Choosing \( \gamma = 1 \), (3.14) may be written as

\[ \gamma = \frac{\|\epsilon\|}{\delta_0} - 1 > \alpha \]  

(3.15)

In view of (3.14) the homogeneous part of (3.13) is exponentially stable, therefore, there exists constants \( \alpha > 0 \) such that the transition matrix \( \epsilon(t) \) of (3.16) satisfies

\[ \|\epsilon(t)\| \leq \alpha \exp \left[ -\alpha_1 (t-t_0) \right], \forall t \geq t_0 \]  

(3.17)

Hence it follows from (3.15), (3.17) that

\[ \|\epsilon(t)\| \leq \alpha_1 \sup \{ \|\epsilon(t)\| : t \geq t_0 \} \leq \epsilon_0, \forall t \geq t_0 \]  

(3.18)

\[ \epsilon(t) \]  

where \( \epsilon(t) \) is exponentially decaying term.

It can be shown [Ioannou and Sun 1986] that

\[ \epsilon_0 = \omega_0 \left[ \frac{\|\epsilon\|}{\delta_0} \right] \]  

(3.19)

and that \( \delta(t) \) is bounded. Since \( \omega_0 \) has stable poles and \( \epsilon(t) \) is bounded it follows from (3.19) that

\[ \|\epsilon(t)\| \leq \alpha_1 \leq \omega_0, \forall t \geq t_0 \]  

(3.20)

In view of (3.15), (3.16), (3.18), (3.19) it follows and the proof is complete.

Proof of Corollary 3.2. From (3.15), (3.17), (3.18), (3.19) we have

\[ \|\epsilon(t)\| \leq \alpha_1 \]  

(3.21)

As in Theorem 5.4 of Ioannou and Sun 1986 it can be shown that

\[ \int_0^t \epsilon^2(t) dt \leq b_0 \]  

(3.22)

for some \( b_0 \). Since \( \|\epsilon\| = \delta_0 \geq 0 \) then it follows from (3.22) that

\[ \int_0^t \epsilon^2(t) dt \leq b_0 \]  

(3.23)

Using the Bellman-Gronwall Lemma as in Lemma 2.5 of Ioannou and Sun 1986 it follows from (3.23), (3.22) that there exists a \( 0 < \alpha < 0 \) such that for \( \epsilon(t) \)

\[ \epsilon(t) \]  

(3.24)

Choosing \( \alpha_0 \) so that \( \alpha_0 \leq \omega_0 \) \( \forall t \geq t_0 \)

\[ \|\epsilon(t)\| \leq \alpha_0 \]  

(3.25)

which implies that \( \{ \epsilon(t) \} : t = 0 \rightarrow t \) because

\[ \|\epsilon\| \leq \alpha_0 \]  

(3.26)

Hence the proof is complete.

Remark 3.1. It should be noted that the dead zone \( f = 0 \) in (3.13) and therefore the bound for the residual tracking error depends on the design parameters \( \delta_0 \) or \( \alpha_0 \) and is not zero even when \( \omega = 0 \) (i.e., in the absence of unmodeled dynamics).

Corollary 3.2 guarantees that if \( f(t) \) is designed properly (as in the case of the switching-0), \( f = 0 \) in (3.13) and therefore the bound for the tracking error is qualitatively the same as the one obtained in the known parameter case.
The persistence of excitation condition used to obtain the result of Theorem 3.2 and Corollary 3.2 is imposed on $c$ which is an integral signal in the adaptive loop. In the following theorem we will show that $c_n$ will be SPE if the reference input signal $r(t)$ is properly designed.

**Theorem 3.3.** Let

$$c_n = H(s)\; s = \left[ \begin{array}{c} \frac{F_x(s)}{A(s)} \large(\frac{x(s)}{x(s)} \large) d(s) \Large(\frac{E(s)}{1} \large) \end{array} \right]$$

(3.27)

If $c_n$ is SPE then there exists $0 < \omega_n < \omega^*$ such that for $H(s)$ to be SPE with $\omega_n$.

**Proof.** From Ioannou and Sun (1986) it can be shown that

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.29)

for all $s > 0$ and $T > 0$ independent of any PE condition. If $c_n$ is SPE we have that

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.30)

for all $T > 0$. Noting that $c_n^2(s) \leq c_n^2(s)$ for some $k_2 > 0$ we have

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.31)

Since

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.32)

It follows from (3.29), (3.31), (3.32) that

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.33)

Choosing $a = k_2^2(1/2) + k_2^2(1/2)$ and

$$\alpha = k_2^2(1/2)$$

(3.34)

we have

$$\int_{s_0}^{s_1} \left( c_n^2(s) + c_n^2(s) \right) ds \geq \alpha c_n^2$$

(3.35)

Setting $F_{ad} = \omega_n$ and $\omega = \omega_n/\alpha$ and noting that $0 < \omega (0, \omega), > (0, \omega)$ because of the property of $\omega_n$, the proof is complete.

**Remark 3.2.** The proof of Theorem 3.3 is a generalization of the proof in Boyd and Sastry (1986) where $c_n$. The signal vector $c_n$ is the state of a linear time invariant system with input $r(t)$. Hence the SPE property of $c_n$ can be satisfied by choosing the reference input signal $r(t)$ properly. The class of reference input signals which guarantees that $c_n$ is SPE with $\omega_n$ is presented in Ioannou and Tao (1982). These signals have to be dominantly rich in [Ioannou and Kokotovic (1982)] i.e., they have to be PE for the system (3.27) but with frequencies $0 < \omega_n < (0, \omega)$.

4. CONCLUSION

In this paper, we derive a general adaptive law for updating the parameters of a model reference controller. We show that this adaptive law guarantees global stability and residual tracking errors which are "small," i.e., of the order of the modeling error in the mean without requiring any persistence of excitation condition. If, however, the reference input signal is dominantly rich so that a certain vector signal within the adaptive loop is strongly persistently exciting then the parameter and tracking error converge exponentially to bounded residual sets whose size depends on the modeling error and a design parameter. This design parameter is equal to zero if the switching-an-modified adaptive law, a special case of the developed general adaptive law, is used. Thus if the switching-an is used the stability properties of the closed-loop adaptive control scheme are qualitatively the same as those which can be achieved in the known parameter case.

5. REFERENCES


ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation under Grant ECS-8312233 and in part by Industrial matching funds from General Motors Foundation, Inc. and Ford Motor Co. under a presidential Young Investigator Award.

The author would like to thank Mr. G. Tao and Ms. J. Sun for many useful discussions.