Autonomous Intelligent Cruise Control Using Front and Back Information for Tight Vehicle Following Maneuvers

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Abstract—During manual driving, most human drivers often use information about the speed and position of the preceding and following vehicles in order to adjust the position and speed of their vehicles. The purpose of this paper is to design an autonomous intelligent cruise control (AICC) which mimics this human driving behavior. The proposed AICC law uses relative speed and spacing information from the preceding and following vehicles in order to choose the proper control action for smooth vehicle following and for maintaining a desired intervehicle spacing specified by the driver. The vehicle stability and platoon stability (in the case of multiple vehicles) in both directions (backward and forward) are guaranteed by the proposed AICC law. Furthermore, platoon stability is guaranteed for a speed-dependent desired spacing (time headway approach) as well as for a constant desired spacing at all speeds.

Index Terms—Automated highway systems (AHS’s), IC control, platoon stability, string stability.

I. INTRODUCTION

During automatic vehicle following, the control objective is to maintain a desired spacing from its preceding vehicle as well as driving comfort. Numerous vehicle following controllers have been proposed in the literature [1]–[10]. The common feature of these controllers is that only information from the preceding vehicle is used to compute the proper control action. In other words, these controllers use no information about the status of the following vehicle and take no action to avoid rear-end collisions due to the actions of the vehicle behind.

Since a good human driver uses information from the vehicles ahead and behind, it is expected that the vehicle controller will behave safer and better if it uses information from the vehicle in front as well as from the vehicle behind. This approach couples all following vehicles and leads to a dynamical system that resembles a series of mass/spring/damper systems. It is well known that a system consisting of a series of mass/spring/damper systems is dissipative, that is, the energy of the system decays to zero eventually (i.e., there is no velocity difference between masses, and the distance between neighboring masses is equal to the normal length of the spring). Therefore, it is expected that if the vehicle controller is designed based on this framework, this large-scale system will be stable and free of slinky-type effects. The feasibility of this approach has been shown in a preliminary study in [12], where it is applied to cooperative platoon control. However, only simulation results were presented, and no analytical results about the performance and stability of the platoon system are presented.

In this paper, we propose an autonomous intelligent cruise control (AICC) law which uses information not only from the controlled vehicle’s immediate predecessor, but also from the controlled vehicle’s immediate follower. The proposed AICC law guarantees both the individual vehicle stability, and platoon stability in both directions (forward and backward). Therefore, the potential disadvantage of this approach, that disturbances may propagate both forward and backward within a platoon, is eliminated.

In the design and analysis of the proposed AICC law, the constant time headway policy in the interval of view, the intervehicle spacing should be as small as possible. This consideration prompted several researchers in the field to suggest constant spacing headway of about 1 m [6], [7]. With 1-m spacing, the capacity of the freeway will increase considerably. However, it has been demonstrated in [1] and [5] that it is impossible to design a vehicle-follower control, which achieves platoon stability, with constant spacing safety policy using only information about relative speed and spacing between the controlled and of the preceding vehicle. In [5], Shladover proposes a new approach that uses additional information that includes preview information from the first vehicle in the platoon. This approach makes it possible to have platoon stability with constant spacing by designing the vehicle controller appropriately. This approach, however, needs an intraplatoon communication protocol to supply the information of the platoon leader to all the vehicles in the platoon. In this paper, we show that our approach guarantees individual vehicle stability as well as platoon stability under the constant spacing safety policy. This result leads to the following important conclusion: the design of the platoon stable vehicle follower controller under constant spacing policy is possible through the use of the relative speed and spacing information from both the controlled vehicle’s immediate predecessor and follower.
The paper is organized as follows. The problem statement is given in Section II. In Section III, a longitudinal vehicle model is reviewed. The AICC controller using both front and back information is proposed in Section IV. In Section V, simulation results are presented to demonstrate the effectiveness of the proposed controller.

II. PROBLEM STATEMENT

For safe vehicle following, each vehicle is required to keep a safe distance from its preceding vehicle. The required safety distance should take into account the vehicle’s performance and braking capabilities, rider’s comfort constraints, road conditions, reaction times, sensor/actuator errors, etc. In this paper, we adopt a safety distance policy [2] for the $v_{th}$ following vehicle given by

$$S_{d_i} = h_1(v_{i-2}^2 - v_{i-1}^2) + h_2v_i + h_3$$

where $v_{i-1}$ and $v_i$ are the velocities of vehicles $i - 1$ and $i$, respectively, and $h_1$, $h_2$, $h_3$ are positive constants that depend on the braking capabilities of the vehicles involved.

For tight vehicle following maneuver, the velocity of the following vehicle is approximately equal to the velocity of its preceding vehicle. Therefore, the safety distance policy (1) can be approximated by

$$S_{d_i} = h_2v_i + h_3,$$

which is known as the “constant time headway policy.” Let $x_i$ ($v_{i-1}$, respectively) and $v_i$ ($v_{i-1}$, respectively) be the position and velocity of the $i$th ($i-1$th, respectively) vehicle as shown in Fig. 1. The spacing deviation of the $i$th vehicle from the desired safety distance is defined as follows:

$$\epsilon_i := x_{i-1} - x_i - l_i - S_{d_i}$$

where $l_i$ is the length of the controlled vehicle.

In this paper, we consider the case where a group of vehicles, is performing tight vehicle following. Our control objective is to design a controller for each vehicle $i$ using information from both the controlled vehicle’s immediate predecessor and the controlled vehicle’s immediate follower to achieve automatic vehicle following.

We assume that the following states are available for measurement to the controlled vehicle:

- M1: The relative distance and the relative velocity between the $i-1$th and $i$th vehicle, i.e., $x_{i-1} - x_i - l_i, v_{i-1} - v_i$.
- M2: The relative distance and the relative velocity between the $i$th and $i+1$th vehicle, i.e., $x_i - x_{i+1} - l_{i+1}, v_i - v_{i+1}$.
- M3: The velocity of the controlled vehicle $i$, i.e., $v_i$.

Remark: Currently available on-board radar and sensors can generate the measurements given in M1 and M3. A similar type of sensors can be used to obtain the measurements given in M2.

Remark: For the last vehicle $N$ with no vehicle behind it (or the vehicle behind it is far away from it), the vehicle’s controller will use the information available from preceding vehicle and assume a virtual vehicle as a follower. The spacing deviation and relative velocity between the vehicle $N$ and the virtual vehicle are zero at any time. For the vehicle with no vehicle in front of it, the vehicle following objective is replaced with a cruise control objective.

The control objective is to choose the control input $u_i(t)$ of the controlled vehicle $i$ such as follows:

- C1: The spacing deviation is regulated to zero, i.e.,
  $$\lim_{t \to \infty} [\epsilon_i(t)] = 0.$$
- C2: The relative velocity between the $i-1$th vehicle and the $i$th vehicle, when the $i$th vehicle cruises at constant speed, is regulated to zero, i.e.,
  $$\lim_{t \to \infty} |v_i(t) - v_{i-1}(t)| = 0$$
  for constant $v_{i-1}$.

In addition, groups of vehicles following each other in the longitudinal direction by forming a long strong or platoon should also be free of slinky effects. The slinky-type effects are well known in vehicle following where the control actions of the lead vehicle cause an amplification in the values of spacing, velocities and accelerations of the following vehicles. A platoon of moving vehicles is considered to be stable if it is free of slinky effects. We refer to this form of stability as platoon stability. Platoon stability is not automatically guaranteed by the individual vehicle stability. The vehicle controller has to be designed to guarantee both vehicle and platoon stability.

Therefore, in addition to the performance requirements C1 and C2, we should add an additional requirement as follows.

- C3: A platoon of individual controlled vehicles should be stable in both forward and backward directions.

III. LONGITUDINAL VEHICLE MODEL

A longitudinal vehicle system is mainly composed of an engine, transmission, drive train and brake system. A simple functional description of such a system is shown in Fig. 2. In this figure, the major system components are connected by the internal states of the system such that the interactions between components are identified.

Each block can be considered as a subsystem with various inputs and outputs. The output of the engine subsystem is the engine torque that is a nonlinear function of the air/fuel ratio, exhaust gas recirculation (EGR), cylinder total mass charge, spark advance, engine speed and the drivetrain load, as well as the throttle angle. The spark advance EGR and air-to-fuel ratio are the outputs of an internal controller (inside the engine block) whose inputs are the throttle position, engine

![Fig. 1. Configuration of vehicle following maneuver.](image-url)
speed and drivetrain load. The transmission subsystem is responsible for transferring engine torque to the drivetrain depending on the vehicle speed and engine condition. The transmission considered in Fig. 2 is an automatic transmission with hydraulic torque coupling and four forward transmission gears. For a certain gear state, the transmission torque output is a linear function of the engine torque. The gear state is a nonlinear function of the throttle angle, engine speed, and vehicle speed. The transmission subsystem receives transmission torque and/or braking torque input and outputs vehicle speed, acceleration, or deceleration. The vehicle speed and acceleration are affected by the road condition, aerodynamic drag, and vehicle mass. The relationship between vehicle speed and transmission torque is also nonlinear.

For longitudinal control, the system in Fig. 2 may be considered as a two-input (throttle angle command and brake torque command) one-output (vehicle speed) system. The other inputs such as aerodynamic drag, road condition, and vehicle mass are treated as disturbances. This two-input single-output system can be subdivided into two major parts. The first part consists of the engine and transmission systems, and the second part is the drivetrain system. Based on several tests conducted using a validated nonlinear vehicle model, it is observed that the engine and transmission systems can be modeled as a fast dynamical system and the drivetrain system as a slow dynamical system.

Since the nonlinear model described above is considerably complicated and highly nonlinear, it is difficult to design a longitudinal controller directly. A simplified vehicle longitudinal model which well represents the original system's dynamics can be obtained to simplify the task of controller design. A simplified longitudinal vehicle model, in which fast dynamics are neglected and are replaced by a quasi-steady-state response, is obtained as follows:

\[ \dot{x} = v \]  
\[ \dot{v} = \frac{1}{m} \left[ -c_v v^2 - c_p v - d_m + f_1(v, y) - RT_{br} \right] \]  
\[ y = \eta(v, u) \]

where \( y = [y_1, y_2]^T \) is the engine speed, \( y_2 \) is the manifold pressure, \( u \) is the throttle angle, \( d_m \) is the mechanical drag, \( c_v \) is the coefficient of aerodynamic drag, \( c_p \) is the coefficient of friction force, \( T_{br} \) is the brake torque, \( R \) is the effective gear ratio from the engine to the wheel, \( v \) is the velocity of the vehicle, \( z \) is the position of the vehicle, \( m \) is the vehicle mass, \( f_1(v, y) \) is the ideal tire force which depends mainly on the vehicle velocity and the throttle angle which is a nonlinear mapping from \( v, y_1, y_2 \) to \( f_1 \) and is generally measured by steady-state tests and supplied in tabular form, and \( \eta(v, u) \) is the steady-state characteristics of engine and transmission systems. The above vehicle dynamics (4)–(6) have the following property.

**Property 1:** For any given desired ideal-tire force and vehicle velocity \( y_1 \), there exists a unique throttle angle input that achieves the desired ideal tire force. In other words, for any \( v \) and \( z \), the equation \( f_1(v, \eta(v, u)) = z \) has a unique solution \( u = \varphi(v, z) \).

In general, a vehicle controller consists of a throttle controller, brake controller, and switching logic. The brake controller is used for deceleration that cannot be achieved by engine torque alone. The throttle controller is responsible for performing the accelerating and decelerating maneuvers when braking is not activated. The role of the switching logic is to properly activate and deactivate the throttle and brake controllers based on the needed control action at the current operating state. The controller continuously computes the throttle angle required to achieve a certain control action. If the required throttle angle is greater than the minimum throttle angle, say \( u_0 \), the logic determines that the throttle controller alone is capable of handling the desired maneuver, and no brake torque is to be applied. Otherwise, the logic will deactivate the throttle controller, i.e., keep the throttle angle at \( u_0 \), and activate the brake controller to generate the proper brake torque \([11]\).

Under the condition that the brake controller is deactivated, the vehicle longitudinal dynamics are governed by

\[ \dot{x} = v \]  
\[ \dot{v} = \frac{1}{m} \left[ -c_v v^2 - c_p v - d_m + f_1(v, y) \right] \]  
\[ y = \eta(v, u) \]

When the brake controller is activated, the throttle angle is kept at the minimum value \( u_0 \). In this case, the vehicle’s dynamics are governed by

\[ \dot{x} = v \]  
\[ \dot{v} = \frac{1}{m} \left[ -c_v v^2 - c_p v - d_m + f_1(v, y) \right] \]  
\[ y = \eta(v, u_0) \]
In the following sections, we use (7)–(9) to design the throttle controller and (10)–(12) to design the brake controller.

It is worth noting that one can replace the two-dimensional (2-D) system dynamics presented in this section by the three-dimensional (3-D) model we adopted in [1] for the case where the brake controller is deactivated. The same ideas proposed in the following section can be applied (under slight modifications) in the case where we replace model (7)–(9) by the model in [1], since both models are feedback linearizable.

IV. AICC WITH TWO-SIDE INFORMATION

Consider a group of \( N \) vehicles performing automatic vehicle following and forming a platoon as shown in Fig. 3. In Fig. 3, \( x_i, \dot{x}_i, \) and \( \ddot{x}_i \) are the position, velocity, and acceleration of vehicle \( i \); \( v_i, \dot{v}_i, \) and \( \ddot{v}_i \) are the position, velocity, and acceleration of the platoon leader. Our objective is to design a controller for vehicle \( i \), using information from its preceding and following vehicles, such that the control objectives C1–C3 are satisfied.

A. Vehicle Following Throttle Controller

When the brake controller is deactivated, the throttle input \( u_i \) is chosen as follows:

\[
u_i = \varphi_i \left( \begin{array}{c}
\dot{x}_i \\
\dot{v}_i \\
\ddot{v}_i
\end{array} \right) = \left( \begin{array}{c}
\dot{x}_i \\
\dot{v}_i \\
\ddot{v}_i
\end{array} \right)
\]

where \( k_{i1}, k_{i2}, q > 0 \) are design constants and

\[
\varphi_i \left( \begin{array}{c}
v_i \\
\dot{v}_i \\
\ddot{v}_i
\end{array} \right) = \eta_i^{-1} \left( \begin{array}{c}
s_i \\
\dot{s}_i \\
\ddot{s}_i
\end{array} \right)
\]

is guaranteed to exist by Property 1.

B. Vehicle Following Brake Controller

When the brake controller is activated, the throttle angle is kept at the minimum \( \theta_0 \) and the brake torque \( T_{br} \) to be applied is chosen as follows:

\[
T_{br} = \frac{1}{R_i} \left( f(\theta_i) \right) \left( \begin{array}{c}
v_i \\
\dot{v}_i \\
\ddot{v}_i
\end{array} \right)
\]

Remark: Note that there is no vehicle behind the last vehicle \( N \) in the platoon. Hence, no back information is available to the last vehicle \( N \). In this situation, we assign the \( \epsilon_{N+1} \) and \( v_{N+1} \) to be zero. In other words, the last vehicle in the platoon uses only information from the preceding vehicle. Therefore, the vehicle following controller for the last vehicle is

\[
u_N = \left( \begin{array}{c}
v_N \\
\dot{v}_N \\
\ddot{v}_N
\end{array} \right) = \left( \begin{array}{c}
v_i \\
\dot{v}_i \\
\ddot{v}_i
\end{array} \right)
\]

where we dropped the index \( i \) for ease of presentation.

The properties of the platoon system based on the above throttle and brake controllers are given by the following theorem.

Theorem 4.1: Consider the system (7)–(9) with the proposed throttle controller (13), or the system (10)–(12) with the proposed brake controller (15). If the following condition holds:

\[
k_{i1} \geq \frac{1}{2} k_{i2} > 0, \quad \forall i
\]

then we have the following.

1) All the closed-loop signals including the spacing deviations \( \epsilon_i(t) \), velocities \( \dot{v}_i(t) \), accelerations \( \ddot{v}_i(t) \) are uniformly bounded.

2) If the leading vehicle is cruising at a constant speed or accelerating or decelerating with an exponentially decaying rate, then the spacing deviations \( \epsilon_i \) and velocity deviations \( v_i - \dot{v}_i \) converge to zero exponentially fast; if the leading vehicle is accelerating or decelerating with an asymptotically decaying rate, then \( v_i - \dot{v}_i \) converge to zero asymptotically.

3) The proposed control laws guarantee platoon stability in the sense that any change in the control actions of the leading vehicle does not cause any slinky effects.

Remark: It can be observed directly from (14) that if we choose \( h = 0 \) (the safety distance policy becomes the
so-called “constant spacing safety policy”), the control objectives C1–C3 are still achieved.

Proof: The proof of this theorem is quite long and tedious and is given in the Appendix.

For safe longitudinal operations, the following vehicle is required to keep a safe distance from its preceding vehicle. From the traffic capacity point of view, the safe distance should be as small as possible. This consideration prompted several researchers in the field to suggest constraint spacing headway of about 1 m [6], [7]. With 1-m spacing, the capacity of the freeway will increase considerably. However, it has been demonstrated in [1] and [5] that it is impossible to design a platoon-stable vehicle-follower control with constant spacing safety policy, using only relative speed and spacing information from the controlled vehicle’s immediate predecessor. In [5], Shaldower proposed a new approach that uses information from the first vehicle in the platoon. This approach makes it possible to have platoon stability with constant-spacing policy, but it requires intraplatoon communication to supply all the vehicles in the platoon with information about the actions of the platoon leader. Our approach provides a way to design a platoon-stable constant-spacing vehicle follower controller without the use of vehicle to vehicle communication. It simply requires relative speed and spacing measurements from the preceding and following vehicles.

Remark: The usual way of establishing platoon stability is by using the notion of string stability (see, e.g., [13]–[16]) and using a frequency-domain analysis of the linear operators that relate the spacing errors of the $i$th and the $i+1$th (or $i-1$th) vehicles. In our case, such an analysis is very difficult to be carried out. As it can be seen in the proof of Theorem 4.1, instead of using frequency domain analytical tools, we establish platoon stability by making use of Lyapunov-stability arguments and by proving that the spacing and velocity errors remain bounded—in the case of changes in the leading vehicle control actions—by bounds that are independent of the platoon size.

Remark: We should note that the leader in the platoon does not use any back information and is cruising at a speed of its own choice.

Remark: In vehicle following, a certain vehicle may behave erratically, causing other vehicles following it to respond in a similar manner. This will be an unpleasant behavior and should be eliminated. The use of filtering techniques to filter out erratic responses has been shown in [11] to be an effective method for smoothing vehicle following responses. Erratic behavior could be a rapid acceleration with high jerk of the preceding vehicle or a deceleration and slowing down of the following vehicle. In both cases, the vehicle controller should filter these responses and allow the possibility of giving up following the preceding vehicle and/or neglecting back information.

Remark: The controller of the $i$th vehicle must be provided with the following signals/variables:

- engine speed $y_1$, manifold pressure $y_2$, mechanical drag $d_m$, coefficients of aerodynamic drag $c_D$ and friction force $c_P$, respectively, brake torque $T_{br}$, effective gear ratio from the engine to the wheel $R_e$ and the mass of the vehicle;

- position $x_i$, speed $u_i$, and length $l_i$ of the vehicle as well as the positions $x_{i-1}$ and $x_{i+1}$ and speeds $u_{i-1}$ and $u_{i+1}$ of the preceding and following vehicles as well as the length $l_{i+1}$ of the following vehicle;

- headway constants $h$ and $h_c$.

V. SIMULATION RESULTS

Consider a platoon consisting of a platoon leader and four following vehicles. Suppose the initial conditions of these vehicles: 1) have initial spacing deviations of all vehicles; 2) are zero; and 3) have velocities of 0 m/s. Then, the platoon leader starts speeding smoothly from 0 to 25 m/s. The design parameters are chosen to be $k_{a1} = 5$, $k_{a2} = 1$, and $q = 3$. Two cases are considered. In the first case, the time headway is chosen as $h = 0.4$ s. In the second case, the constant spacing safety policy is adopted (i.e., $h = 0$). With the proposed vehicle following controller, the behavior of the following vehicles are shown in Figs. 4–9. Excellent velocity tracking with very small transient spacing error are achieved for both cases. Moreover, no slinky-type effects exist in either cases.
VI. CONCLUSION

An AICC law is proposed that uses relative speed and spacing measurements from both the vehicle in front and vehicle behind. The controller guarantees vehicle stability as well as platoon stability for both time headway and space headway policies without using preview information about the platoon leader.

APPENDIX

PROOF OF THEOREM 1

Let us start by considering that the leading vehicle is cruising at a constant speed, i.e., \( \dot{u}_t = 0 \).

We introduce the following variables:

\[
\begin{align*}
\dot{v}_i &\triangleq v_i - u \\
\delta_i &\triangleq x_{i-1} - x_i - l_i - h_c
\end{align*}
\]  

which satisfy

\[
\dot{\delta}_i = v_{i-1} - v_i - \delta_i - h_w.
\]  

We apply the control law given by (13) to (7)–(9) or (15) to (10)–(12) to obtain the following:

\[
\begin{align*}
m_i \dot{v}_i &= k_{d1}(\epsilon_i + q \hat{\delta}_i) - k_{d2}(\epsilon_{i+1} + q \hat{\delta}_{i+1}), \\
m_N \dot{\delta}_N &= k_{N1}(\epsilon_N - q \hat{\delta}_N),
\end{align*}
\]  

Let us consider the following quadratic function as a Lyapunov candidate:

\[
V = \sum_{i=1}^{N} \left[ \frac{\gamma_i}{2} m_i \dot{v}_i^2 + \frac{\kappa_i}{2} (\delta_i - h_w)^2 \right]
\]  

where \( \gamma_i, \kappa_i > 0 \) are to be chosen.
The time derivative of $V$ along the solution of (A.1) and (A.3) is given by

\[ \dot{V} = \sum_{i=1}^{N-1} \gamma_i v_r \left[ k_{i1} (c_i + q \delta_i) - k_{i2} (c_{i+1} + q \delta_{i+1}) \right] + \gamma_N v_r k_{N1} (c_N + q \delta_N) + k_1 (d_1 - hw) (-v_r) \]

\[ + \sum_{i=1}^{N} \kappa_i \delta_i - hw (v_{r,-1} - v_r) \]

\[ = \sum_{i=1}^{N-1} \gamma_i v_r \left[ k_{i1} (c_i + q \delta_i) - k_{i2} (c_{i+1} + q \delta_{i+1}) \right] + \gamma_N v_r k_{N1} (c_N + q \delta_N) - \sum_{i=1}^{N-1} \gamma_i \cdot v_r (k_{i1} h w_i - k_{i2} h w_{i+1}) - \gamma_N v_r k_{N1} h w_N \]

\[ + \kappa_1 \delta_1 (-v_r) + \sum_{i=2}^{N} \kappa_i \delta_i (v_{r,-1} - v_r) \]

\[ + \kappa_1 h w v_r - \sum_{i=2}^{N} \kappa_i h w (v_{r,-1} - v_r) \]

\[ = \dot{V}_0 + q \dot{V}_q + h \dot{V}_h \]

where

\[ \dot{V}_0 = \sum_{i=1}^{N-1} \gamma_i v_r (k_{i1} \delta_i - k_{i2} \delta_{i+1}) + \gamma_N v_r k_{N1} \delta_N \]

\[ - \kappa_1 \delta_1 v_r + \sum_{i=2}^{N} \kappa_i \delta_i (v_{r,-1} - v_r) \]

\[ \dot{V}_q = \sum_{i=1}^{N-1} \gamma_i v_r (k_{i1} \delta_i - k_{i2} \delta_{i+1}) + \gamma_N v_r k_{N1} \delta_N \]

\[ \dot{V}_h = - \kappa_1 \delta_1 v_r + \sum_{i=2}^{N} \kappa_i \delta_i (v_{r,-1} - v_r) \]

We evaluate the above three quantities separately. First, we rewrite $\dot{V}_0$ as follows:

\[ \dot{V}_0 = \gamma_1 k_{11} \delta_1 v_r + \sum_{i=2}^{N} \delta_i (-\gamma_{i-1} k_{i2} v_{r,-1} + \gamma_i k_{i2} v_r) - \kappa_1 \delta_1 v_r + \sum_{i=2}^{N} \kappa_i \delta_i (v_{r,-1} - v_r) \]

Second, let us consider $\dot{V}_q$

\[ \dot{V}_q = \sum_{i=1}^{N-1} \gamma_i v_r (k_{i1} (v_{r,-1} - v_r) - k_{i2} (v_{r,-1} - v_{r+1})) + \gamma_N v_r k_{N1} (v_{r,-1} - v_{r+1}) \]

\[ = -\gamma_1 k_{11} v_r^2 + \sum_{i=2}^{N-1} \gamma_i v_r (k_{i1} v_{r,-1} - v_r) - \sum_{i=1}^{N-1} \gamma_i v_r k_{i2} (v_r - v_{r+1}) \]

\[ = -\kappa_1 v_r^2 + \sum_{i=2}^{N-1} \gamma_i v_r k_{i1} (v_{r,-1} - v_r) \]

\[ = \kappa_{i+1} v_r (v_r - v_{r+1}) \]

Third, the term $\dot{V}_h$ can be computed as

\[ \dot{V}_h = \sum_{i=1}^{N-1} \gamma_i v_r (-\kappa_i v_i + \kappa_{i+1} v_{i+1}) - v_r N \]

\[ A_q = \begin{pmatrix} \kappa_1 + \kappa_2 & -\kappa_2 & 0 & \cdots & 0 \\ -\kappa_2 & \kappa_2 + \kappa_3 & -\kappa_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_{N-1} + \kappa_N & -\kappa_{N-1} \end{pmatrix} \]

\[ U_T = (v_r, v_r, v_r, \ldots, v_r) \]

\[ U_T = \begin{pmatrix} \kappa_1 + \kappa_2 & -\kappa_2 & 0 & \cdots & 0 \\ -\kappa_2 & \kappa_2 + \kappa_3 & -\kappa_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_{N-1} + \kappa_N & -\kappa_{N-1} \end{pmatrix} \]

\[ A_q = \begin{pmatrix} \kappa_1 v_r \delta_1 v_r + \sum_{i=2}^{N} \kappa_i v_i (v_{r,-1} - v_r) + \kappa_{i+1} v_{i+1} v_r \end{pmatrix} \]

\[ = \sum_{i=1}^{N} \kappa_i v_i (v_{r,-1} - v_r) \]

\[ = -\kappa_1 v_r - \sum_{i=2}^{N} \kappa_i v_i (v_{r,-1} - v_r) \]

\[ = -\kappa_1 v_r^2 + \sum_{i=2}^{N} \kappa_i v_i (v_{r,-1} - v_r) \]

\[ = -\kappa_1 v_r^2 + \sum_{i=2}^{N} \kappa_i v_i (v_{r,-1} - v_r) \]

\[ = -\kappa_1 v_r^2 + \sum_{i=2}^{N} \kappa_i v_i (v_{r,-1} - v_r) \]

If we choose $\gamma_i, \kappa_i$ such that

\[ \gamma_i = \gamma_{i+1} = \gamma_i, \quad i = 2, 3, \ldots, N \]

then it is easy to see that all the $\gamma$ and $\kappa$ terms cancel out and we end up with

\[ \dot{V}_0 = 0 \]
where
\[
A_h = \begin{pmatrix}
\frac{-k_2}{2} & 0 & \cdots & 0 \\
\frac{k_2}{2} & \frac{-k_3}{2} & \cdots & 0 \\
0 & \frac{k_3}{2} & \ddots & \vdots \\
& & \ddots & -\frac{k_{N-1}}{2}
\end{pmatrix}
\]

Finally, we combine (A.6)–(A.8) and get
\[
\dot{V} = -U^T(qA_q + hA_h)U. \tag{A.9}
\]

Apparently, if \(qA_q + hA_h > 0\), then the stability of the system [(A.1) and (A.3)] is guaranteed by the single Lyapunov function \(V\).

We proceed by establishing the following properties for the matrices \(A_q\) and \(A_h\).

**Lemma A.1:** If \(k_i > 0\), \(\forall i\), then we have the following.
- The matrix \(A_q\) is positive definite, i.e., there exist a scalar constant \(\alpha > 0\) such that \(A_q \geq \alpha I\).
- If \(k_i\)'s satisfy either \(k_1 \geq k_2 \geq \cdots \geq k_N\) or \(k_1 \leq k_2 \leq \cdots \leq k_N\), then the matrix \(A_h\) is positive semidefinite, i.e., \(A_h \geq 0\).

**Proof:** Consider the quadratic function \(x^T A_q x\), where \(x = (x_1, x_2, \cdots, x_N)^T\) is an arbitrary \(N\) by 1 vector. Using the fact that
\[
x_i^2 + (x_i - x_j)^2 \geq \frac{1}{2} (x_i^2 - (x_i - x_j)^2) = \frac{1}{2} x_j^2
\]
we have
\[
x^T A_q x = k_1 x_1^2 + \sum_{i=2}^{N} k_i (x_{i-1} - x_i)^2 \\
\geq k_{\min} \left[ x_1^2 + \sum_{i=2}^{N} (x_{i-1} - x_i)^2 \right] \\
\geq k_{\min} \left[ x_1^2 + \frac{1}{2} \sum_{i=2}^{N} (x_{i-1} - x_i)^2 \right] \\
\geq \frac{k_{\min}}{2^N} \sum_{i=1}^{N} x_i^2 = \frac{k_{\min}}{2^N} x^T x
\]
where \(k_{\min} = \min \{k_1, k_2, \cdots, k_N\}\). Hence, \(A_q \geq \alpha I\) with \(\alpha = \frac{k_{\min}}{2^N-1}\).

Now consider \(x^T A_h x\). Assuming \(k_1 \geq k_2 \geq \cdots \geq k_N\), we have
\[
x^T A_h x = \frac{k_1}{2} x_1^2 + \frac{1}{2} \sum_{i=1}^{N-1} (k_i x_i^2 - k_{i+1} x_i x_{i+1} + k_{i+1} x_{i+1}^2) \\
\geq \frac{k_1}{2} x_1^2 + \frac{1}{2} \sum_{i=1}^{N-1} k_{i+1} (x_i - x_{i+1})^2 \\
\geq 0.
\]

Hence, \(A_h \geq 0\). Due to the symmetry of \(A_h\), similarly we can obtain \(A_h \geq 0\) if \(k_1 \leq k_2 \leq \cdots \leq k_N\).

From the definition of \(k_i\) which is stated in (A.5), we see that \(k_1 \geq k_2 \geq \cdots \geq k_N\) or \(k_1 \leq k_2 \leq \cdots \leq k_N\) holds if and only if \(k_{i1} \geq k_{2i} \) or \(k_{i1} \leq k_{2i} \), \(\forall i\), respectively. However, from the condition of the theorem, we have that \(k_{i1} \geq k_{2i}\), which in turn implies that \(k_1 \geq k_2 \geq \cdots \geq k_N\). Therefore, from Lemma A.2, we have that the derivative of the Lyapunov function \(V\) given in (A.4) satisfies
\[
\dot{V} \leq -q \alpha \sum_{i=1}^{N} x_i^2,
\]
which further indicates that \(x_i \rightarrow 0\) as \(t \rightarrow \infty\). For now, we have proved part 1 of Theorem 1 and partially part 2.

To prove the convergence of \(x_i\), let us consider the following quadratic functions:
\[
\dot{x}_i = \frac{1}{2} c_i^2, \quad i = 1, 2, \cdots, N.
\]

The derivative of \(\dot{x}_i\) along the solution of (A.1) and (A.3) is
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i,
\]
\[
\dot{\dot{x}}_i = \left( 1 - \frac{h}{m_i} k_{i1} q_i \right) x_i + \frac{h}{m_i} k_{i2} x_i + \frac{h}{m_i} k_{i2} q_i + \frac{h}{m_i} k_{i2} q_i - \frac{h}{m_i} k_{2i} \dot{x}_i + \frac{h}{m_i} k_{2i} \dot{x}_i.
\]

Then, \(\dot{\dot{x}}_i\) can be expressed as
\[
\dot{\dot{x}}_i = -d_i e_i^2 + g_i x_i + h_i x_i + E^T \Delta_i U,
\]
where
\[
E^T = (e_1, e_2, \cdots, e_N).
\]
It is worth mentioning that $d_i > 0$ and the signs of the other coefficients are not important.

Now consider
\[
V_T = \sum_{i=1}^{N} T_i
\]
and we have
\[
\dot{V}_T = -\sum_{i=1}^{N} d_i \epsilon_i^2 + \sum_{i=1}^{N} g_i \epsilon_i \zeta_{i+1} + \sum_{i=1}^{N} E^T \Delta_i U
\]
\[
\leq -\sum_{i=1}^{N} d_i \epsilon_i^2 + \frac{1}{2} \sum_{i=1}^{N} g_i \epsilon_i^2 + \frac{1}{2} \sum_{i=1}^{N-1} g_i \epsilon_i^2 + E^T \Delta U
\]
where $\Delta = \sum_{i=1}^{N} \Delta_i$ and, for notational convenience, $\epsilon_{N+1} = 0$. Using the definitions of $d_i$ and $g_i$ and the condition of the theorem (note that from the condition of the theorem and the definitions of $d_i g_i$ we have that $g_i - \frac{1}{2} d_i < 0$ for all $i$), it can be easily seen that the above inequality can be rewritten as follows:
\[
\dot{V}_T = -\frac{1}{2} \sum_{i=1}^{N} d_i \epsilon_i^2 + E^T \Delta U.
\]
Denote $d_{\min} = \min \{d_1/2, d_2/2, \ldots, d_N/2\}$ and then
\[
\dot{V}_T \leq -d_{\min} E^T E + E^T \Delta U
\]
\[
\leq -d_{\min} E^2 + \frac{1}{2} \sum_{i=1}^{N} g_i \epsilon_i^2 + \frac{1}{2} \sum_{i=1}^{N} g_i \epsilon_i^2 + \frac{1}{2} \sum_{i=1}^{N-1} g_i \epsilon_i^2 + E^T \Delta U
\]
Finally, we choose
\[
V_N = V_T + \beta \mathcal{V}
\]
where $V$ is as defined in (A.4), and $\beta = (\|\Delta\|^2/2q_0 d_{\min}) + (d_{\min}/2q_0)$. Then
\[
\dot{V}_N \leq -d_{\min} \frac{1}{2} (|E|^2 + |U|^2).
\]
It is also clear from (A.4) that
\[
V = \frac{1}{2} \sum_{i=1}^{N} \left[ m_i \dot{x}_i^2 + \kappa_i (\epsilon_i - h \nu_i)^2 \right] \leq \frac{1}{2} \alpha_0 (|E|^2 + |U|^2)
\]
where
\[
\alpha_0 = \max \{m_i + 2\kappa_i h^2, 2\kappa_i\}.
\]
Hence
\[
V_N \leq \frac{\alpha_{\max}}{2} (|E|^2 + |U|^2)
\]
where
\[
\alpha_{\max} = \alpha_0 + 1.
\]
Combining (A.11) and (A.12)
\[
\dot{V}_N \leq -\frac{d_{\min}}{\alpha_{\max}} V_N
\]
which implies that $V_N \to 0$ as $t \to \infty$ exponentially, and all the components of $V_N$, i.e., the spacing and velocity deviations $\epsilon_i, \nu_i$, all converge to zero exponentially. Note that the constants $\alpha_{\max}$ and $d_{\min}$ are independent of the size $N$ of the platoon.

Now let us consider the case where $\dot{\nu}_i \to 0$. Then, the vehicle dynamics (A.3) will be
\[
m_i \ddot{x}_i = k_{22} (\epsilon_i + q_i \dot{\nu}_i) - k_{22} (\epsilon_{i+1} + q_i \dot{\nu}_{i+1}) + m_i \ddot{\nu}_i,
\]
\[
\forall i = 1, 2, \ldots, N = 1
\]
\[
m_N \ddot{\nu}_N = k_{N1} (\epsilon_N + q_N \dot{\nu}_N) + m_N \ddot{\nu}_N
\]
and the derivatives of all the functions $V, V_T$ will contain cross terms of $\dot{\nu}_i$ with $\epsilon_i$ or $\nu_i$. That is, we will have
\[
\dot{V}_N \leq -\frac{d_{\min}}{\alpha_{\max}} V_N + (E^T \Delta_x + U^T \Delta_v) \dot{\nu}_i
\]
where $\Delta_x$ and $\Delta_v$ are $N \times 1$ constant vectors which depend only on the known parameters. Considering the fact that
\[
|E^T \Delta_x + U^T \Delta_v| \leq c (|E|^2 + |U|^2)
\]
where $c > 0$ is a constant which is independent of the size of the platoon, it is readily obtained by completing the squares as we have done before that
\[
\dot{V}_N \leq -\tilde{\alpha}_1 V_N + \tilde{\alpha}_2 \dot{\nu}_i^2
\]
for some $\tilde{\alpha}_1, \tilde{\alpha}_2 > 0$. Therefore, if $\dot{\nu}_i$ converges to zero exponentially, $V_N$ still converges to zeros exponentially and the same result for $\dot{\nu}_i = 0$ will hold. If, however, $\dot{\nu}_i \to 0$ only asymptotically, then we still have $\epsilon_i, \nu_i \to 0$ as $t \to \infty$, but with the same rate as $\dot{\nu}_i$ instead of exponentially.

We will now prove part three of the theorem. In order to do so, let us consider again inequality (A.15). Then we have that for any $t$ and $t_0$ such that $t > t_0$
\[
V_N(t) \leq e^{-\tilde{\alpha}_1 (t-t_0)} V_N(t_0) + \tilde{\alpha}_2 \int_{t_0}^{t} e^{-\tilde{\alpha}_1 (t-t')} \dot{\nu}_i^2(t') \, dt'.
\]
It suffices to show that when the system is in steady state, any bounded change in the control actions of the leading vehicle will have the effect that all spacing and velocities of the vehicles in the platoon are bounded by bounds that are independent of the size of the platoon. Suppose that such a change in the control actions of the leading vehicle occurs at time $t_0$; since the system is in steady state we have that $V_N(t_0) = 0$ and therefore
\[
V_N(t) \leq \tilde{\alpha}_2 \int_{t_0}^{t} e^{-\tilde{\alpha}_1 (t-t')} \dot{\nu}_i^2(t') \, dt'.
\]
Since we are considering only bounded changes in the control actions of the leading vehicle, we have that $\dot{\nu}_i(t) < \overline{\nu}$ for all $t$ and some positive constant $\overline{\nu}$ and therefore
\[
V_N(t) \leq \tilde{\alpha}_2 \overline{\nu} \int_{t_0}^{t} e^{-\tilde{\alpha}_1 (t-t')} \, dt' \leq \frac{\tilde{\alpha}_2 \overline{\nu}}{\tilde{\alpha}_1}.
\]
It can be easily seen that the constants \( \overline{c_1} \) and \( \overline{c_2} \) are independent of the size of the platoon (in fact, the constants \( \overline{c_1} \) and \( \overline{c_2} \) depend on the variables \( d_{\text{main}}, \delta_{\text{max}}, \) and \( \phi \)). Therefore, we have that \( V_N \) is bounded from above by a constant that is independent of the size of the platoon, which, in turn, implies that the variables \( \delta_Y \), and \( c_2 \) are bounded by constants that are independent of the size of the platoon, which concludes the proof of part three.

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