of the processors in such a manner that if it is not optimal to assign a job to processor \( i \) then it is not optimal to make an assignment to processor \( i \) until such time as there is at least one more arrival. That is, the state cannot change to one in which it becomes optimal to assign a job to processor \( i \) simply by the departure of jobs. This is the behavior we have observed numerically and argued for above.

REFERENCES


On the Stability Proof of Adaptive Schemes with Static Normalizing Signals and Parameter Projection

T. Tsao and P. A. Ioannou

Abstract—Recently, it has been shown [8] that robust stability of model-reference adaptive control (MRAC) is guaranteed by simply using a static, instead of a dynamic, normalizing signal together with a parameter projection in the adaptive law. In this note, we show that the stability proof associated with such modification follows directly from the proof of schemes with dynamic normalization. Consequently, the proofs and stability arguments used in [8] can be simplified considerably.

I. INTRODUCTION

Robustness has been an important issue in the study of adaptive control theory, and many successful modified schemes have been proposed to improve robustness with respect to unmodeled dynamics and bounded disturbances, and guarantee signal boundedness for any given initial conditions. One common feature of most robust and globally stable schemes is that the adaptive law employs a model-reference adaptive control (MRAC) scheme. The stability analysis used to establish this result is long and rather complicated and deviates from that used in the ideal case or in the case of schemes with a dynamic normalizing signal. In this note, we establish the same result by slightly modifying the proof associated with the dynamic normalizing signal. The proof is a direct application of the approach in [9], which unifies the design and analysis of various modified robust adaptive controllers. The approach of [9] is a simplification of that taken in [4], on which [8] relies to a large extent. The following notation is used in this note.

We consider the following plant:

\[ y(t) = k_p \frac{Z_m(s)}{R_m(s)} (1 + \Delta_m(s))u(t) \]  

and reference model

\[ y_r(t) = \frac{Z_m(s)}{R_m(s)} u_r(t) \]  

where \( k_p \) is a constant; \( Z_m(s), R_m(s) \) are monic Hurwitz polynomials of degree \( n_m, n_r \), respectively; and the poles and zeros of \( W_m(s) \) are all in \( \Re \{s\} < -(p/2) \) for some \( p > 0 \). The following assumptions are made for the plant-transfer function.

1) \( Z_m(s) \) and \( R_m(s) \) are monic polynomials of degree \( m \) and \( n \), respectively, \( Z_m(s) \) is Hurwitz, \( m \leq n - 1 \), and \( k_p \) is a constant gain with known sign.
2) The relative degree of \( W_m(s) \) satisfies \( n_r - n_m = n - m \) and \( n_r \leq n \).
3) \( \Delta_m(s) \) is a stable transfer function such that \( \Delta_m W_m \) is strictly proper and analytic in \( \Re \{s\} \geq -(p/2) \).

Additive plant perturbations and bounded disturbances may also be included in (1) without altering the analysis and results of this note. Consequently, such perturbations are omitted without loss of generality.

The model-reference adaptive control law is given by

\[ u(t) = \theta_i^*(t) w(t) = \theta_i^*(t) \frac{a(s)}{A(s)} u(t) + \theta_i^*(t) \frac{a(s)}{A(s)} y(t) + \theta_i^*(t) y_i(t) + c_i(t) r(t) \]  

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where
\[ a^T(s) = [s^{n-2}, s^{n-3}, \ldots, 1] \]
\[ \Lambda(s) = \Lambda_0(s)Z_2(s) \quad \text{and} \]
\[ \Lambda_0(s) \] is a monic Hurwitz polynomial of degree \( n - m - 1 \)
\[ \theta^T(t) = [\theta^1(t), \theta^2(t), \theta^3(t), c_0(t)] . \]

The parameter update law is
\[ \dot{\rho}(t) = \rho^2(t) - \gamma(\rho(t) \xi(t)) \]
\[ \dot{\theta}(t) = \rho(t) - \Gamma(e(t) \xi(t) \text{sgn} \, \rho^*) \quad (4) \]
where
\[ \rho^* \] is defined as the upper bound of \( \rho^* - \frac{1}{\rho_0} = (k_0/K_0) \), respectively; \( M \) is the unknown upper bound of \( ||\theta^*|| \).

Following [9], we choose
\[ v(t) = \frac{1}{2} (\frac{1}{\rho_0^2} + \frac{1}{\gamma^2}) \rho^* \xi(t) e(t) \]
whose time derivative along (1)-(4) satisfies
\[ \frac{d}{dt} V = \frac{1}{2} (\frac{1}{\rho_0^2} + \frac{1}{\gamma^2}) \rho^* \xi(t) \dot{e}(t) \]
and by following the approach of [9], we can show that
\[ \theta^T(t) \xi(t) + c_0^*(\gamma(t) \xi(t)) = \theta^* \eta(t) + k_1(t) \]
\[ y(t) = W_m(s) \eta(t) + \frac{1}{c_0^*} W_m(s) \xi(t) \eta(t) + \theta^* \eta(t) \]
\[ \dot{\theta}(t) = \frac{\Delta_0(s)W_m(s)\theta^T g(s) - \theta^* \lambda(s)}{c_0^* \lambda(s)} \quad (5) \]
where
\[ \rho_{\text{max}}, \rho_{\text{min}} \] are the known upper and lower bounds of \( \rho^* - \frac{1}{\rho_0} = (k_0/K_0) \), respectively; \( M \) is the unknown upper bound of \( ||\theta^*|| \).

Theorem 1: For all \( \Delta_m \) satisfying
\[ \| \Delta_m(s)W_m(s) (\frac{\theta^T g(s) + \theta^* \lambda(s)}{c_0^* \lambda(s)}) \| < 1, \]
\[ \| \Delta_m(s)W_m(s) (1 - \theta^T a(s)/\lambda(s)) \| \leq c_1 \]
for some constants \( c_1 > 0 \) and \( 0 < \delta < \rho \), we have that all signals in the MRAC system, described by (1)-(4), are uniformly bounded.

Proof: From (1), (3), and
\[ W_m(s) = \frac{k_0Z_2(s)\lambda(s)}{\Lambda_0(s)\lambda(s) - \theta^T g(s) - k_0Z_2(s)(\theta^T g(s) + \theta^* \lambda(s))} \]
where $\mathcal{N}(t) \subseteq \mathcal{F}, \forall y \geq 0$, and $\mathcal{F}(t) \geq 0$ is a finite constant since $\rho, \psi \in L_0$, due to the use of projection in (4). So, from the definition of "small" in the mean property, we have

$$\dot{\varphi} + \epsilon \varphi \leq \frac{\eta^2 + \kappa_2^2}{\kappa_1}.$$  \hspace{1cm} (7)

Define the fictitious normalizing signal $m(t)$ to be

$$m^2(t) = 1 + m(t),$$  \hspace{1cm} (8)

where

$$m(t) = -\delta m(t) + u^2(t) + y^2(t), \quad m(T_0) = 0, 0 < \delta < \rho$$

and $T_0$ is some nonnegative real number whose value will be determined later. Since $\rho(t), \psi(t) \in L_0$, all signals are finite in the finite time interval $[0, T_0]$ it is sufficient to show boundedness of signals for $t \geq T_0$. In the following, we are going to show that $m(t)$ is uniformly bounded in $[T_0, \infty)$ when the upper-bound conditions on uncertainty (5) are satisfied and that $c_i$ is independent of the size of initial conditions at $t = 0$.

From (6) and the definition of $\|x\|_x$, we have

$$\|x(t)\|_x + \|y(t)\|_y \leq \frac{\Delta_m(s)((\epsilon^2 + \kappa_2^2)\Lambda(s))^{1/2}}{\epsilon^2 + \kappa_2^2} \|x(t)\|_x + \frac{\epsilon}{\kappa_1} \|x(t)\|_x + \frac{\kappa_2}{\kappa_1} \|x(t)\|_x$$

where $c$ and $k$ are generic symbols for nonnegative constants; and $k$ is used to denote those quantities dependent on the size of initial conditions. When (5) is satisfied, we have

$$\|x(t)\|_x + \|y(t)\|_y \leq \frac{\Delta_m(s)((\epsilon^2 + \kappa_2^2)\Lambda(s))^{1/2}}{\epsilon^2 + \kappa_2^2} \|x(t)\|_x + \frac{c}{\kappa_1} \|x(t)\|_x + \frac{k}{\kappa_1} \|x(t)\|_x$$

(9)

where $c$ and $k$ are generic symbols for nonnegative constants; and $k$ is used to denote those quantities dependent on the size of initial conditions. When (5) is satisfied, we have

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(10)

Applying the swapping lemma to $\dot{\theta}^2$, and using the properties of the $\|x\|-\|x\|$ norm given in [9], we have

$$\|\dot{\theta}^2(t)\|_2 + \|y^2(t)\|_2 \leq \frac{\Delta_m(s)(\epsilon^2 + \kappa_2^2)\Lambda(s))^{1/2}}{\epsilon^2 + \kappa_2^2} \|x(t)\|_x + \frac{c}{\kappa_1} \|x(t)\|_x + \frac{k}{\kappa_1} \|x(t)\|_x$$

(11)

where $c$ and $k$ are generic symbols for nonnegative constants; and $k$ is used to denote those quantities dependent on the size of initial conditions. When (5) is satisfied, we have

$$\|\dot{\theta}^2(t)\|_2 + \|y^2(t)\|_2 \leq \frac{\Delta_m(s)(\epsilon^2 + \kappa_2^2)\Lambda(s))^{1/2}}{\epsilon^2 + \kappa_2^2} \|x(t)\|_x + \frac{c}{\kappa_1} \|x(t)\|_x + \frac{k}{\kappa_1} \|x(t)\|_x$$

(12)

where $W_1(s) = (-C(sI-A)^{-1})$, $W_2(s) = -(sI-A)^{-1}B$, and $W_3(s) = C(sI-A)^{-1}B$. When $\alpha$ is chosen large enough, (10)-(12) give

$$m^2(t) \leq c + ke^{-2\alpha T_0} + \|\dot{\theta}^2(t)\|_2 + c\|\dot{\theta}(t)\|_2 + c\|\dot{\theta}(t)\|_2 + c\|\dot{\theta}(t)\|_2$$

$$\|x(t)\|_x + \|y(t)\|_y \leq \frac{\Delta_m(s)(\epsilon^2 + \kappa_2^2)\Lambda(s))^{1/2}}{\epsilon^2 + \kappa_2^2} \|x(t)\|_x + \frac{c}{\kappa_1} \|x(t)\|_x + \frac{k}{\kappa_1} \|x(t)\|_x$$

(13)

which is independent of initial conditions. Up to (13), except for the consideration of initial conditions, the analysis is the same as that in [6] and [10] and does not depend on the form of normalization used in the projection. If $n^2$ is a dynamic normalizing signal of the type considered in [9] and [10], then $(m/n) \in L_n$ and, therefore,

$$\int_{T_0}^{\infty} m^2(t) \leq 0$$

(14)

which together with (13) and (7) imply that $m \in L_n$ and $-c(m(t) - c)$ provided $1 - \mu + \Delta\Delta^2 > 0$.

The above analysis for the static normalization $n^2 = 1 + \|\alpha\|^2 + \|\alpha\|^2$ is only slightly different. The argument is as follows.
Since \( m_f(T_0) = 0 \) we have \( m_f(t) \geq 0 \) for \( t \in [T_0, T_1] \) for some \( T_1 > 0 \), which, together with \( \theta \in L_1 \), implies that

\[
m_f(t) \leq \frac{u^2(t)y^2(t)}{\delta} = \frac{(\theta^2(t)ao(t)^2 + y^2(t)}{\delta} \leq \epsilon n^2(t)
\]

for some \( c > 0 \).

\[
\text{i.e., } \left[ m^2(t)/n^2(t) \right] = [1 + M\theta(t)/n^2(t)] \leq c_{\delta} \text{ for some } c_{\delta} > 0 \forall t \in [T_0, T_1].
\]

Applying the Gronwall lemma for \( t \in [T_0, T_1] \) we obtain

\[
m(t) \leq c_{\epsilon} \forall t \in [T_0, T_1].
\]

Provided \( \Delta^2 \leq (\beta/c_{\delta}) \) for some small positive \( \beta \) satisfying \( 1 - \mu_2 - \epsilon \beta > 0 \), where \( c_\delta \) is a constant independent of \( T_1 \). Although \( T_0 \) is a function of the size of initial conditions, \( \mu_1, \mu_2 \) are not; hence \( c_{\delta} \) is also independent of initial conditions at \( t = 0 \). If \( T_0 < \infty \), we are done. If \( T_0 > \infty \), then we have \( m_0(t) > 0 \) \( \forall t \in [T_1, T_2] \) and some \( T_2 > T_1 \). But \( m_0(t) < 0 \) implies that \( m_0(t) < m_0(T_1) \forall t \in [T_1, T_2] \) or \( [m_0^2(t)/n_0^2(t)] \leq m_0(t) \leq m_0(T_1) \leq c_{\delta} \forall t \in [T_1, T_2] \). If \( T_2 = \infty \), we are done. If \( T_2 < \infty \), then we have \( m_0(t) \geq 0 \forall t \in [T_1, T_2] \) and some \( T_2 > T_1 \) which implies that \( [m_0^2(t)/n_0^2(t)] \leq \max(c_{\delta}, c_\gamma) \forall t \in [T_1, T_2] \), which in turn implies that for

\[
\Delta^2 \leq -\frac{\beta}{\max(c_{\delta}, c_\gamma) + c_\gamma^2}
\]

\( m(t) \leq c_{\epsilon} \forall t \in [T_0, T_1] \). Since \( c_{\epsilon} \) and \( c_\gamma \) are independent of the interval, we can conclude that \( m(t) \leq c_{\epsilon} \forall t \geq T_0 \) provided \( \Delta \leq \epsilon \) by following the same procedure. Also, \( c_\epsilon \) is independent of initial conditions at \( t = 0 \) because neither are \( c_\delta \) and \( c_\gamma \).

Since \( m(t) = 1 + \|u(t)\|^2 - (\|y(t)\|^2) \geq 0 \) for \( t \geq T_0 \) and \( m \in L_1[T_0, \infty) \), we can establish for \( t \geq T_0 \) the boundedness of \( u(t), y(t) \), and all other signals in the closed-loop system from the boundedness of \( m(t) \) by following the same arguments as in the case of a dynamic normalization [9].

Remark: If the plant (1) is changed to include the kind of uncertainty considered in [8], i.e.,

\[
y(t) = \frac{Z(s)}{R_0(s)} \left( 1 - \Delta_u(s) \right) u(t) + \sum \Delta_u(s) \varepsilon(t) + \frac{\Delta_u(s)}{c(t)}
\]

where \( \Delta_u(s) \) and \( \Delta_u(s) \) are stable and strictly proper, and \( \varepsilon(t) \) is a bounded disturbance, then with minor modifications on (6) we can still show the boundedness of all signals provided \( ||\Delta_u||^2 \) and \( ||\Delta_u||^2 \) are small enough.

Remark: The proof of [8] relies on the results of [4] and includes additional arguments to handle the static normalization signal. The results of this note rely on those of [9], which are simplified versions of those in [4]. The simplifications emphasized by this note concerns the additional arguments to handle the static normalization signal. The complex arguments of [8] result from the fact that the static normalization signal \( n(t) \) employed does not contain the term \( y^2(t) \), which is essential in the establishment of the inequality (14) for the \( m_f(t) \leq 0 \) situation.

III. Conclusion

In this note, we show that the stability proof for the MRAC scheme with static normalization given in [8] can be simplified considerably by adding the term \( y^2 \) in the static normalization signal.