EXTERNAL THREAT AND COLLECTIVE ACTION

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This article studies how players allocate their endowed resources between productive and conflictual activities in the context of rivalry between two groups. We show that the suboptimality and exploitation propositions established by Olson (1965) do not necessarily apply when external threat is endogenized. We also illustrate that it does not always pay to take an offensive stance. When competing with an offensive group, it might be better for members of a defensive group to remain defensive. Furthermore, in the context of rivalry between two groups, free riding can actually benefit everyone in the system. (*JEL* D70, D74)

I. INTRODUCTION

Individuals with common interests usually attempt to further their welfare by acting collectively. Examples of collective action include the provision of a public good or collective good, the establishment of clubs, and the formation of alliances or political parties. In his seminal work *The Logic of Collective Action*, Olson (1965) uses the provision of collective goods as an example to discuss collective action problem or coordination failure. In particular, he analyzes how group size and the distribution of individual benefits of a collective good may affect the provision level of the good. Two of the most celebrated propositions from his book are the suboptimality proposition—"the larger the group, the farther it will fall short of providing an optimal amount of a collective good" (p. 35), and the exploitation proposition—"there is a systematic tendency for "exploitation" of the great by the small" (p. 29). These propositions have generated a broad research effort in collective action problems in economics and political science. Many scholars have examined the validity of these propositions.[1] A limitation of Olson's theory is that he analyzed collective action problems as phenomena within a single group, while assuming that external threat to a group either does not exist or is treated as a constant. In many cases, however, two or more groups compete against each other. In these situations, a group's provision of collective goods might be affected by the configurations of its own group and the competing groups and relative strength of the groups.

To illustrate, assume that two alliances of states, say, A and B, are competing for supremacy. An increase in A's military spending poses a greater threat to B's security. In response to the heightened threat, members of B might feel the need to increase their military spending, which in turn could affect the military spending decisions of members of A. Bruce (1990) develops a model that endogenizes external threat in a three-country model with two allies and one adversary. Interest

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1. An implicit assumption in Olson's analysis is that the total group endowments (or benefits) are fixed and that the group's pareto-optimal level of the collective good is independent of group size. As the number of members in the group increases, an individual's share of endowments (or benefits) decreases and the total provision of the collective good actually declines. When the group's total endowments are not fixed, the total provision of the collective good can actually increase as the group size grows. McGuire (1974), for example, shows that if individual members have identical endowments and utility functions, as the number of group members increases, the total provision level of the collective good does not necessarily fall, though it falls even shorter of the pareto-optimal level. See Sandler (1992, 1993) for a general discussion of these situations.

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group lobbying is another example. To understand a group's lobbying behavior, scholars often focus on the number and size of firms within the group and their geographical distribution. However, the configuration of a competing interest group and the two groups' relative strengths should also figure into an individual firm's decision concerning the appropriate contribution to its group's lobbying efforts. Other examples include party discipline, voter turnout (Ledyard 1981, 1984; Palfrey and Rosenthal 1983), electoral coalitional politics (Tsebelis 1990; Hausken 1995a, b), negative campaigning in primary elections (Newman and Niou 2000), factional rivalry in civil wars, and joint ventures for research and development in industrial organizations. All these examples point to the importance of external threat in explaining the logic of collective action.

In this article, to study simultaneously the way intergroup competition and intragroup collective action interact, we develop a collective good model that endogenizes external threat. In the context of competition between two groups, a group can be either defensive or offensive in nature. We define a group as defensive if its fighting capacity can be used only to defend its own resources; a group is offensive if its fighting capacity can be used to expropriate resources from its rivals as well as protect its own resources. We realize that empirically it is difficult to determine whether a group is offensive or defensive. But theoretically it is useful to know how the offensive or defensive nature of a group can affect the collective action problem. We analyze collective action problems when: (1) one of the groups is offensive whereas the other is defensive, and (2) both groups are offensive, respectively. Also, in group competition, if conquests are economically profitable, the way spoils of victory are divided among group members can also affect collective action. To study the effects of different division rules on collective action, we consider two types of division rules: proportional and equal.

The organization of this article is as follows. In the next section we develop a collective good model that endogenizes external threat, assuming that one of the groups is offensive, whereas the other is defensive. In section III we study collective action problems when both groups are offensive. In section IV we analyze the effects of proportional and equal division rules on collective action. Section V concludes with several remarks on some insights drawn from our theoretical analysis.

II. DEFENSIVE VERSUS OFFENSIVE GROUPS

Suppose groups A and B have m and n members, respectively. Let i index members of A, $i = 1, \ldots, m$, and j index members of B, $j = 1, \ldots, n$. Furthermore, suppose that (1) member i in A is endowed with resources w_A^i, where $w_A^i > 0$; (2) each i divides his endowed resources between conflictual activity, q_A^i, and productive activity $w_A^i - q_A^i$; and (3) depending on technologies of production and conflict, endowed resources are converted into income and fighting force. For simplicity, we assume that the conversion rate is one to one. These assumptions are also applied to members of B. Furthermore, we assume that (4) every member in the system seeks to maximize his or her expected net income; and (5) fighting capabilities are not productive and can only be used to defend members against aggression or to commit aggression against the other group (Viner 1948; Hirshleifer, 1991a, b, 1995; Skaperdas, 1992, 1998).

Notationally, let

$$W_A = \sum_{i=1}^{m} w_A^i, \quad W_B = \sum_{j=1}^{n} w_B^j$$

be the total endowed resources groups A and B possess, respectively. Let

$$Q_A = \sum_{i=1}^{m} q_A^i, \quad Q_B = \sum_{j=1}^{n} q_B^j$$

be the total amount of endowed resources A an B devote to fighting, respectively.[2]

Suppose that A is defensive and that each member i in A has the following expected net income,

$$(1) \qquad I_A^i(q_A^i, Q_A, Q_B) = [Q_A/(Q_A + Q_B)] \times (w_A^i - q_A^i)$$

2. In the literature on collective goods, other methods of aggregating individual members' contributions have been proposed: the weakest-link technology, the best-shot technology, and the constant elasticity of substitution technology (see Hirshleifer 1983).

if $Q_A > 0$ and $Q_B \geq 0$. In addition $I_A^i = w_A^i$ if $Q_A = Q_B = 0$, and $I_A^i = 0$ if $Q_A = 0$ and $Q_B > 0$.

Suppose that group B is offensive and that each member j in B has the following expected net income,

$$(2) \quad I_B^j(q_B^j, Q_A, Q_B) = w_B^j - q_B^j + [Q_B/(Q_A + Q_B)] \times (q_B^j/Q_B)(W_A - Q_A).$$

The payoff functions (1) and (2) have a number of distinct features that need further discussion.

1. We assume that conflict between two groups can be resolved through either fighting or bargaining, and that each group member's expected net income is positively associated with his or her group's total fighting capabilities but negatively associated with its adversary's fighting capabilities. The specific functional relation we use takes the following ratio form,[3]

$$Q_A/(Q_A + Q_B).$$

2. We assume that the resources spent on fighting are sunk costs (Hirshleifer 1991b; Skaperdas 1992). A more general specification of the utility function would be to assume that only a fraction, γ, $0 \leq \gamma \leq 1$, of the resources devoted to fighting are sunk. To simplify our presentation we assume that $\gamma = 1$. The theoretical results we establish in the following sections, however, hold for any $0 < \gamma \leq 1$. If $\gamma = 0$, then Q_A and Q_B are fully recoverable fixed costs. Then, trivially, all of the members in B allocate all their resources to fighting.

3. Equation (2) assumes that when two groups are competing, the spoil of victory for group B is the amount of resources group A devotes to productive purpose, $(W_A - Q_A)$. That is, the prize of conflict is endogenously determined (Hirshleifer 1989, 1991a, b, 1995). Alternatively, groups might be competing for a fixed prize (Katz et al. 1990; Esteban and Ray 2001). For instance, Esteban and Ray (2001) provide a thorough analysis of group competition when the prize of conflict is fixed. The appropriateness of each assumption depends on the context of application.

4. Equation (2) specifies that the portion of A's endowed resources $(W_A - Q_A)$ that member j expects to receive if B wins is determined by j's share of B's total spending on fighting, q_B^j/Q_B. This assumption can be justified by the following arguments: (a) during competition, group members compete with each other to acquire the adversary's resources; (b) their relative contributions to the conflict determine their relative successes. Similarly, if a conflict is settled through negotiation, we assume that j's relative contribution to the conflict determines the portion of $(W_A - Q_A)$ that he or she expects to gain. An alternative to the proportional division rule is to divide the spoils of victory equally among individual members regardless of their contribution to the group's fighting effort (Esteban and Ray 2001). In section IV we will compare the effects of different division rules on collective good provision.

5. For each member i in the defensive group, equation (1) implies that an increase in q_A^i has two effects: It improves group A's chance of winning and reduces the amount of resources that i can allocate to production. The trade-offs between these two effects determine i's optimal allocation of resources between production and fighting. But for each member j in the offensive group B, an increase in the amount of resources j devotes to fighting has an additional effect: It increases j's share of the spoils of victory. In general, the trade-offs between these three effects determine j's optimal resource allocation between production and fighting.

6. Players in our model simultaneously make their optimal once-and-for-all decisions between productive and conflictual allocations. In reality, players can often adjust their decisions in response to interaction outcomes. The task of determining the equilibrium in such a dynamic setting for members of both groups poses a challenging analytical problem that we do not attempt to address in this article.[4]

7. We assume that there are only two groups and that group members are not allowed to switch from one group to the other.

3. Other functional forms have been discussed in the literature on conflicts and rent-seeking. See Hirshleifer (1989, 1991b), Skaperdas (1992), and Neary (1997a, b).

4. Powell (1993) develops a dynamic game of this nature to study countries' choices between guns and butter in two-country systems. See Hirshleifer (1995) for an exposition of different choices in modeling conflicts.

A more general model should allow group members to choose their preferred group.[5]

We are now ready to determine the Nash equilibrium of the game. Each player chooses his or her fighting effort to maximize his or her net income in equation (1) or (2), subject to the resource constraint, and given the choices of other players. We focus on the interior solution for which resource constraints are not binding.

Taking the derivative of I_A^i with respect to q_A^i and setting the derivative equal to zero, we obtain the following first-order condition for an interior solution,

$$(3) \qquad [Q_B/(Q_A+Q_B)^2](w_A^i - q_A^i) - Q_A/(Q_A+Q_B) = 0.$$

It can be verified that the second-order condition for player i is satisfied, and as such equation (3) is sufficient for player i' optimization problem. Equation (3) implies that all of the members of group A keep the same amount of resources for production. An immediate implication of (1) and (3) is that all of the members of A receive the same payoff and that the wealthy members allocate more resources to fighting.

Aggregating equation (3) over $i = 1, \ldots, m$, we obtain

$$(4) \qquad Q_B(W_A - Q_A) = mQ_A(Q_A + Q_B)$$

from which we can solve for Q_A as $Q_A = R(Q_B, W_A, m)$, where

$$R(Q_B, W_A, m) = \sqrt{(1+1/m)^2 Q_B^2/4 + (W_A/m)Q_B} - (1+1/m)Q_B/2.$$

This is the aggregate best-reply function for group A in response to the aggregate spending by group B. It can be verified that this best-reply curve is upward-sloping and concave and intersects the 45-degree line at $Q_A = Q_B = W_A/(2m+1)$. Moreover, the best-reply function increases with W_A and decreases with m.

5. Linster (1993) and Skaperdas (1998) have already made important contributions along this line.

Similarly, the first-order condition for members of the offensive group B is

$$[(Q_A + Q_B - q_B^j)/(Q_A+Q_B)^2](W_A - Q_A) = 1,$$

which implies that all of the members of B spend the same amount on conflict. Because we assume the spoils of victory are distributed in proportion to a member's contribution to the group's fighting capacity, this result also implies that all of the members of B expect to receive equal portions of the victory spoils.

Aggregating over j, we obtain

$$(5) \qquad n(Q_A+Q_B)^2 = (nQ_A + [n-1]Q_B) \times (W_A - Q_A),$$

which determines the aggregate best-reply function for the offensive group B. Equation (5) can be equivalently written as the following quadratic equation

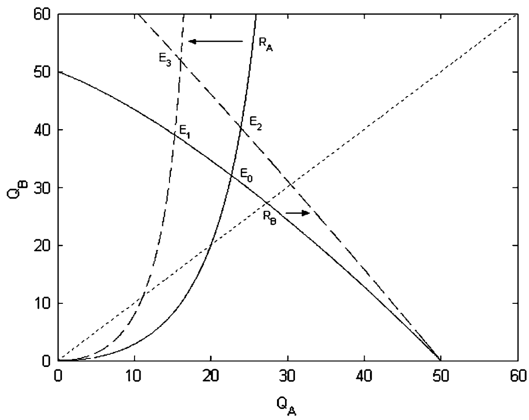
$$(6) \qquad Q_B^2 + [(3-1/n)Q_A - (1-1/n)W_A]Q_B + (2Q_A - W_A)Q_A = 0.$$

Note that when $Q_A = W_A/2$, $Q_B = 0$, and that when $Q_A > W_A/2$, equation (6) does not have a positive solution. Thus, we restrict $Q_A \in [0, W_A/2]$, for which the quadratic equation has only one positive solution.

It can be easily verified that the following properties of the positive solution hold. First, as Q_A increases, the positive solution of (6) decreases, so that the aggregate best-reply curve for the offensive group B is downward-sloping. Second, as (a) the number of members of group B, n, increases, or (b) A's total resources, w_A, increase, the quadratic function on the left-hand side of (6) decreases so that the positive solution of (6) increases, and hence the best-reply curve for B shifts upward. Third, the best-reply curve for B intersects the 45-degree line at $Q_A = Q_B = W_A(2n-1)/(6n-1)$.

In Figure 1, we illustrate the aggregate best-reply curves for groups A and B, labeled as R_A and R_B, for values $W_A = 100$ and $W_B = 60$, where the horizontal axis represents Q_A and the vertical axis represents Q_B. E_0 is the equilibrium point for $m = n = 2$. As m increases from 2 to 4, R_A shifts toward left, but R_B does not change, so the new equilibrium point (located at E_1) constitutes a drop in A's aggregate fighting effort while B's aggregate fighting

FIGURE 1
Aggregate Best-Reply Curves: Defensive (A) versus Offensive (B)



effort rises. On the other hand, as n goes up from 2 to 4, the R_A curve does not change while R_B shifts upward. The new equilibrium point is located at E_2 where both groups' fighting efforts rise. When $m = n = 4$, the new equilibrium is located at E_3.

To summarize, the aggregate best-reply curve for the offensive group is downward-sloping and moves up as either n or W_A increases. On the other hand, the aggregate best-reply curve for the defensive group is upward-sloping, decreases with m, and increases with W_A. The intersection of these two curves determines the equilibrium levels of resources devoted to conflictual activities. Proposition 1 summarizes these general properties of the equilibrium in words.

PROPOSITION 1. *Suppose that (i) only one of the groups is offensive; (ii) external threat is endogenously determined; (iii) a proportional division rule is used by members of the offensive group; and (iv) resource constraints are not binding. Then the following results hold. (a) As the number of members of the defensive group increases, the defensive group spends less on fighting but the offensive group spends more; (b) all of the members of the defensive group receive the same payoff, but wealthy members allocate more resources to fighting; (c) as the number of members of the offensive group increases, both groups increase their spending on fighting; and (d) all of the members of the offensive group devote the same amount of resources to fighting and expect to receive equal portions of the victory spoils.*

It is worth noting that the difference in spending on fighting for members of the defensive and offensive groups is determined by whether fighting capability is a pure public good and not by endogenous external threat. When i in A unilaterally increases his or her fighting effort, it increases A's chance of prevailing in the conflict. Thus, because spending on fighting is a pure public good, the incentives to free-ride by members of A rise with m, as illustrated in Figure 1, regardless whether external threat is fixed or endogenized. For each member j in B, however, an increase in j's spending on fighting not only increases B's chance of winning but also increases j's own share of the spoils of victory. So *j's* spending on fighting has a private good component that gives j a stronger incentive to contribute to B's fighting capacity.

Another interesting feature of the equilibrium is that the defensive group spends less on fighting than does the offensive group whenever $m > 2n/(2n-1)$. This can be easily demonstrated by comparing how each best-reply curve intersects with the 45-degree line. As long as the defensive group is not too small in size (i.e., $m > 2$ and $n > 1$), it always spends less than does the offensive group.

III. OFFENSIVE VERSUS OFFENSIVE GROUPS

So far we have analyzed collective action problems when only one of the groups is offensive. In this section we investigate whether the properties established in Proposition 1 can be different when the two competing groups are offensive in nature.

We assume that each member i in A has the following payoff function,

$$(7) \quad I_A^i(q_A^i, Q_A, Q_B) = [Q_A/(Q_A + Q_B)] \times (w_A^i - q_A^i + [q_A^i/Q_A] \times [W_B - Q_B])$$

if $Q_A > 0$ and $Q_B \geq 0$. In addition $I_A^i = w_A^i$ if $Q_A = Q_B = 0$, and $I_A^i = 0$ if $Q_A = 0$ and $Q_B > 0$. The payoff function for each member j in B can be specified analogously,

$$(8)\quad I_B^j(q_B^j, Q_A, Q_B) = [Q_B/(Q_A + Q_B)] \times [w_B^j - q_B^j + (q_B^j/Q_B \times [W_A - Q_A])]$$

if $Q_A \geq 0$ and $Q_B > 0$. Furthermore, $I_B^j = w_B^j$ if $Q_A = Q_B = 0$, and $I_B^j = 0$ if $Q_A > 0$ and $Q_B = 0$.

For member i in A the first-order condition for an interior solution is given by

$$(9)\quad [Q_B/(Q_A + Q_B)^2] \times (w_A^i - q_A^i + [q_A^i/Q_A][W_B - Q_B]) - Q_A/(Q_A + Q_B) + \{(Q_A - q_A^i)/[(Q_A + Q_B)Q_A]\} \times (W_B - Q_B) = 0.$$

It can be verified that I_A^i is strictly concave in q_A^i so that (9) is sufficient for player i's optimality.

Aggregating equation (9) over $i = 1, \ldots, m$, we obtain

$$[Q_B/(Q_A + Q_B)^2](W_A + W_B - Q_A - Q_B) - [mQ_A/(Q_A + Q_B)] + [(m-1)/(Q_A + Q_B)] \times (W_B - Q_B) = 0,$$

or equivalently,

$$(10)\quad -m(Q_A + Q_B)^2 + (m-1) \times W_B(Q_A + Q_B) + (W_A + W_B)Q_B = 0.$$

This equation implicitly determines the aggregate spending by group A, Q_A, in response to the aggregate spending by group B, Q_B.

Analogously, the first-order conditions for members of B yield

$$(11)\quad -n(Q_A + Q_B)^2 + (n-1) \times W_A(Q_A + Q_B) + (W_A + W_B)Q_A = 0,$$

which determines the aggregate spending by group B in response to the aggregate spending by group A.

If there exists an interior solution for which all resource constraints are not binding, then the equilibrium levels of conflictual spending, $\bar{Q}_A$ and $\bar{Q}_B$, are simultaneously determined by equations (10) and (11) as follows

$$(12)\quad \bar{Q}_A = [(W_A/m + W_B/n)(W_B - \beta W_A)] /[(1-\beta)^2(W_A + W_B)],$$
$$\bar{Q}_B = [(W_A/m + W_B/n)(W_A - \beta W_B)] /[(1-\beta)^2(W_A + W_B)],$$

where $\beta = 1 - 1/m - 1/n$. Notice that when the resource constraints are not binding, $\bar{Q}_A$ and $\bar{Q}_B$ depend on m, n, W_A, and W_B, but not on the internal distribution of resources within each group. Furthermore, $\bar{Q}_A \leq \bar{Q}_B$ if and only if $W_A \geq W_B$. This implies that the resource-laden group spends less on fighting than does the resource-poor group. From (11), the total spending by the two groups can be written as

$$\bar{Q}_A + \bar{Q}_B = (nW_A + mW_B)/(m+n),$$

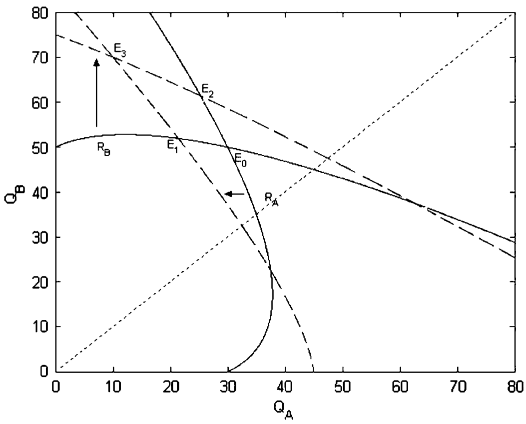
which is a linear combination of W_A and W_B.

An immediate implication of (12) is that if $m = n = 1$, then $\bar{Q}_A = \bar{Q}_B = (W_A + W_B)/4$. In this case, both groups spend the same amount of resources on fighting, independent of the initial resource distribution and conditional on the resource constraints being nonbinding. This is consistent with the result obtained by Tullock (1980) in a rent-seeking model where two individuals compete for a single, private good prize. Hirshleifer (1989, 1991b) discusses a similar model of conflicts and rent-seeking.

When a group has more than one member, the collective action problem within the group affects its members' incentives to spend on fighting. As implied by equation (12), when A and B have equal amount of initial resources, the two groups' total marginal gains and marginal costs are symmetric. Thus, in equilibrium, both groups' fighting efforts are the same and independent of group size.[6] When the two groups have unequal amounts of endowed resources, however, we show that the resource-laden group actually spends less on fighting than does the resource-poor group. Furthermore, as the number of members of the resource-laden group increases, its total spending on fighting decreases, whereas the resource-poor group's spending increases. On the contrary, as the number of members in the resource-poor group increases, its total spending increases, but its rival's spending

6. See Katz et al. (1990) for a similar result in a rent-seeking model where two groups compete for a pure public good.

FIGURE 2
Aggregate Best-Reply Curves: Offensive versus Offensive



continues to decline. This finding shows that the incentives are different for members of the two competing groups because the spoils of victory differ. Members in the resource-poor group have stronger incentives to spend on military capability because its rival's initial endowed resources are relatively larger than its own.

In Figure 2, we show the two groups' aggregate best-reply curves for values of $W_A = 100$, $W_B = 60$, and $m = n = 2$. Both curves are initially increasing and then downward-sloping. They intersect at E_0, which determines the aggregate equilibrium spending by the two groups. As group A's size increases from 2 to 4, the new equilibrium point E_1 indicates that group A spends less while group B spends more than before. But as the less endowed group, B, increases its size from 2 to 4, the new equilibrium point E_2 indicates that group B's aggregate spending on conflict increase while A's spending decreases.

It is worth pointing out that when proportional division rule is used, the assumption of endogenous external threat is crucial to our results. If external threat is fixed instead, whether a group's total defense spending will increase or decrease with the group size depends on how severe the threat level is. We can use Figure 2 to illustrate this point. If group B's threat level to A is not severe, roughly below 22.5, then the total spending will increase with the group's size; but if the threat level is over 22.5, then the total spending will actually decrease with the group's size. The same is true for group B. In contrast, if external threat is endogenously determined, the suboptimality proposition only holds true for the better endowed group.

In the following we compute individual members' spending on fighting and expected net income in equilibrium to understand individual members' strategic thinking. Using the first-order condition (9), we can calculate member i's fighting effort as

$$\bar{q}_A^i = w_A^i \bar{Q}_B / W_B + (\bar{Q}_A + \bar{Q}_B) \times (W_B - \bar{Q}_A - \bar{Q}_B)/W_B. \tag{13}$$

When resource constraints are nonbinding, the equilibrium expected net income for member i in A can be calculated from equations (7) and (13) as

$$\bar{I}_A^i = (W_B - \bar{Q}_A - \bar{Q}_B)^2 / W_B + (W_B - \bar{Q}_B) w_A^i / W_B. \tag{14}$$

A similar solution can be derived for members of group B.

It follows from equation (13) that an individual member's equilibrium spending on conflict is a linear function of his or her initial resources. Thus, in general, a group's wealthy members allocate more resources to fighting than its poor members. But in terms of the percentage of resources that a member allocates to fighting, wealthy members of the better endowed group spend more than the poor, while the opposite is true for the less endowed group.

In Proposition 2, we summarize the theoretical findings established in this section.

PROPOSITION 2. *Suppose that (i) both groups are offensive, (ii) external threat is endogenously determined, and (iii) a proportional division rule is used to distribute the spoils of victory. The following conclusions hold. (a) If the two competing groups have equal resource endowments, their fighting efforts are also the same and independent of group size. Furthermore, members within each group spend the same share of their resources on fighting. (b) If the two competing groups' endowed resources are unequal, the resource-laden group actually* *spends less on fighting than the rivaling group does. Furthermore, the wealthy members of the resource-laden group allocate a higher share of their resources on fighting than the poor members, while the wealthy members of the less-endowed group allocate a lower share of their resources to fighting. (c) As the number of members in the resource-laden group increases, its spending on fighting decreases but the rivaling group's total spending increases. On the contrary, as the number of members in the resource-poor group increases, its total expenditure increases but the rivaling group's spending continues to decline.*

Before we turn our attention to the effects of division rules on collective action problems in the next section, we present an interesting and unexpected finding based on the results in sections II and III. Intuitively, it seems that it would always benefit a group to be offensive when it faces an offensive opponent. The following example shows that this claim is not always true. Suppose that $(w_A^1, w_A^2) = (55, 45)$ and $(w_B^1, w_B^2) = (35, 25)$.

Case 1: Defensive versus Offensive

We use the result in section II to compute the equilibrium fighting efforts and net incomes as follows.

- A is defensive: $(Q_A = 22.65)$, $(q_A^1 = 16.32$, $I_A^1 = 16.01)$, $(q_A^2 = 6.32,\ I_A^2 = 16.01)$.
- B is offensive: $(Q_B = 32.04)$, $(q_B^1 = 16.02$, $I_B^1 = 41.64)$, $(q_B^2 = 16.02,\ I_B^2 = 31.64)$.

Case 2: Offensive versus Offensive

We use the results in this section to compute the equilibrium fighting efforts and net incomes.

- A is offensive: $(Q_A = 30)$, $(q_A^1 = 19.17$, $I_A^1 = 15.83)$, $(q_A^2 = 10.83, I_A^2 = 14.17)$.
- B is offensive: $(Q_B = 50)$, $(q_B^1 = 26.50$, $I_B^1 = 28.50)$, $(q_B^2 = 23.50, I_B^2 = 21.50)$.

This example illustrates that for members of the defensive group, it is not to their benefit to become offensive if their opponent is offensive. In the conclusion we will provide an explanation to this result. To endogenize group type, however, is beyond the scope of this article.

IV. EFFECTS OF THE DIVISION OF THE SPOILS OF VICTORY

Olson and Zeckhauser (1966) argue that "the problem of disproportionality and suboptimality in international organizations ... should be met instead through institutional changes that alter the pattern of incentives" (pp. 278–79). In this section we pursue this suggestion by analyzing the effects of different division rules on fighting efforts.

An alternative to the proportional division rule employed in the previous section is to divide the spoils of victory equally among individual members regardless of their contribution to the group's fighting effort.[7] The imperial partition of China at the end of the nineteenth century provides an illustrative example of this equal division rule in action. The larger imperial powers (Britain and Russia) were to get larger shares, and smaller shares would go to Japan, Germany, France, and possibly the United States. As a latecomer to China, the United States did not support the *proportional* division rule. Instead, the United States advocated an "open-door" policy allowing each imperial power equal access to China's market and resources. Eventually, the other imperial powers acquiesced to American demands.

In this section, first, we derive the equilibrium results when both groups use equal division; second, we derive the equilibrium results when one of the groups uses proportional division and the other group uses equal division. Last, we study the conditions under which a group would prefer the proportional over the equal division rules.

When both groups are offensive and use the equal division rule, the expected net income for i in A is

$$(15) \qquad I_A^i(q_A^i, Q_A, Q_B) = [Q_A/(Q_A + Q_B)] \times (w_A^i - q_A^i + [W_B - Q_B]/m).$$

Analogously, the expected net income for j in B is

7. See Moulin (1996) for other specifications of division rules.

$$(16)\qquad I_B^j(q_B^j, Q_A, Q_B) = [Q_B/(Q_A + Q_B)] \times (w_B^j - q_B^j + [W_A - Q_A]/n).$$

In equilibrium, i chooses q_A^i to maximize his or her payoff in (15) subject to resource constraints $0 \leq q_A^i \leq w_A^i$, and given the choices of other group members. Analogously, j chooses q_B^j to maximize payoff in (16) subject to resource constraints $0 \leq q_B^j \leq w_B^j$, and given the choices of other group members. We focus on the interior solution in which resource constraints are not binding. For member i in A the first-order condition for an interior solution is given by

$$(17)\qquad [Q_B/(Q_A + Q_B)^2] \times (w_A^i - q_A^i + [W_B - Q_B]/m) - Q_A/(Q_A + Q_B) = 0.$$

It can be easily verified that the second-order condition is satisfied. Aggregating equation (17) over $i = 1, \ldots, m$, we obtain

$$(18)\qquad -m(Q_A + Q_B)^2 + (m-1)Q_B \times (Q_A + Q_B) + (W_A + W_B)Q_B = 0.$$

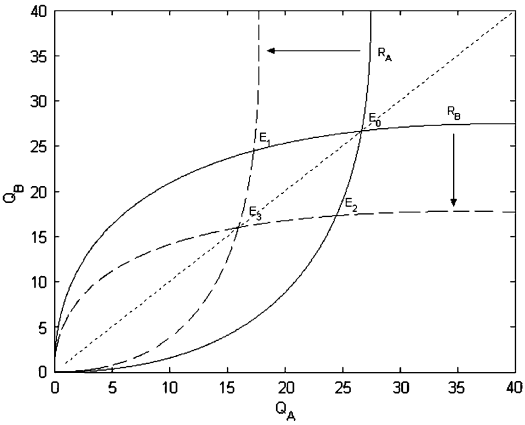
Analogously, the first-order condition for members in B yields

$$(19)\qquad -n(Q_A + Q_B)^2 + (n-1)Q_A \times (Q_A + Q_B) + (W_A + W_B)Q_A = 0.$$

Figure 3 illustrates the two aggregate best-reply curves for values of $W_A = 100$, $W_B = 60$, and $m = n = 2$, which are symmetric and upward-sloping. The equilibrium point is located at E_0. Note that each group's best reply curve shifts downward when its own size increases. For instance, as m goes up from 2 to 4, the R_A curve shifts toward left while R_B remains the same, and the new equilibrium point is located at E_1. The new equilibrium point shows that both groups spend less on fighting than before, and the group with more members, group A, spends less than group B.

In the interior solution, $\bar{Q}_A$ and $\bar{Q}_B$, are determined by (18) and (19) and computed as follows

FIGURE 3
Aggregate Best-Reply Curves: Equal Sharing



$$(20)\qquad \bar{Q}_A = \sqrt{n}(W_A + W_B) / [(\sqrt{n} + \sqrt{m})(\sqrt{mn} + 1)],$$
$$\bar{Q}_B = \sqrt{m}(W_A + W_B) / [(\sqrt{m} + \sqrt{n})(\sqrt{mn} + 1)].$$

From equations (17) and (20), each member's fighting effort, $\bar{q}_A^i$, can be computed as

$$(21)\qquad \bar{q}_A^i = w_A^i - [(2\sqrt{mn} + 1)W_A - \sqrt{mn}W_B] / [m(\sqrt{mn} + 1)].$$

The equilibrium payoffs can be calculated from (15)–(16) and (20)–(21), and are given by

$$(22)\qquad \bar{I}_A^i = n(W_A + W_B) / [(\sqrt{mn} + m)(\sqrt{mn} + 1)],$$
$$\bar{I}_B^j = m(W_A + W_B) / [(\sqrt{mn} + n)(\sqrt{mn} + 1)],$$

which are independent of i and j, respectively.

In Proposition 3, we summarize the implications of these equilibrium results.

PROPOSITION 3. *Suppose that (i) both groups are offensive, (ii) external threat is endogenously determined, and (iii) both use an equal division rule to divide the spoils of victory. Then for any m, n, W_A, and W_B, the following conclusions hold. (a) Wealthy members allocate a higher percentage of their resources to fighting, but receive the same expected net* *incomes as less endowed members. (b) Both groups' fighting efforts decrease as either group's size increases. (c) The group with more members allocates less resources to fighting, and its members also receive lower expected net incomes than members of the smaller group. (d) A group member's expected net income is negatively associated with the size of its own group but positively associated with the size of the competing group.*

From the results summarized in Propositions 2 and 3, we learn that division rules play an essential role in determining whether the suboptimality and the exploitation propositions are valid. When the equal division rule is used by both groups, both propositions continue to hold regardless of whether external threat is endogenously determined. The nature of external threat becomes influential only if both groups use a proportional division rule.

Because different division rules can generate different outcomes for individual group members, it is interesting to know whether individual members have preferences over division rules. To explore this question, in addition to the results in Propositions 2 and 3, we still need to establish the equilibrium results when group A uses the proportional division rule and B adopts the equal division rule. In this case, the equilibrium fighting efforts for A and B are determined by (10) and (19). Accordingly, we can calculate the equilibrium payoffs. To simplify the analysis, we consider only the case in which $m = n = 2$.

Given the (p, e) sharing rules adopted by A and B respectively, the equilibrium total contributions are computed as

$$Q_A = -4(W_A + W_B) + \Delta,$$
$$Q_B = (13W_A + 14W_B - 3\Delta)/4,$$

where $\Delta = \sqrt{16(W_A + W_B)^2 + (W_A + 2W_B)^2}$. The equilibrium expected net income for a member with resource-endowment w in A is

$$I_{pe}(W_A, W_B, w) = (3W_A + 6W_B - \Delta)^2/(16W_B) + (-13W_A - 10W_B + 3\Delta)w/(4W_B)$$

and the equilibrium expected net income for a member of B is

$$I_{ep}(W_A, W_B) = (13W_A + 14W_B - 3\Delta)^2/[16(-4W_A - 4W_B + \Delta)].$$

When A adopts the equal division rule and B uses the proportional division rule, the equilibrium payoffs for individual members can be computed symmetrically. We are now ready to study group members' preferences over the two division rules. Consider the following example: $w_A^1 = w_A^2 = 50$ and $w_B^1 = w_B^2 = 30$. The payoffs are presented in Table 1.

TABLE 1
Proportional versus Equal Division Rules: Symmetric Resource Distribution ($w_A^1 = w_A^2 = 50$ and $w_B^1 = w_B^2 = 30$)

		Group B	
	\	*e*	*p*
Group A	*e*	(26.67,26.67); (26.67,26.67)	(14.25,14.25); (26.90,26.90)
	p	(27.43,27.43); (20.47,20.47)	(15.00,15.00); (25.00,25.00)

Notice that this is a prisoners' dilemma situation because $I_{pe} > I_{ee} > I_{pp} > I_{ep}$ for members of both groups. In the game, (p, p) is the unique pure strategy Nash equilibrium (in dominant strategy). However, players are collectively better off when both groups adopt equal division rules.

As our example illustrates, when the distribution of resources is relatively symmetric, it is a dominant strategy for all group members to choose a proportional division. However, when resource distribution becomes relatively asymmetric, a proportional division is no longer the dominant strategy, and group members might have conflicting preferences over division rules.

Consider the following distribution of resources $(w_A^1, w_A^2) = (60, 40)$ and $(w_B^1, w_B^2) = (35, 25)$. The equilibrium payoffs are presented in Table 2.

We make two observations. First, regardless the decision of the weaker (less endowed) group, B, members of the stronger group, A, have conflicting preferences over division rules. A wealthy member prefers proportional division over equal division, while a poor member prefers the opposite. In this case, the poor member has more incentives to free ride. Second, for members of the weaker group, their preferences over division rules

TABLE 2
Proportional versus Equal Division Rules: Asymmetric Resource Distribution
$[(w_A^1, w_A^2) = (60, 40)$ and $(w_B^1, w_B^2) = (35, 25)]$

		Group B	
	\	*e*	*p*
Group A	*e*	(26.67,26.67); (26.67,26.67)	(14.25,14.25); (30.56,23.25)
	p	(32.86,22.00); (20.47,20.47)	(16.67,13.33); (28.50,21.50)

are partially aligned. Indeed, when the stronger group chooses equal division, members of the weaker group have conflicting preferences. On the other hand, when the stronger group chooses the proportional division rule both members in the weaker group prefer the proportional division as well. To predict which division rule will be chosen by a group, we need to model the intragroup negotiation in greater details, which is beyond the scope of this article.

V. CONCLUDING REMARKS

In this article we study intragroup collective action problems when two groups compete. Conceptually, our model fits into the two-level game framework proposed by Putnam (1988) and Tsebelis's (1990) nested game approach. Our framework enriches our understanding of collective action problems because it expands the domain of the theory to include factors such as the configuration of the competing group, the relative strength of the two groups, the defensive or offensive nature of the groups, and the division rules that the groups use to divide the spoils of victory. We now conclude by highlighting some of the interesting findings we draw from our theoretical analysis. It is our hope that these findings provide new insights for future empirical research on collective action.

Finding 1: It Does Not Always Pay to Be Offensive

When competing with an offensive group, members of the defensive group do not necessarily benefit from becoming offensive. This is due to the fact that, unlike members of an offensive group, members of a defensive group do not expect to receive any income from victory. If they decide to become offensive, the total amount of resources that each group devotes to fighting will increase, which lowers the amount of resources available for redistribution at the conflict's end. Thus, members of a defensive group might receive a lower payoff if they become offensive.

Finding 2: The Weaker Side Might Be More Aggressive

If the proportional division rule is used, in equilibrium, the weaker (less endowed) group allocates more resources to fighting, has a higher probability of winning, and receives higher total payoffs than the stronger group. This finding is consistent with Hirshleifer's observation that "No. 2 tries harder" (Hirshleifer 1991a, b). Because the two contending groups are not unitary actors and because the spoils of victory are divided proportionally to each member's contribution to the group's fighting efforts, members of the weaker group have stronger incentives to allocate resources to fighting because the spoils of victory are relatively greater than that for members of the rich group. For members of the stronger group to improve their payoffs, they must increase the degree of intragroup cooperation to curtail free riding. The group's total payoff reaches its maximum when its members behave as a unitary actor.

Finding 3: Free Riding Can Be Pareto Improving

In section IV we showed that the equal division rule induces more severe free riding than does the proportional division rule, which results in lower spending on fighting. Lower expenditure by one group implies a lesser external threat to the other group. In response, the other group will reduce its own spending on conflict. In equilibrium, equal division rule leads to lower spending on fighting and higher payoffs for members of both groups.[8]

8. This result is consistent with the finding in Bruce (1990). He shows that "a cooperative treaty to increase defense spending in an alliance may actually make it worse off. Further, even where the treaty makes an alliance better off, ceteris paribus, such treaty making may constitute a form of the prisoner's dilemma for the world as a whole with both sides worse than in the absence of cooperative treaties." Ihori (2000), however, develops a model that does not support this finding.

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