

THE AFFILIATION EFFECT IN FIRST-PRICE AUCTIONS

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We study the monotonicity of the equilibrium bid with respect to the number of bidders n in affiliated private-value models of first-price sealed-bid auctions and prove the existence of a large class of such models in which the equilibrium bid function is not increasing in n . We moreover decompose the effect of a change in n on the bid level into a competition effect and an *affiliation effect*. The latter suggests to the winner of the auction that competition is less intense than she had thought before the auction. Since the affiliation effect can occur in both private- and common-value models, a negative relationship between the bid level and n does not allow one to distinguish between the two models and is also not necessarily (only) due to bidders taking account of the winner's curse.

KEYWORDS: Affiliation effect, first-price auctions, affiliated private values, the winner's curse.

1. INTRODUCTION

THERE ARE TWO PARADIGMS in auction theory: the private- and common-value (CV) paradigms. In the private-value paradigm, each bidder knows her own valuation of the object to be auctioned and the only uncertainty is how much other bidders value the object. The valuations of bidders can be either independent (IPV) or affiliated (APV). In a pure CV setting, all bidders attach the same value to the object ex-post, but the exact value of the object is unknown ex-ante. More generally, in the affiliated value (AV) model, the valuations can depend on a common unknown factor as well as on an idiosyncratic private signal where all the signals and the common factor are affiliated.²

A fundamental question in auction theory is whether it is possible to distinguish these two paradigms empirically. This question is important from both positive and normative perspectives. A bidder's optimal strategy is different under the two paradigms and bidders hence behave differently in the two environments. The predictions of auction theory regarding the optimal design of auctions therefore depend on the paradigm. For instance, if the auction environment is CV, policy instruments may be introduced by the seller to minimize

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²See Wilson (1977) and Milgrom and Weber (1982) for pioneering studies of general AV models. The APV model has been studied by a number of authors. See, for instance, Matthews (1987), Laffont and Vuong (1996), and Li, Perrigne, and Vuong (2000, 2002).

the impact of the winner's curse. Being able to identify the applicable paradigm will enable one to test game theoretic predictions and provide better policy/strategy advice.

Laffont and Vuong (1996) have shown that for any *fixed* number of bidders, any symmetric AV model is observationally equivalent to some symmetric APV model if only bids are available. This result illustrates some difficulties of using structural models of auctions to recover the underlying distributions of signals and values among bidders. It also raises the question of whether one can instead exploit exogenous variation in the number of bidders to differentiate between the two paradigms.

Numerous studies have provided evidence for a negative relationship between the bid level and the number of bidders in first-price sealed-bid auctions, particularly in the context of the sale of oil and gas leases. For instance, Gilley and Karels (1981) found that individual bids still decrease with the number of bids after taking sample selection bias into account. Hong and Shum (2002) found a similar relationship between the bid level and the number of bidders in the context of first-price auctions for construction contracts. Such results may lead researchers to attribute a nonmonotonically increasing relationship between (average) bid and number of bidders to the auction being of the common-value type.³

The justification for the above method of inference appears to be straightforward. In the CV and AV models, the equilibrium bid in first-price auctions can decrease in the number of bidders since the winner's curse effect can dominate the competition effect.⁴ This implies that restricting the number of bidders can raise the price and benefit the seller. In APV models, where the winner's curse is absent, however, it is commonly believed that in first-price auctions the presence of additional bidders always increases the price as is the case in IPV models. The only theoretical discussion on the monotonicity of bids in the number of bidders in APV models of which we are aware is in Matthews (1987). When comparing the seller's policies of revealing or concealing information on the number of bidders, Matthews explicitly makes the assumption that the equilibrium bid is monotonically increasing in the number of bidders.⁵

³For a survey on this literature, see Laffont (1997) and Hendricks and Porter (1998). Laffont (1997) in fact states that "Despite the rather imprecise nature of predictions [...] and [...], observed bids decreasing in the number of bidders and the winning bid increasing in $1/n$ have been considered as support of the theory when they should at best be viewed as rejection of the private value model" (p. 12).

⁴See, for instance, Rothkopf (1969), Smiley (1979), Matthews (1984), and Wilson (1992) for illustrations. The winner's curse is usually associated with the fact that the value of the object to the winner of the auction is less than what she had expected before the auction. A rational bidder takes into account the winner's curse effect. See Bulow and Klemperer (2002) for an interesting discussion on the relationship between price and the winner's curse in common-value models.

⁵He argues that "This assumption is not only intuitive but apparently weak: I have not found an example satisfying [the other assumptions in his paper] that violates [it]" (p. 642).

He further provides a sufficient condition for monotonicity, namely that conditional on a bidder's valuation, the number of bidders and the maximum of all rivals' valuations are affiliated.

In this paper we study a class of APV models with an exogenous number of bidders in which bidders' private valuations are affiliated through a common unknown factor but are independent conditional on that factor. We show that in this class of models the sufficient condition provided by Matthews (1987) can be violated and the equilibrium bid function in a first-price auction can decrease with the number of bidders.

The driving force for our results is what we call an *affiliation effect*. In the class of APV models we consider, the distribution of private valuations is a mixture of distributions indexed by an unknown common component z . Higher values of z imply that bidders are more likely to draw higher private values. A bidder knows that winning the auction means that the z -value is probably smaller than her a priori information suggests that it is. She takes this phenomenon into account by reducing her bid accordingly. The greater is the number of bidders, the greater is the posterior probability of z being small conditional on winning, and hence the lower is her bid relative to the IPV case. We also formally separate the affiliation effect from the standard competition effect and show that the two effects work in opposite directions.

The affiliation effect can occur in both private- and common-value models. It is different from, but similar to, the winner's curse effect, which only occurs in common-value models. With the winner's curse effect, a larger number of bidders means that the value of the object to the winner of the auction is less than she had believed on the basis of her private information. The affiliation effect is strategic in nature. It does not bear on the ex-post value of the object to any bidder, but suggests to a winning bidder that the intensity of competition is less than she had thought. Moreover, without further assumptions on the auction environment, the relationship between bid level and the number of bidders does not allow one to distinguish between the private- and common-value paradigms in first-price sealed-bid auctions.

A natural question to ask is whether the expected winning bid (i.e., price) can also be decreasing in the number of bidders; this remains an open question. Developing an example (should one exist) is complicated by the existence of an additional effect, the *sampling effect*, which arises because the presence of one more bidder results in an extra draw from the value distribution. In Pinkse and Tan (2004) (henceforth referred to as PT) we provide several sufficient conditions for price to be *increasing* in the number of bidders.

The paper proceeds as follows. Section 2 compares the monotonicity of the equilibrium bid function with respect to the number of bidders to that of the *reverse hazard* in the context of APV models of first-price sealed-bid auctions. Section 3 presents our main finding that the equilibrium bid can be locally decreasing in the number of bidders. Section 4 concludes.

2. BID FUNCTIONS

We consider the following symmetric APV model and a first-price sealed-bid auction mechanism with a binding reserve price r . A single indivisible object is auctioned to n potential buyers who are risk-neutral, $n \geq 2$. The value to buyer i , $i = 1, \dots, n$, of the object is x_i , which is known only to buyer i . The vector (x_1, \dots, x_n) is a random vector with distribution function F_n^* and density f_n^* with support $[\underline{x}, \bar{x}]^n$. The number n and the function f_n^* are common knowledge. We assume that f_n^* is affiliated and symmetric in (x_1, \dots, x_n) .

The equilibrium bid function in this setting has been characterized by Milgrom and Weber (1982), who cover all symmetric affiliated value models. Following Milgrom and Weber, we first show that the monotonicity of bids as a function of n depends on the monotonicity of the reverse hazard

$$R(x, n) = \frac{f_{n-1}(x|x)}{F_{n-1}(x|x)},$$

where $F_{n-1}(\cdot|x)$ denotes the conditional distribution function of $\max_{j \neq i} x_j$ given $x_i = x$ and $f_{n-1}(\cdot|x)$ the corresponding density. Affiliation of f_n^* implies that $F_{n-1}(\cdot|x)$ has the monotone likelihood ratio property (MLRP) with respect to x . Moreover, because f_n^* is symmetric in its arguments, R is independent of the identity of the bidders.

To gain some intuition for how an increase in the number of bidders may change a bidder's optimal bidding strategy, we conduct the following exercise. When the $n - 1$ rivals use the same monotone bidding strategy $B(x)$, the expected payoff for bidder i by submitting b is

$$\pi(x_i, b, n) = (x_i - b)F_{n-1}\{\phi(b)|x_i\},$$

with ϕ the inverse of the bid function B and hence a monotone function, also. Note that

$$\left. \frac{\partial \log \pi}{\partial b} \{x_i, b, n\} \right|_{b=B(x_i)} = R(x_i, n)\phi'(B(x_i)) - \frac{1}{x_i - B(x_i)}.$$

When R is increasing (decreasing) in n , bidder i will bid more (less) aggressively as more rivals follow the same bidding strategy B . In equilibrium, all bidders take this into account.

To determine the symmetric equilibrium bid function $B(\cdot, n)$, we use the direct revelation approach. Suppose that a bidder of type x reports her type to be x' . Equilibrium requires that her expected payoff $\pi\{x, B(x', n), n\}$ be maximized at $x' = x$. The first-order condition for this maximization problem yields the differential equation

$$(1) \quad \frac{\partial B}{\partial x}(x, n) = \{x - B(x, n)\}R(x, n),$$

subject to the initial condition $B(r, n) = r$. The solution to (1) is

$$(2) \quad B(x, n) = x - \int_r^x \exp \left\{ - \int_t^x R(s, n) ds \right\} dt,$$

which depends on x, r , and n . The following two lemmas, which follow from (1) or (2), show formally that the monotonicity of the equilibrium bid with respect to n is closely related to the monotonicity of $R(x, n)$ with respect to n .

LEMMA 1: *Suppose, for all $x \in (\underline{x}, \bar{x})$, $R(x, n)$ is strictly increasing in n . Then, $\forall r \in (\underline{x}, \bar{x}), \forall x \in (r, \bar{x}), B(x, n)$ is strictly increasing in n .*

PROOF: Suppose that, for some n, r and some $\tilde{x}, B(\tilde{x}, n) \geq B(\tilde{x}, n + 1)$. Since

$$\frac{\partial B}{\partial x}(r, n) = 0, \quad \frac{\partial^2 B}{\partial x^2}(r, n) = R(r, n), \quad \text{and} \quad R(r, n) < R(r, n + 1),$$

$B(x, n) < B(x, n + 1)$ for x in a neighborhood of r . Therefore, for some $x^* \in (r, \tilde{x}]$,

$$B(x^*, n) = B(x^*, n + 1) \quad \text{and} \quad \frac{\partial B}{\partial x}(x^*, n) \geq \frac{\partial B}{\partial x}(x^*, n + 1),$$

which violates (1).

Q.E.D.

In the IPV model, an extreme case of the APV model, $R(x, n) = (n - 1)f_1^*(x)/F_1^*(x)$, where F_1^*, f_1^* are the unconditional marginal distribution and density function of valuations. Clearly, R is strictly increasing in n for all x and the sufficient condition in Lemma 1 is satisfied. It is well known that in the IPV model both equilibrium bids and expected winning bids are increasing functions of n .

Lemma 1 provides sufficient conditions for $B(x, n)$ to be strictly increasing in n for any pair (x, r) . Matthews (1987) assumes that conditional on $x_i, \max_{j \neq i} x_j$ and n are (strictly) affiliated, implying that $F_{n-1}(y|x)$ satisfies the strict MLRP with respect to n . This in turn implies that $R(x, n)$ is strictly increasing in n for all x . Lemma 1 suggests that to generate nonmonotonicity of $B(\cdot, n)$ in n somewhere, $R(x, n)$ cannot be strictly increasing in n , for all $x > r$. So the condition that $R(x, n)$ is decreasing in n somewhere, is necessary for the nonmonotonicity of $B(x, n)$ in n . Lemma 2 shows that the condition is also sufficient.

LEMMA 2: *Suppose that, for some value of $n, R(r, n) > R(r, n + 1)$. Then, for some $x > r, B(x, n) > B(x, n + 1)$.*

PROOF: Because

$$B(r, n) = B(r, n + 1) = r, \quad \frac{\partial B}{\partial x}(r, n) = \frac{\partial B}{\partial x}(r, n + 1) = 0, \quad \text{and}$$

$$\frac{\partial^2 B}{\partial x^2}(r, n) = R(r, n) > R(r, n + 1) = \frac{\partial^2 B}{\partial x^2}(r, n + 1),$$

$B(x, n) > B(x, n + 1)$ for all x in some neighborhood of r . *Q.E.D.*

In Lemma 2 a reserve price r is introduced to simplify the proof. Perhaps there are circumstances in which $R(\underline{x}, n) > R(\underline{x}, n + 1)$, but in most cases both $R(\underline{x}, n)$ and $R(\underline{x}, n + 1)$ are infinite, which unnecessarily complicates matters.

In summary, in the APV model, affiliation and symmetry of f_n^* are not very restrictive on how R and B vary with n . Indeed, our main contribution is to show that it is fairly easy to construct reasonable examples in which B is decreasing in n . We do so in the next section.

3. CONDITIONALLY INDEPENDENT PRIVATE VALUES

In this section we focus on conditionally independent private-value (CIPV) models, i.e., a class of APV models, in which bidders' private valuations, x_1, \dots, x_n , are affiliated through a random variable z , but in which they are independent conditional on z . Denote the conditional distribution and density function of x_i given z at x by $H(x|z)$ and $h(x|z)$, which have support $[\underline{x}, \bar{x}]$. Let G, g be the distribution and density function of z with support $[\underline{z}, \bar{z}]$. Assume that $h(x|z)$ satisfies the strict MLRP, so that x_1, \dots, x_n are strictly affiliated.

The CIPV model is a special case of the conditionally independent private information (CIPI) model studied by Li, Perrigne, and Vuong (2000).⁶ The CIPV model has been studied by Milgrom (1981) and Levin and Smith (1994). It has also been used by Wang (1998) to compare auctions with posted-price selling mechanisms.

Li, Perrigne, and Vuong (2000) provide an economic interpretation of the CIPV model and note that by de Finetti's theorem (see, e.g., Kingman (1978)), any APV model is a CIPV model under "exchangeability," or "symmetry" in auction theory parlance. Symmetry requires that the joint distribution function of valuations be symmetric in its arguments. While it is true that all symmetric APV models can be expressed as CIPV models, in most cases the conditioning variable z and the distribution H will depend on the number of bidders n . We restrict ourselves to CIPV models in which z and H are not dependent on n . We do impose that z be affiliated with the valuations to ensure our CIPV model is also an APV model.

⁶They consider a general model in which a bidder's utility depends on her private information and a common component. If bidders' valuations are conditionally independent given the common component, the resulting model is the CIPI model.

We now continue to analyze the effect of changes in n on B by examining the behavior of the reverse hazard. In the CIPV model,

$$(3) \quad R(x, n) = (n - 1) \frac{\int_{\underline{z}}^{\bar{z}} H^{n-2}(x|z)h^2(x|z)g(z) dz}{\int_{\underline{z}}^{\bar{z}} H^{n-1}(x|z)h(x|z)g(z) dz},$$

which can equivalently be written as

$$R(x, n) = \int_{\underline{z}}^{\bar{z}} R(x, n; z) p_n(z|\mathcal{W}, x) dz,$$

where $R(x, n; z) = (n - 1)h(x|z)/H(x|z)$ is the reverse hazard function in an IPV model when z is known and

$$p_n(z|\mathcal{W}, x) = \frac{H^{n-1}(x|z)h(x|z)g(z)}{\int_{\underline{z}}^{\bar{z}} H^{n-1}(x|z)h(x|z)g(z) dz}$$

is the posterior density function of z conditional on a bidder of type x winning the auction; here $\mathcal{W} = "x_i \geq \max_{j \neq i} x_j"$ denotes the event that bidder i wins the auction. So the reverse hazard function when z is unknown is a linear combination of the reverse hazard functions for specific z , where the weights are the posterior densities of z conditional on a bidder of type x winning the auction.

Note that $R(x, n; z)$ increases with both n and z for all x, n , and z , which follows from the strict MLRP. However, $p_n(z|\mathcal{W}, x)$ can decrease with n , depending on z .

3.1. A Special CIPV Model

To understand why R , and hence the equilibrium bids B , may decrease in n in the CIPV model, first consider the two-point distribution of z . When z can take two values, z_1 and z_2 , with probabilities g_1 and g_2 , respectively, where $z_1 < z_2$, the posterior probabilities z conditional on a bidder of type x winning the auction can be written as

$$p_n(z_1|\mathcal{W}, x) = \frac{h(x|z_1)g_1}{h(x|z_1)g_1 + [\frac{H(x|z_2)}{H(x|z_1)}]^{n-1}h(x|z_2)g_2}$$

and $p_n(z_2|\mathcal{W}, x) = 1 - p_n(z_1|\mathcal{W}, x)$.

Note that given the strict MLRP, $H(x|z_1) > H(x|z_2)$ for almost all $x \in (\underline{x}, \bar{x})$. Consider one such x . The posterior probabilities $p_n(z_k|\mathcal{W}, x)$, $k = 1, 2$, have two interesting properties. First, for any $n \geq 2$, the posterior probability of $z = z_1$ conditional on winning exceeds the corresponding prior probability, i.e.,

$$p_n(z_1|\mathcal{W}, x) > p_1(z_1|x) = \frac{h(x|z_1)g_1}{h(x|z_1)g_1 + h(x|z_2)g_2}.$$

Hence, also $p_n(z_2|\mathcal{W}, x) < p_1(z_2|x)$. Therefore, the fact that bidder i wins the auction with valuation x makes it more likely to her that $z = z_1$ than before the auction commenced.

Second, the posterior probability $p_n(z_1|\mathcal{W}, x)$ increases with n and $p_n(z_2|\mathcal{W}, x)$ decreases with n . That is, conditional on a bidder of type x winning the auction, the posterior probability of the true state z being z_1 (z_2) increases (decreases) with n . So the greater is the number of bidders n , the greater is the discrepancy between the probability a given bidder attaches to the state being z_1 before the auction and upon winning the auction.

Furthermore, the reverse hazard defined in (3) can be written as

$$(4) \quad R(x, n) = R(x, n; z_1)p_n(z_1|\mathcal{W}, x) + R(x, n; z_2)p_n(z_2|\mathcal{W}, x).$$

Since both $R(x, n; z_1)$ and $R(x, n; z_2)$ increase with n and $R(x, n; z_1) < R(x, n; z_2)$ for all x, n , in the expression of R in (4) the decreasing component, $p_n(z_2|\mathcal{W}, x)$, can decrease with n (much) faster than the other components increase. If we select the ratio $[h(x|z_1)g_1]/[h(x|z_2)g_2]$ appropriately, then $R(x, n)$ can decrease with n . Note, however, that as n becomes large, $\partial R(x, n)/\partial n$ is approximately equal to $\partial R(x, n; z_1)/\partial n$ which is nonnegative. Therefore, $R(x, n)$ eventually increases with n . This is formally shown in PT, Proposition 1.

We now offer a concrete example in which R , and hence B , is not increasing in n . The distribution function in the example originates from Wilson (1992), who uses the class of distributions in a common-value framework.

EXAMPLE 1: Suppose that z takes two values z_1 and z_2 with probabilities g_1 and g_2 , respectively, where $z_1 < z_2$, and that, for $x \in (0, 1]$,

$$H(x|z_k) = \exp\{z_k(1 - x^{-\beta})\}, \quad \text{where } \beta > 0.$$

It can be verified that H satisfies the strict MLRP.

We choose $z_1 = .01$, $z_2 = 2$, $g_1 = .85$, $g_2 = .15$, $\beta = .5$, and $r = .1$. The equilibrium bid functions for $n = 2, 6$ are computed and presented in Figure 1. Note that the bidding curve for $n = 6$ is below the one for $n = 2$ between $x = .11$ and $x = .63$. Hence, equilibrium bids are not monotone in n .

The intuition for our result can be described as follows. Consider a first-price auction of an abstract painting in which there are two states of the world. In the first state (z_1) people are likely to detest the painting and in the second state (z_2) tastes are distributed more evenly. Imagine that a bidder somewhat likes the painting ($x = .4$, say). Because of strict affiliation, for any fixed number of bidders she bids less when she knows that $z = z_1$ than when $z = z_2$. If she knew the value of z , then she would bid more if the number of bidders n was larger. When she does not know z , however, a larger number of bidders implies that winning the auction (with the same x) makes it more likely that $z = z_1$ (i.e., $p_n(z_1|\mathcal{W}, x)$ increases with n). But if she knew that $z = z_1$, she

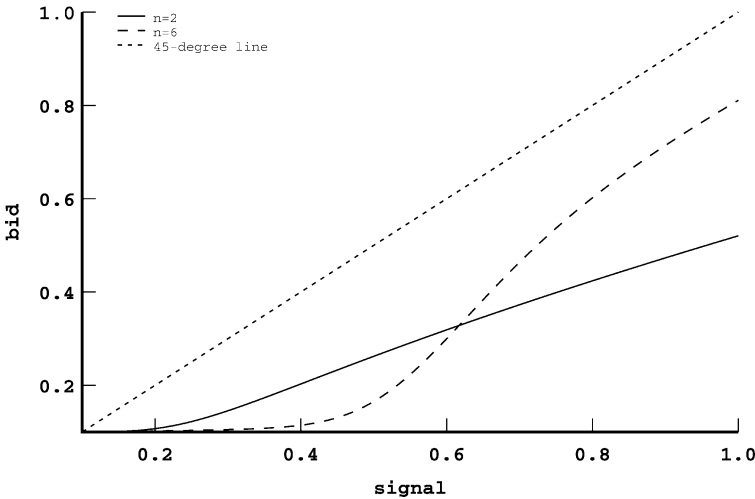


FIGURE 1.—Bid functions.

could bid less and still win the auction. Therefore, with affiliation she may bid less when n is greater. In a common-value auction the event of winning the auction informs a bidder that the common value of the object is less than she had expected. In our private-value example the event of winning the auction tells her that a smaller fraction of individuals (in the population, not just of the actual bidders) like the object than her prior information had her believe.

3.2. The Affiliation Effect

In this section, we decompose changes in bids with respect to the number of bidders into an affiliation effect and a competition effect and show that the affiliation effect is negative and can offset the competition effect.

The effect of a change in the number of bidders in a CIPV model can be decomposed as follows. Let $Q(\cdot|x)$ and $q(\cdot|x)$ denote the distribution and density function of x_1 conditional on $x_2 = x$, i.e.,

$$Q(t|x) = P(x_1 \leq t|x_2 = x) = \frac{\int_{\underline{z}}^{\bar{z}} H(t|z)h(x|z)g(z) dz}{\int_{\underline{z}}^{\bar{z}} h(x|z)g(z) dz},$$

$$q(t|x) = \frac{\int_{\underline{z}}^{\bar{z}} h(t|z)h(x|z)g(z) dz}{\int_{\underline{z}}^{\bar{z}} h(x|z)g(z) dz},$$

and let $R_Q(x, n) = (n - 1)q(x|x)/Q(x|x) = (n - 1)R(x, 2)$. So $R_Q(x, n)$ is the reverse hazard function when the $n - 1$ rivals of bidder i draw independently

from the marginal distribution of private valuations conditional on bidder i 's private valuation being equal to x . Equation (2) implies that

$$(5) \quad \frac{\partial B}{\partial n}(x, n) = \underbrace{\int_r^x \left(\int_t^x \frac{\partial \Delta R}{\partial n}(s, n) ds \exp \left\{ - \int_t^x R(s, n) ds \right\} \right) dt}_{\text{affiliation effect}} + \underbrace{\int_r^x \left(\int_t^x \frac{\partial R_Q}{\partial n}(s, n) ds \exp \left\{ - \int_t^x R(s, n) ds \right\} \right) dt}_{\text{competition effect}},$$

where $\Delta R(x, n) = R(x, n) - R_Q(x, n)$. Since q, Q are positive, the second term on the right-hand side of (5) is positive and measures the effect of increased competition due to larger n on the bid function at private valuation x . The first term measures the corresponding effect of affiliation and is necessarily negative (except at r), as established in Proposition 1. We call the effect measured by the first term in (5) the *affiliation effect*. Note that since $\Delta R(x, 2) = 0$, $\Delta R(x, n)$ itself is therefore also negative for $n > 2$.⁷

In Proposition 1 and other theoretical results that follow, we make repeated use of the inequality stated in Lemma 3.

LEMMA 3: For any random variable ξ , if ϵ_1 and ϵ_2 are nondecreasing functions with $E\epsilon_j^2(\xi) < \infty$ for $j = 1, 2$, then the correlation between $\epsilon_1(\xi)$ and $\epsilon_2(\xi)$ is nonnegative.

PROOF: Let $\tilde{\epsilon}_1(\xi) = \epsilon_1(\xi) - E\epsilon_1(\xi)$, and let ξ_0 be the smallest value such that $\tilde{\epsilon}_1(\xi_0) = 0$. Then

$$\begin{aligned} \text{cov}(\epsilon_1(\xi), \epsilon_2(\xi)) &= E(\tilde{\epsilon}_1(\xi)\epsilon_2(\xi)) \\ &= E((\tilde{\epsilon}_1(\xi) - \tilde{\epsilon}_1(\xi_0))(\epsilon_2(\xi) - \epsilon_2(\xi_0))) \geq 0, \end{aligned}$$

since $\tilde{\epsilon}_1(\xi) \geq \tilde{\epsilon}_1(\xi_0)$ if and only if $\epsilon_2(\xi) \geq \epsilon_2(\xi_0)$. Q.E.D.

Kingman (1978) coined the inequality in Lemma 3 “the other Chebyshev inequality.” If ξ is vector-valued, then the conditions of Lemma 3 constitute the definition of *association*. Lemma 3 also holds if ξ is a vector of affiliated random variables.⁸

PROPOSITION 1: In the CIPV model with H satisfying the strict MLRP, for all $x \in (r, \bar{x})$, $\Delta R(x, n)$ is **decreasing** in n for all $n \geq 2$.⁹

⁷Both the decomposition and the result that the affiliation effect is necessarily negative hold for general APV models. See PT, Section 3.

⁸See Theorem 4.2 in Karlin and Rinott (1980) and Theorem 23 in Milgrom and Weber (1982).

⁹The proof shows that $\Delta R(x, n + 1) < \Delta R(x, n)$ for all integers $n \geq 2$ whereas (5) relies on derivatives. The proof can be adapted to show that $\Delta R(x, n + t) < \Delta R(x, n)$ for all real $t > 0$.

PROOF: Note that, for all $n \geq 2$,

$$\begin{aligned} &\Delta R(x, n + 1) - \Delta R(x, n) \\ &= n \left(\frac{R(x, n + 1)}{n} - \frac{R(x, n)}{n - 1} \right) + \frac{R(x, n)}{n - 1} - R(x, 2). \end{aligned}$$

It suffices to show that $\forall x \in (r, \bar{x})$, n : $R(x, n + 1)/n < R(x, n)/(n - 1)$, or equivalently (omitting arguments) that

$$(6) \quad \int (h/H)Hk_n \int k_n < \int (h/H)k_n \int Hk_n \quad \text{with} \quad k_n = H^{n-1}hg.$$

But by the strict MLRP, for all $x \in (r, \bar{x})$, h/H is strictly increasing in z and H is strictly decreasing in z . By Lemma 3, noting that k_n is proportional to a density, (6) holds. Q.E.D.

The affiliation effect also occurs in common-value models and is distinct from the winner’s curse effect.¹⁰

The key to Example 1 is the nonmonotonicity of $R(x, n)$ with respect to n . Our proposition below shows that in the CIPV model R , and hence B , can easily decrease with n .

PROPOSITION 2: *In the CIPV model, for any $H(x|z)$ that satisfies the strict MLRP and for any $x \in (\underline{x}, \bar{x})$, for which $\min_z\{H(x|z)h(x|z)\} > 0$, there exist n and g such that $R(x, n) > R(x, n + 1)$.*

PROOF: Note that the strict MLRP implies that, for any $x \in (\underline{x}, \bar{x})$ such that $\min_z\{H(x|z)h(x|z)\} > 0$, both $H(x|z)$ and $H(x|z)/h(x|z)$ are strictly decreasing in z . Hence, by Lemma 3 (omitting arguments),

$$(\bar{z} - \underline{z}) \int_{\underline{z}}^{\bar{z}} H(H/h) > \int_{\underline{z}}^{\bar{z}} H \int_{\underline{z}}^{\bar{z}} (H/h).$$

For our choice of n to be made later, let $g(z) = \alpha/\{H(x|z)^{n-2}h(x|z)^2\}$, where α is such that $\int_{\underline{z}}^{\bar{z}} g(z) dz = 1$. Hence,

$$R(x, n) = (n - 1) \frac{\bar{z} - \underline{z}}{\int_{\underline{z}}^{\bar{z}} (H/h)} \quad \text{and} \quad R(x, n + 1) = n \frac{\int_{\underline{z}}^{\bar{z}} H}{\int_{\underline{z}}^{\bar{z}} H(H/h)}.$$

¹⁰In CV and AV models (5) would have a third term, namely $\partial E_n(U(V, x)|\mathcal{W}, x)/\partial n$, with $U(V, x)$ the unknown ex-post value of the object to a bidder with private valuation x . When U is strictly increasing in its first argument, the additional term is negative and measures the winner’s curse effect.

Then $R(x, n) > R(x, n + 1)$ is equivalent to

$$(7) \quad 1 - \frac{1}{n} > \frac{\int_{\underline{z}}^{\bar{z}} H \int_{\underline{z}}^{\bar{z}} (H/h)}{(\bar{z} - \underline{z}) \int_{\underline{z}}^{\bar{z}} H(H/h)}.$$

Since the right-hand side of (7) is strictly less than 1 and independent of n , n can be chosen large enough such that (7) is satisfied. *Q.E.D.*

From Proposition 2 it follows that for any given $x_0 \in (\underline{x}, \bar{x})$, there exist (n^*, g^*) (which generally depend on x_0) such that $R(x_0, n)$ is strictly decreasing in n at $n = n^*$. If r is set to x_0 or close to x_0 , then by Lemma 2 $B(x, n^* + 1) < B(x, n^*)$ for x in a neighborhood of x_0 . This implies our main result stated in Corollary 1.

COROLLARY 1: *In the CIPV model the equilibrium bid can be decreasing in n .*

Proposition 2 requires that the support of x is independent of z and that the strict MLRP is satisfied. An example that violates both conditions simultaneously is the following. Let $H(x|z) = (x/z)^\gamma$ for $x \in [0, z]$ and for some $\gamma > 0$. For any given $x > 0$ and $z \geq x$, $H(x|z)/h(x|z)$ is independent of z and $R(x, n) = (n - 1)\gamma/x$, which is increasing in n .

Note that Propositions 1 and 2 rely on Lemma 3. Since Lemma 3 also holds for affiliated vectors ξ , both propositions also hold when z is an affiliated vector.

While Corollary 1 states that B can be decreasing in n , it is necessarily increasing in n at $x = \bar{x}$ as Proposition 3 shows.

PROPOSITION 3: *In the CIPV model, $B(\bar{x}, n)$ is increasing in n .*

PROOF: Note that

$$R(s, n) = \frac{d \log F_{n-1}(s|s)}{ds} - \frac{\partial \log F_{n-1}(s|x)}{\partial x} \Big|_{x=s},$$

implying that

$$\int_t^{\bar{x}} R(s, n) ds = -\log F_{n-1}(t|t) - \int_t^{\bar{x}} \frac{\partial \log F_{n-1}(s|x)}{\partial x} \Big|_{x=s} ds.$$

But $\forall t: \partial \log F_{n-1}(t|t)/\partial n \leq 0$ and (omitting arguments and setting $k_n = H^{n-1}hg$ as before)

$$\frac{\partial^2 \log F_{n-1}(s|x)}{\partial x \partial n} \Big|_{x=s} = \frac{\int k_n \frac{h'}{h} \log H \int k_n - \int k_n \frac{h'}{h} \int k_n \log H}{(\int k_n)^2} \leq 0$$

by Lemma 3 since $\log H$ is decreasing in z and h'/h is increasing in z by affiliation. The claim follows from (2). *Q.E.D.*

Because in equilibrium a bidder with value \bar{x} always wins the auction, the event of winning the auction is not informative to her about the value of z . However, since type \bar{x} bidders know that other bidders will be shading their bids, it is still optimal for type \bar{x} bidders to bid less than they would have in the IPV case, so there is an affiliation effect even at the top of the value distribution. However, this affiliation effect is no longer sufficient to offset the competition effect at \bar{x} .

Proposition 3 also holds for general APV models, as is shown in Proposition 5 of PT. This result can potentially be used to distinguish between common- and private-value auctions if data on a number of similar auctions is available since $B(\bar{x}, n)$ is necessarily increasing in n in private-value auctions but can be decreasing in n in common-value auctions.

By Proposition 2, nonmonotonicity can occur at any value of n . However, since $p_n(z_1|\mathcal{W}, x) \rightarrow 1$ as $n \rightarrow \infty$, $R(x, n)$ behaves similarly to $R(x, n; z_1)$ for large n . This suggests that $R(x, n)$ increases with n when n is sufficiently large. This is shown formally in PT, Proposition 1. Therefore, for any fixed g and H , if n is sufficiently large, then the bid function is again increasing in n . That is, in the CIPV model the competition effect will eventually dominate the affiliation effect. But the number of bidders n beyond which the bid functions are increasing in n is unknown and hence it is unclear if this will apply to empirical work.

4. CONCLUSION

It has long been known that the equilibrium bid in first-price common-value auctions can decrease with the number of bidders since the winner's curse effect can dominate the competition effect. In this paper, we have shown that a nonmonotonically increasing relationship between the equilibrium bid and the number of bidders can also occur in an affiliated private-value model, in which the winner's curse is absent. We have identified an affiliation effect which is similar to, but different from, the winner's curse effect. If a bidder wins an auction, it implies that the distribution of rivals' valuations has more mass at lower valuations than what the bidder had expected prior to bidding. This affiliation effect can offset the competition effect since more competition implies that, conditional on winning, the rivals' valuations are more likely to be low. The affiliation effect also occurs in common-value models. This implies that a negative relationship between bid level and the number of bidders in common-value models is not necessarily only due to bidders taking account of the winner's curse.

Our result demonstrates that one cannot use a reduced-form test on the relationship between the bid level and the number of bids to distinguish between

the private- and common-value paradigms in first-price auctions. However, such a reduced-form test works well in second-price and ascending private-value auctions since it is then a dominant strategy for a bidder to bid her true valuation. An alternative approach is taken by Athey and Haile (2002), who provide conditions for identification and nonparametric structural tests of standard auction models using a variety of available information including one or more bids, the transaction price, and exogenous variation in the number of bidders. Haile, Hong, and Shum (2002) pursue yet another possibility. Following Li, Perrigne, and Vuong (2002), they use a test that is based on the fact that the bidders' expected value distributions are first-order stochastic dominance-ordered in terms of the number of bidders under the common-value paradigm, but are independent of the number of bidders under the private-value paradigm. Hendricks, Pinkse, and Porter (2003) suggest that the private-value hypothesis can be tested by examining bidding behavior close to the reserve price. Finally, as pointed out in Section 3, one could use a test based on our Proposition 3.

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REFERENCES

- ATHEY, S., AND P. HAILE (2002): "Identification of Standard Auction Models," *Econometrica*, 70, 2107–2140.
- BULOW, J., AND P. KLEMPERER (2002): "Prices and the Winner's Curse," *RAND Journal of Economics*, 33, 1–21.
- GILLEY, O. W., AND G. KARELS (1981): "The Competitive Effect in Bonus Bidding: New Evidence," *Bell Journal of Economics*, 12, 637–648.
- HAILE, P., H. HONG, AND M. SHUM (2002): "Nonparametric Tests for Common Values in First-Price Sealed-Bid Auctions," Mimeo, Yale University.
- HENDRICKS, K., J. PINKSE, AND R. PORTER (2003): "Empirical Implications of Equilibrium Bidding in First-Price, Symmetric, Common-Value Auctions," *Review of Economic Studies*, 70, 33–58.
- HENDRICKS, K., AND R. PORTER (1998): "Lectures on Auctions: An Empirical Perspective," Mimeo, Northwestern University.
- HONG, H., AND M. SHUM (2002): "Increasing Competition and the Winner's Curse: Evidence from Procurement," *Review of Economic Studies*, 69, 871–898.
- KARLIN, S., AND Y. RINOTT (1980): "Classes of Orderings of Measures and Related Correlation Inequalities. I. Multivariate Totally Positive Distributions," *Journal of Multivariate Analysis*, 10, 467–498.
- KINGMAN, J. F. C. (1978): "Uses of Exchangeability," *The Annals of Probability*, 6, 183–197.
- LAFFONT, J. J. (1997): "Game Theory and Empirical Economics: The Case of Auction Data," *European Economic Review*, 41, 1–35.
- LAFFONT, J. J., AND Q. VUONG (1996): "Structural Analysis of Auction Data," *American Economic Review*, 86, 414–420.

- LEVIN, D., AND J. SMITH (1994): "Equilibrium in Auctions with Entry," *American Economic Review*, 84, 585–599.
- LI, T., I. PERRIGNE, AND Q. VUONG (2000): "Conditionally Independent Private Information in OCS Wildcat Auctions," *Journal of Econometrics*, 98, 129–161.
- (2002): "Structural Estimation of the Affiliated Private Value Model," *RAND Journal of Economics*, 33, 171–193.
- MATTHEWS, S. A. (1984): "Information Acquisition in Discriminatory Auctions," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. E. Kihlstrom. New York: North-Holland, 181–207.
- (1987): "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View," *Econometrica*, 55, 633–646.
- MILGROM, P. (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49, 921–943.
- MILGROM, P., AND R. WEBER (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.
- PINKSE, J., AND G. TAN (2004): "The Affiliation Effect in First-Price Auctions: Supplementary Material," *Econometrica Supplementary Material*, <http://www.econometricsociety.org/ecta/supmat/3142extension.pdf>.
- ROTHKOPF, M. (1969): "A Model of Rational Competitive Bidding," *Management Science*, 15, 774–777.
- SMILEY, A. (1979): *Competitive Bidding Under Uncertainty: The Case of Offshore Oil*. Cambridge, MA: Ballinger Publishing Company.
- WANG, R. (1998): "Auctions versus Posted-Price Selling: The Case of Correlated Private Valuations," *Canadian Journal of Economics*, 31, 395–410.
- WILSON, R. (1977): "A Bidding Model of Perfect Competition," *Review of Economic Studies*, 44, 511–518.
- (1992): "Strategic Analysis of Auctions," in *Handbook of Game Theory*, Vol. 1, ed. by R. J. Aumann and S. Hart. Amsterdam: Elsevier Science Publishers, 228–279.