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# Optimal procurement mechanisms for an informed buyer

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*Abstract.* This paper studies a buyer (e.g., a government agency) offering a procurement contract to a number of privately informed suppliers. The buyer has private information about her demand for the product to be procured. The optimal mechanisms for all types of the buyer are examined. It is optimal for the buyer to reveal her demand information through the contract offer and use a first-price sealed-bid auction procedure to award the contract, announcing her reserve price in advance. Any second-price auction is shown to yield less expected surplus for the buyer than the optimal first-price auction does when the buyer's marginal willingness to pay decreases with quantity.

*Mécanismes optimaux d'approvisionnement pour un acheteur informé.* Ce mémoire étudie le comportement d'un acheteur (e.g., une agence gouvernementale) offrant un contrat d'approvisionnement à un nombre de fournisseurs privés informés. L'acheteur a des renseignements privilégiés sur la demande de produits qu'on doit fournir. On examine les mécanismes optimaux pour toutes sortes de types d'acheteurs. Il est optimal pour l'acheteur de révéler son information sur la demande via l'offre de contrat, puis de mettre en place une procédure d'appel d'offres closes qui accorde le contrat au plus bas soumissionnaire et au prix qu'il a soumissionné (first-price auction), tout en annonçant d'avance ses contraintes sur les prix. Toute mise aux enchères qui accorde le contrat au plus bas soumissionnaire mais au prix du soumissionnaire qui est arrivé en seconde place (second-price auction) produit un surplus moindre pour l'acheteur que ce n'est le cas quand la procédure optimale antérieure (first-price auction) est utilisée, quand la volonté marginale de payer décroît avec la quantité.

## 1. INTRODUCTION

Governments often purchase goods or services from private suppliers using various procurement mechanisms. For instance, a large proportion of government procure-

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ment is conducted through competitive bidding schemes where potential suppliers submit sealed bids. The contract is then awarded to the supplier with the lowest bid. A key advantage of such an auction-type mechanism is that by eliciting the suppliers' private information, the net revenue for the government is increased. In many cases the government is better informed than the suppliers are about its demand for the goods to be procured. The objective of this paper is to study the optimal procurement mechanism for the government in a simple environment where the government has an informational advantage over potential suppliers. We analyse how the government uses its information strategically through a procurement contract offer and whether standard competitive bidding schemes are the optimal mechanisms. We discuss further how the optimal mechanism can be implemented using simple schemes.

In many circumstances the government (or the buyer) possesses procurement information that is not observed by potential suppliers. For example, in a defence procurement the government is typically well informed about the weapon's strategic value as well as its budget allocation. Potential suppliers, however, face uncertainty concerning the budget and the government's purchasing decisions. This informational asymmetry is also present in the private sector. A downstream firm is often better informed than the input suppliers about the profitability of the input purchase.

Standard analysis suggests that in a general procurement environment the privacy of the buyer's valuation matters. When offering a procurement mechanism, the buyer faces the dilemma of whether to conceal or to reveal her private information to the suppliers through the mechanism. Low-demand buyers may have incentives to reveal their demand information and to distinguish themselves from high-demand buyers, so a low price can be expected. At the same time, high-demand buyers may pretend to be low-demand buyers and hence pay lower prices. On the other hand, revealing low-demand information could discourage the participation of potential suppliers in the competition for the procurement contract. Therefore, in a general buyer-sellers environment, the incentives to reveal information typically vary with different types of buyers, and conflicts often exist among them in selecting procurement mechanisms. It is important to understand how these conflicts are resolved and how the information is used in the best interests of the buyers.

The issues of optimal procurement contracts can be studied in a theoretical framework of mechanism design. Informational asymmetries between the buyer and sellers can prevent agreement from being reached even when such an agreement would be efficient under complete information. Either the buyer or the sellers may have incentives to misrepresent their private information to obtain a better price. Mechanism design theory provides a useful framework in which these incentives can be analysed. An important result of this theory is the Revelation Principle.<sup>1</sup> Roughly speaking, in our procurement context it states that regardless

1 For further discussion of the Revelation Principle, see Myerson (1991). Many applications of the Revelation Principle to procurement and regulation problems can be found in Laffont and Tirole (1993).

of the bargaining format that might describe the process of the interaction between the buyer and sellers, the bargaining outcome can always be achieved by a direct revelation mechanism. Such a mechanism is a specification of the terms of exchange as functions of the information that is available to all participants. Given this mechanism, the buyer and sellers simultaneously announce their private information, and then goods are exchanged according to the terms specified in the mechanism. The Revelation Principle also requires the direct revelation mechanism to be incentive compatible and individually rational. Individual rationality implies that the buyer and sellers have incentives to participate in trading, whereas incentive compatibility means that the participants are given incentives to announce their information truthfully. Therefore, the optimal procurement mechanism for the buyer can be determined by choosing a direct revelation mechanism subject to these incentive constraints. Since the set of all direct revelation mechanisms with the above incentive properties often has a simple structure, the buyer's optimization problem can be easily solved.

In a general principal-agents decisions problem with informational asymmetries, Myerson (1983) has studied the issues of information revelation through a mechanism offer.<sup>2</sup> He provides a theory of how privately informed principals facing many informed agents resolve their conflicts and select mechanisms. The Revelation Principle also applies to this case, and hence there is no loss in generality by requiring that all types of the principal select the same direct revelation mechanism that is incentive compatible and individually rational for both the agents and the principal. The principal can build any communication into the process of the mechanism itself. He proposes a mechanism with the following two properties. First, it is Pareto efficient among all types of the principal. That is, there is no other incentive-compatible and individually rational mechanism that would make one type of principal better off without reducing the pay-off for some other type of principal. Second, it is incentive compatible for all agents even if the agents knew the principal's type. In other words, the agents would report their types honestly when the principal's type was revealed. The buyer can always implement the mechanism no matter what the firms might infer about her type. Clearly, all types of principal should agree upon this mechanism if it exists. Therefore, what Myerson has proposed is a self-enforcing agreement among different types of principal. It is in this sense that we call the above agreement an equilibrium mechanism. We apply this approach to our specific procurement environment to determine the equilibrium mechanism for the privately informed buyer.

Our procurement framework is similar to the independent private values model investigated by Myerson (1981) in optimal auction design, except that the principal has private information. In our model, the buyer (the principal) is privately informed about her valuation of the product to be procured. We also allow the quantity choice to be variable. We determine the equilibrium procurement mechanism and show

<sup>2</sup> Other studies on this issue are Maskin and Tirole (1990) on a three-stage principal-agent game, and Spulber (1988) in the context of government regulation.

that this mechanism can be implemented by a first-price sealed-bid auction with an announced price-quantity schedule and a reserve price. Therefore, a variation of the optimal auction mechanism characterized by Myerson (1981) turns out to be an equilibrium mechanism, when the buyer has an informational advantage about her valuation of the product over the suppliers. That is, different types of buyer have incentives to offer the same type of auction mechanism. However, the optimal reserve price and the price-quantity schedule depend on the buyer's valuation. The buyer reveals her information to the potential suppliers through the auction mechanism. To the buyer, a secret reserve price policy is not better than a public reserve price policy.

Moreover, we show that when the buyer's marginal willingness to pay decreases with quantity, the equilibrium mechanism cannot be implemented by any form of second-price sealed-bid auction schemes. In other words, a second-price auction with the optimal choice of the price-quantity schedule and reserve price yields less expected consumer's surplus than the optimal first-price auction does. This finding provides an explanation for why first-price auctions, not second-price auctions, are widely used in procurement.

Issues related to information revelation have been discussed previously in the auction literature. In this approach an auction scheme (a particular mechanism) is given and the major concern is whether the auctioneer should reveal or conceal any private information. The case where the auctioneer has private information about the number of bidders has been analysed by McAfee and McMillan (1987) and Matthews (1987). They find that, in a first-price sealed-bid auction with an independent private values environment, whether the auctioneer reveals or conceals the number of bidders depends on the degree of absolute risk aversion by bidders. In particular, if the bidders' utility function exhibits constant or decreasing absolute risk aversion, then the seller prefers the concealing policy to the revealing policy. Milgrom and Weber (1982) find that, in standard auction schemes within a common value environment, the revelation of the auctioneer's information on the item's value generates a higher expected revenue for the auctioneer.<sup>3</sup> Whether the auction schemes are equilibrium mechanisms is not considered in these studies. Our focus is on examining equilibrium mechanisms for the buyer in a specific procurement environment where the buyer is privately informed.

We describe our procurement model in the next section. In section III we characterize the equilibrium procurement mechanisms. In section IV the issues of implementing the equilibrium mechanisms using simple auction schemes are discussed. Concluding remarks follow.

3 On the other hand, Hendricks, Porter, and Spady (1989) have found some evidence suggesting that a random reservation price is used in the Outer Continental Shelf drainage auctions. In particular, the government often reveals its reservation price only after the bidding in these auctions is completed. Recent work by Hendricks, Porter, and Wilson (1994) shows that a common value auction model with a random reservation price is consistent with the bidding behaviour in the U.S. federal auctions of oil and gas leases.

## II. THE MODEL

We consider a single buyer who is interested in procuring a variable number of units of a product or service from  $n$  potential firms ( $i = 1, \dots, n$ ). The buyer's benefit of consuming  $Q$  units of the product is  $\theta B(Q)$ , where  $\theta$  is a parameter that affects the buyer's willingness to pay. We assume that  $\theta$  is known to the buyer but is unobserved by the firms. Each firm believes that  $\theta$  is drawn randomly from a cumulative distribution function  $G(\theta)$  for which a positive density function,  $g(\theta)$ , on a bounded interval,  $[0, \bar{\theta}]$ , exists, where  $\bar{\theta} > 0$  and  $g(\theta)$  is the first-order derivative of  $G(\theta)$ . We assume that  $g(\theta)$  is continuously differentiable over  $[0, \bar{\theta}]$  and that  $B(Q)$  is twice continuously differentiable, increasing, and concave in  $Q \in R_+^1$ . In private procurement, for example, we can think of the buyer as a downstream firm that uses input  $Q$  to produce an output according to the production function  $B(Q)$ . Thus,  $\theta$  can be interpreted as the unit price of the output that is known to the downstream firm but may not be observed by upstream firms. In other cases  $\theta$  can also be viewed as a technology factor or utility parameter.

Firm  $i$  can produce the product at a unit cost  $c_i$ , which is known by firm  $i$  prior to contracting,  $i = 1, \dots, n$ . The other firms and the buyer believe that  $c_i$  is independently drawn from a cumulative distribution function  $F(c_i)$  with a positive density function  $f(c_i)$  on support  $[\underline{c}, \bar{c}]$ , where  $\bar{c} > \underline{c} > 0$  and  $f(c_i)$  is the first-order derivative of  $F(c_i)$  and is assumed to be continuously differentiable on  $[\underline{c}, \bar{c}]$ . We assume that  $\theta$  and  $c_i$  are independent and that  $\bar{\theta} B'(0) > \underline{c}$ , where  $B'(0)$  is the derivative of  $B(Q)$  at  $Q = 0$ . The latter assumption means that for the highest value of  $\theta$  and for the lowest value of  $c_i$ , it is always potentially efficient for the buyer to procure a positive amount of the product from the firms.

Both the buyer and the firms are assumed to be risk neutral. Each firm's objective is to maximize its expected profits. The buyer maximizes her expected surplus. Moreover, we define  $J(\theta) = \theta - [1 - G(\theta)]/g(\theta)$  for  $\theta \in (0, \bar{\theta})$  and  $I(c_i) = c_i + F(c_i)/f(c_i)$  for  $c_i \in (\underline{c}, \bar{c})$ . Following the literature on the theory of incentives, we assume that the distribution functions satisfy the regularity condition:  $J(\theta)$  and  $I(c_i)$  are strictly increasing in  $\theta$  and in  $c_i$ , respectively. Loosely speaking, this monotonicity condition allows us to design procurement contracts that separate different types of buyer and firms. Uniform, exponential, and many other distribution functions commonly used in the literature satisfy this condition. When this monotonicity condition is violated, optimal contracts involve pooling. Firms with different cost parameters receive the same contract.<sup>4</sup> In the following discussion we use the notation  $c = (c_1, \dots, c_n)$  and  $c_{-i} = (c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_n)$ .

## III. OPTIMAL PROCUREMENT MECHANISMS

In this section we investigate the optimal design of the procurement mechanisms. Since the buyer and the firms have private information, we can model the procurement process as a Bayesian incentive problem in which the buyer and  $n$  firms are

4 For further discussion on this issue, see Guesnerie and Laffont (1984).

the players. According to the Revelation Principle, for any Bayesian equilibrium of any procurement mechanism that the players might choose, there exists an incentive compatible and individually rational direct revelation mechanism that is equivalent. In other words, any pattern of bargaining outcomes that can be achieved by any mechanism that takes into account the players' rational decisions can actually be achieved by a mechanism in which the buyer and firms report their types and are given incentives to do so honestly and to participate voluntarily. Incentive compatibility describes this truth-telling property of the direct revelation mechanism, whereas individual rationality refers to the voluntary participation. Therefore, with no loss of generality, we have to consider only direct revelation mechanisms with these two incentive properties. Since the set of all direct revelation mechanisms with these incentive properties has a simple mathematical structure and is easy to characterize, the optimal procurement mechanism for the buyer can then be determined by solving an optimization problem.

In what follows, we first specify direct revelation mechanisms and characterize these mechanisms with incentive properties. After studying the possibility of implementing ex post efficient allocations, we determine the optimal mechanisms for all types of buyer.

### 1. Incentive compatible mechanisms

A direct revelation mechanism specifies the number of units of the product that each firm produces and the payment that each firm receives from the buyer as functions of the types submitted by the buyer and all the firms. Given this mechanism, the buyer and sellers announce their types and then goods are produced and exchanged according to the terms specified in the mechanism. In particular, for each firm  $i$ , let  $q_i(\theta, c)$  and  $p_i(\theta, c)$  be the number of units of the product that firm  $i$  provides and the payment from the buyer to firm  $i$  conditional on the buyer and firms' submitting their types  $(\theta, c)$ , respectively. A direct revelation procurement mechanism, denoted by  $[p(\theta, c), q(\theta, c)]$ , consists of a vector of quantities  $q(\theta, c) = (q_1(\theta, c), \dots, q_n(\theta, c))$  and a vector of payments  $p(\theta, c) = (p_1(\theta, c), \dots, p_n(\theta, c))$ . To simplify our analysis, let  $Q(\theta, c) = \sum_{i=1}^n q_i(\theta, c)$  be the total quantity and  $P(\theta, c) = \sum_{i=1}^n p_i(\theta, c)$  the total payment. Also, for  $i = 1, \dots, n$ , let  $\bar{q}_i(c_i) = E_{(\theta, c_{-i})} q_i(\theta, c)$  and  $\bar{p}_i(c_i) = E_{(\theta, c_{-i})} p_i(\theta, c)$  be firm  $i$ 's expected quantity and payment from the buyer conditional on its production cost's being equal to  $c_i$ .

Given a direct revelation mechanism, the expected pay-off for the buyer of type  $\theta$  conditional on the buyer and firms' submitting their true types can be written as

$$V(p, q; \theta) \equiv E_c(\theta B(Q(\theta, c)) - P(\theta, c)), \text{ for all } \theta \in [0, \bar{\theta}].$$

For  $i = 1, \dots, n$ , the expected pay-off for firm  $i$  of type  $c_i$  conditional on the buyer and firms' submitting their true types is given by

$$U_i(p, q; c_i) \equiv \bar{p}_i(c_i) - c_i \bar{q}_i(c_i), \text{ for all } c_i \in [\underline{c}, \bar{c}].$$

Incentive compatibility for the buyer requires that

$$V(p, q; \theta) \geq E_c(\theta B(Q(\bar{\theta}), c) - P(\bar{\theta}, c)), \text{ for all } \theta, \bar{\theta} \in [0, \bar{\theta}]. \tag{1}$$

That is, when all the firms submit their true types, a buyer of true type  $\theta$  cannot be better off by reporting a different type  $\bar{\theta}$ . Similarly, each firm should be given incentives to submit its true cost when the buyer and other firms report their true types. Thus, incentive compatibility for the firms requires that

$$U_i(p, q; c_i) \geq \bar{p}_i(\bar{c}_i) - c_i \bar{q}_i(\bar{c}_i), \text{ for all } c_i, \bar{c}_i \in [\underline{c}, \bar{c}], \text{ and } i = 1, \dots, n. \tag{2}$$

Equations (1) and (2) together imply that given the direct revelation mechanism truth-telling by all players is a Bayesian equilibrium. Moreover, individual rationality for the buyer and firms requires that

$$V(p, q; \theta) \geq 0 \text{ for all } \theta \in [0, \bar{\theta}], \text{ and } U_i(p, q; c_i) \geq 0 \text{ for all } c_i \in [\underline{c}, \bar{c}], \\ i = 1, \dots, n. \tag{3}$$

In other words, both the buyer and the firms expect non-negative rents from participation, where the reservation pay-off for each participant is normalized to be zero.

For simplicity, we call  $[p(\theta, c), q(\theta, c)]$  a *regular* mechanism if it satisfies inequalities (1)–(3). In other words, it is an incentive compatible and individually rational direct revelation mechanism. We also define a procurement quantity vector,  $q(\theta, c)$ , to be *implementable* if there exists a payment vector,  $p(\theta, c)$ , such that  $[q(\theta, c), p(\theta, c)]$  is regular. The following lemma characterizes all the regular mechanisms.

LEMMA 1.  $[p(\theta, c), q(\theta, c)]$  is regular if and only if the following hold

- 1)  $E_c B(Q(\theta, c))$  is non-decreasing in  $\theta$  and  $\bar{q}_i(c_i)$  is non-increasing in  $c_i$  for  $i = 1, \dots, n$ ;
- 2) The payments satisfy

$$E_c \left( \theta B(Q(\theta, c)) - \int_0^\theta B(Q(t, c)) dt - P(\theta, c) \right) \\ = V(p, q; 0), \text{ for all } \theta \in [0, \bar{\theta}] \tag{4}$$

and

$$\bar{p}_i(c_i) = c_i \bar{q}_i(c_i) + U_i(p, q; \bar{c}) + \int_{c_i}^{\bar{c}} \bar{q}_i(t) dt, \text{ for all } c_i \in [\underline{c}, \bar{c}], i = 1, \dots, n; \tag{5}$$

- 3)  $V(p, q; 0) \geq 0$  and  $U_i(p, q; \bar{c}) \geq 0$  for  $i = 1, \dots, n$ .

The proof of the lemma consists of standard techniques for characterizing incentive compatibility constraints and is omitted here; see, for example, Myerson and Satterthwaite (1983) for the proof in the case where one unit of a good is traded. Claim (1) in the lemma is the standard monotonicity condition that is required for incentive compatibility. Equations (4) and (5) determine the expected transfers from the buyer to the firms that guarantee truth-telling as a Bayesian equilibrium. Notice that (4) and (5) can be rewritten as

$$\begin{aligned}
 V(p, q; \theta) &= V(p, q; 0) + \int_0^\theta B(Q(t, c))dt, & \theta \in [0, \bar{\theta}] \\
 U_i(p, q; c_i) &= U_i(p, q; \bar{c}) + \int_{c_i}^{\bar{c}} \bar{q}_i(t)dt, & c_i \in [\underline{c}, \bar{c}],
 \end{aligned}$$

for  $i = 1, \dots, n$ . In equilibrium, the expected pay-off for the buyer of type  $\theta$  consists of two terms. The first term is the expected pay-off for the buyer with the lowest willingness to pay. The second represents the information rents that the buyer earns from private information. The buyer with higher demand earns more information rents. Similarly, the firm with lower cost earns more information rents. Claims (1) and (2) together imply that the low-demand buyer does not have an incentive to over-report her demand information and that the high-cost firms have no incentives to under-report their costs. Since the buyer's pay-off is non-decreasing in  $\theta$  and each firm's pay-off is non-increasing in  $c_i$ , individual rationality is then equivalent to the statement (3) in the lemma. If the buyer with the lowest demand and the firm with the highest cost make non-negative pay-offs, then all types of buyer and firms are given incentives to participate in the game.

Using the above two equations, we can write the total expected pay-offs among all the players as

$$\begin{aligned}
 E_\theta V(p, q; \theta) + \sum_{i=1}^n E_{c_i} U_i(p, q; c_i) \\
 = E_{(\theta, c)} \left( \frac{1 - G(\theta)}{g(\theta)} B(Q(\theta, c)) + \sum_{i=1}^n \frac{F(c_i)}{f(c_i)} q_i(\theta, c) \right) + C, \quad (6)
 \end{aligned}$$

where  $C \equiv V(p, q; 0) + \sum_{i=1}^n U_i(p, q; \bar{c}) \geq 0$ . Equation (6) says that the total expected gain of the procurement equals the sum of a non-negative amount and the total expected information rents that are required to guarantee incentive compatibility. For any procurement quantity vector  $q(\theta, c)$ , define the expected *virtual gain* (i.e., the adjusted benefits minus the adjusted costs) from this procurement as

$$\Phi_n(q) \equiv E_{(\theta, c)} \left( J(\theta)B(Q(\theta, c)) - \sum_{i=1}^n I(c_i)q_i(\theta, c) \right). \quad (7)$$

It follows from (6) that  $\Phi_n(q) = C \geq 0$ . A corollary of lemma 1 then follows:

COROLLARY 1. *If  $q(\theta, c)$  is implementable, then  $\Phi_n(q) \geq 0$ .*

Corollary 1 states that, given the asymmetries of information between the buyer and the firms, a procurement quantity vector,  $q(\theta, c)$ , is implementable only if the total expected gain from the trade is no less than the total expected information rents. If the total expected gain from the trade is less than the total expected information rents, then a transfer from an outsider is needed to implement the quantity vector. Next, we analyse whether the efficient quantity vector under complete information is implementable.

2. *Ex post efficiency*

Suppose that the players' private information become public ex post. The total surplus of the procurement between the buyer and the firm is  $\theta B(\sum_{i=1}^n q_i) - \sum_{i=1}^n c_i q_i$ . Let  $q^*(\theta, c)$  maximize this ex post total surplus. Then  $q^*(\theta, c)$  can be calculated from the first-order conditions and is given by

$$q_i^*(\theta, c) = \begin{cases} h(c_i/\theta), & \text{if } c_i < \min \{r^*(\theta), \min_{j \neq i} c_j\} \\ 0, & \text{if } c_i > \min \{r^*(\theta), \min_{j \neq i} c_j\} \end{cases} \tag{8}$$

for  $i = 1, \dots, n$ , where  $h(\cdot)$  is the inverse function of  $B'(Q)$ ,

$$r^*(\theta) = \begin{cases} \underline{c}, & \text{if } \beta\theta \leq \underline{c} \\ \beta\theta, & \text{if } \underline{c} < \beta\theta < \bar{c} \\ \bar{c}, & \text{if } \beta\theta \geq \bar{c} \end{cases}$$

is a cut-off level such that  $\theta B(h(r/\theta)) = rh(r/\theta)$ , and  $\beta$  satisfies  $B(h(\beta)) = \beta h(\beta)$ . If  $c_i = c_j$ , both  $q_i^*(\theta, c)$  and  $q_j^*(\theta, c)$  can be defined as  $h(c_i/\theta)/2$ . Since  $\theta B'(0) > \underline{c}$  by assumption,  $h(c_i/\theta)$  is positive on a set with a positive measure. Note that the cut-off level,  $r^*(\theta)$ , is the maximum number above which the ex post total surplus is non-negative.

The question is whether the ex post efficient quantity level,  $q^*(\theta, c)$ , is implementable when the players' information is not public. The answer is no. Indeed, it can be verified that the expected virtual gain,  $\Phi_n(q^*)$ , is always negative. In other words, the expected potential gain from procuring  $q^*(\theta, c)$  is strictly less than the total expected information rents. It then follows from corollary 1 that there does not exist any payment vector,  $p^*(\theta, c)$ , such that  $[p^*(\theta, c), q^*(\theta, c)]$  is regular. In order to implement the efficient procurement allocation, some outsiders have to provide a minimum subsidy of  $-\Phi_n(q^*)$  to the buyer and firms. The following proposition determines the amount of the minimum subsidy.

PROPOSITION 1.  *$q^*(\theta, c)$  is not implementable unless there is a positive expected transfer from an outsider that equals*

$$-\Phi_n(q^*) = n \int_0^{\bar{\theta}} \int_{\underline{c}}^{r^*(\theta)} h(t/\theta) F(t) [1 - F(t)]^{n-1} g(\theta) dt d\theta. \tag{9}$$

Furthermore,

$$\lim_{n \rightarrow \infty} -\Phi_n(q^*) = 0.$$

The proof of proposition 1 follows from (7), (8), and some calculations. It is not provided here; see Tan (1994a) for the details. Proposition 1 is an extension of the impossibility result obtained by Myerson and Satterthwaite (1983) and extended by McAfee (1991). They provide a formula similar to (9) in a two-person bargaining framework.<sup>5</sup> In our procurement context, the number of firms  $n$  is usually larger than one. As  $n$  increases, the expected minimum production cost decreases and the level of efficient quantities increases. Thus, the expected potential gain from this procurement also increases, but the total expected information rents also increase as  $n$  becomes large. Owing to the trade-off between these two effects, the minimum expected transfer,  $-\Phi_n(q^*)$ , is not necessarily monotonic in  $n$ . As the number of firms goes to infinity, however, the minimum expected production cost reaches the lower bound  $\underline{c}$ , and  $-\Phi_n(q^*)$  approaches zero. Therefore, ex post efficiency can be approximately achieved when there is a large number of firms available. This result has useful implications for procurement policy.

### 3. Equilibrium mechanisms

Since the ex post efficient allocation cannot be implemented, we now search for the second-best mechanisms. In particular, we are concerned with the optimal mechanisms for the buyer when she has the ability to select mechanisms. Applying the Revelation Principle, the buyer only has to consider all possible regular mechanisms that are characterized in lemma 1. Since she knows her type  $\theta$ , the question is whether different types of buyer would choose the same procurement mechanisms. If they choose the same mechanism, which one should they agree upon?

In a general Bayesian incentive problem, Myerson (1983) has developed the Inscrutability Principle, which states that there is no loss in generality by requiring that all types of principal select the *same* regular mechanism. The principal can always build any communication into the process of the mechanism itself. Myerson has also provided a solution concept that is called a strong solution. It is an undominated and safe mechanism. In our context a mechanism,  $[p(\theta, c), q(\theta, c)]$ , is undominated if it is regular and if there is no other regular mechanism  $[\bar{p}(\theta, c), \bar{q}(\theta, c)]$  such that  $V(p, q; \theta) \leq V(\bar{p}, \bar{q}; \theta)$  for all  $\theta \in [0, \bar{\theta}]$ . In other words, an undominated mechanism yields a Pareto-efficient outcome among all the types of buyer. On the other hand, a mechanism is safe if it is incentive compatible for the firms, given any type of buyer,  $\theta$ . That is, the buyer can always implement a safe mechanism no matter what the firms might infer about her type.

Myerson argues that a strong solution, if it exists, is essentially unique and

<sup>5</sup> Spulber (1988) and McAfee (1991) also have illustrated the possibility of ex post efficiency in their models. They consider general pay-off functions for both the buyer and the seller.

In general, the existence of ex post efficient mechanisms depends on the form of the players' pay-off functions and their expectations about others' private information.

reasonable for the principal to select. One of the reasons is that a strong solution can be justified in the context of a non-cooperative mechanism-selection game. In our procurement context, consider the following sequential game. First, the buyer selects and announces a mechanism. Second, each firm makes some inferences about the buyer's type, based upon the announced mechanism, and then chooses its strategy accordingly. Finally, the mechanism is implemented according to the terms of the mechanism. A natural equilibrium concept for this game is the sequential equilibrium. Myerson (1983) shows that a strong solution, if it exists, can be supported as a sequential equilibrium of the mechanism-selection game. Thus, given the rational behaviour of the firms, no type of buyer would do better in any alternative mechanism than in the strong solution.

While a strong solution has nice properties, it may fail to exist in a general Bayesian incentive problem, as Myerson also points out. The major objective of this section is to show the existence of a strong solution in our independent private value model. We construct a strong solution in the following way. We first find the mechanism that maximizes the buyer's ex ante expected pay-off. This is the mechanism that the buyer would choose before she learns her type. More important, since the mechanism maximizes a weighted sum of the buyer's expected pay-off among all the regular mechanisms where the weight  $g(\theta)$  is positive over  $[0, \theta]$ , it follows from the separating hyperplane theorem that the mechanism cannot be dominated by any other regular mechanism. That is, the buyer's ex ante optimal solution is undominated. We then show that this mechanism is in fact a safe mechanism. Therefore, a strong solution is constructed.

To be more specific in our construction, we apply lemma 1. It follows that the expected payment function in a regular mechanism is determined by equations (4) and (5). Without loss of generality, set  $U_i(p, q; \bar{c}) = 0$  for all  $i = 1, \dots, n$ . It then follows from (5) and integration by parts that the buyer's ex ante expected pay-off can be written as

$$EV = E_{(\theta, c)} \left( \theta B(Q(\theta, c)) - \sum_{i=1}^n I(c_i) q_i(\theta, c) \right),$$

which depends only upon quantity vector  $q(\theta, c)$ . We choose  $q(\theta, c)$  to maximize this adjusted expected pay-off subject to the monotonicity conditions (1) in lemma 1. To characterize the solution, we ignore constraints (1) for now and check them later. The solution is determined by the first-order conditions and described as follows. For  $\theta \in [0, \bar{\theta}]$ , let  $\hat{r}(\theta)$  be the adjusted cut-off level which is defined by

$$\hat{r}(\theta) = \begin{cases} \underline{c}, & \text{if } \beta\theta \leq \underline{c} \\ I^{-1}(\beta\theta), & \text{if } \underline{c} < \beta\theta < I(\bar{c}) \\ \bar{c}, & \text{if } \beta\theta \geq I(\bar{c}), \end{cases} \tag{10}$$

where  $\beta > 0$  satisfies  $B(h(\beta)) = \beta h(\beta)$ . For  $(\theta, c) \in [0, \theta] \times [\underline{c}, \bar{c}]^n$ , let  $z(\theta, c_{-i}) = \min \{\hat{r}(\theta), \min_{j \neq i} c_j\}$  and define a procurement quantity vector  $\hat{q}(\theta, c)$  as

$$\hat{q}_i(\theta, c) = \begin{cases} h(I(c_i)/\theta), & \text{if } c_i < z(\theta, c_{-i}) \\ 0, & \text{if } c_i > z(\theta, c_{-i}), \end{cases} \tag{11}$$

for  $i = 1, \dots, n$ . If  $c_i = c_j$ , then  $\hat{q}_i(\theta, c)$  and  $\hat{q}_j(\theta, c)$  can be defined as  $h(I(c_i)/\theta)/2$ . Equation (11) says that, for each  $(\theta, c)$ , the buyer's marginal benefit is equal to each firm's virtual marginal cost conditional on the firm's cost's being the lowest across the firms. It is easy to verify that  $\hat{q}(\theta, c)$  satisfies conditions (1) in lemma 1 ignored before.

We further construct a payment vector  $\hat{p}(\theta, c)$  as follows: For  $i = 1, \dots, n$ ,

$$\hat{p}_i(\theta, c) = c_i \hat{q}_i(\theta, c) + \int_{c_i}^{\bar{c}} \hat{q}_i(\theta, c_{-i}, t) dt. \tag{12}$$

We show that the direct revelation mechanism,  $[\hat{p}(\theta, c), \hat{q}(\theta, c)]$ , defined in (10)–(12) satisfies (1)–(3) in lemma 1 and is a safe mechanism. The details of the proof are provided in Tan (1994a). Therefore, we have

PROPOSITION 2. (i)  $[\hat{p}(\theta, c), \hat{q}(\theta, c)]$  is regular and maximizes the buyer's ex ante expected pay-off among all regular mechanisms, and (ii) for any  $\theta \in [0, \bar{\theta}]$ ,  $[\hat{p}(\theta, c), \hat{q}(\theta, c)]$  is incentive compatible and individually rational for each firm.

The first statement in proposition 2 tells us that the buyer's ex ante optimal commitment solution exists and is determined by (10)–(12). The solution is also an undominated mechanism and hence Pareto efficient among all types of buyer. In other words, there is no other mechanism that would make all types of buyer better off.

Notice that the buyer's ex ante optimal commitment solution depends upon the buyer's type. A natural question is whether the buyer can successfully implement this mechanism given any conjectures of the firms about the buyer's type? The second statement in proposition 2 implies that the answer is yes. The mechanism  $[\hat{p}(\theta, c), \hat{q}(\theta, c)]$  is incentive compatible and individually rational even if the firms knew the buyer's information  $\theta$ . It is also incentive compatible and individually rational given any subset of  $\theta$ . No matter how firms infer the buyer's type, the mechanism always induces the firms to report their true types. This is a safe mechanism in the sense of Myerson (1983).

Combining the two results in proposition 2, we have obtained a strong solution, which is the equilibrium mechanism for the buyer to select. Since the mechanism is safe and incentive compatible for the buyer, the buyer can actually announce the mechanism and her true type at the same time.

It should be noted that a similar result is obtained by Maskin and Tirole (1990) in the case of one agent. Using a non-cooperative-game approach with the concept of a perfect Bayesian equilibrium, they analyse a three-stage extensive-form game in which the informed principal offers a mechanism first, the agent then accepts or rejects the offer, and finally the two parties play the proposed mechanism if it is accepted by the agent. They show that, when the utility functions for both players are quasi-linear, in the perfect Bayesian equilibrium the principal neither gains nor loses if her private information is revealed to the agent prior to contracting.

Therefore, the two approaches yield the same qualitative outcome. Their result depends crucially upon the assumption of quasi-linear preferences. For instance, Maskin and Tirole (1990) also show that when the preferences are not quasi-linear, the principal does not reveal any of her information until the final stage. Tan (1994b) illustrates that the buyer can do better by concealing her private reserve price in a first-price sealed-bid auction when the bidders (or the suppliers) are very risk averse.

Note that the equilibrium quantity vector  $\hat{q}(\theta, c)$  is unique once  $B(Q)$  is strictly concave. The payment vector associated with  $\hat{q}(\theta, c)$  may not be unique. All these mechanisms (safe and undominated), however, provide the buyer with the same expected pay-off. In other words, the buyer's equilibrium mechanism is essentially unique.

IV. IMPLEMENTATION

We turn now to implementation of the equilibrium mechanism characterized in proposition 1. Notice that in the equilibrium mechanism the buyer purchases only from the firm with the lowest cost of production, provided that this cost is also below some cut-off level. This property suggests that the equilibrium mechanism may possibly be implemented using a variation of standard auction procedures in which the buyer's purchases can depend upon the winning bid.

Consider the following first-price sealed-bid auction (FPA) with variable quantities: The buyer proposes a price-quantity schedule,  $Q = Q(b)$ , with a reserve price,  $r$ , and asks the firms to submit sealed bids independently. Each bid is a unit price for the product to be procured. The buyer promises that the firm with the lowest bid,  $b_l$ , wins the contract,  $[b_l Q(b_l), Q(b_l)]$ , if  $b_l$  is no larger than the reserve price  $r$ . The winner produces  $Q(b_l)$  and receives a payment  $b_l Q(b_l)$  from the buyer. Here, we assume that the buyer can credibly precommit to the price-quantity schedule and the reserve price.<sup>6</sup> This auction procedure with a price-quantity schedule captures some of these features of many practical procurement contracts. As Hansen (1988) has pointed out, procurement often involves an open-ended quantity contract, and the actual quantity transacted is affected by the price. 'Requirements' contracts are this sort of open-ended contract.

We now construct a price-quantity schedule and a reserve price that implement the equilibrium mechanism. For any  $\theta \in [0, \theta]$  and  $c_i \in [\underline{c}, \hat{r}(\theta)]$ , define

$$b(c_i, \theta) = c_i + \frac{\int_{c_i}^{\hat{r}(\theta)} [1 - F(t)]^{n-1} h(I(t)/\theta) dt}{[1 - F(c_i)]^{n-1} h(I(c_i)/\theta)}, \tag{13}$$

6 Dasgupta and Spulber (1990) have used similar auctions to implement the optimal procurement contract in the case where the demand information is public. When there is only one firm, this type of auction becomes a take-it-or-leave-it scheme with a non-linear compensation schedule, which is also discussed by Spulber (1988).

where  $h(\cdot)$  is the inverse function of  $B'(Q)$ . It can be verified that  $b(c_i, \theta)$  is strictly increasing in  $c_i$ . For any  $\theta$ , let  $\phi(b, \theta)$  be the inverse function of  $b(c_i, \theta)$  with respect to  $c_i$  and define

$$\hat{Q}(b, \theta) \equiv h(I(\phi(b, \theta))/\theta). \quad (14)$$

It can be verified that  $\hat{Q}(b, \theta)$  is decreasing in  $b$ .

**PROPOSITION 3.**  *$[\hat{p}(\theta, c), \hat{q}(\theta, c)]$  can be implemented using a first-price sealed-bid auction in which the buyer precommits to a downward-sloping price-quantity schedule  $Q = \hat{Q}(b, \theta)$  combined with a reserve price  $\hat{r}(\theta)$ .*

In the FPA specified in proposition 3 the buyer announces her true type  $\theta$  and the price-quantity schedule with the reserve price. Each bidder then observes  $\theta$ . It can be shown that the equilibrium bidding strategy for each bidder is given by (13). Since  $b(c_i, \theta)$  is strictly increasing in  $c_i$ , the firm with the lowest cost  $c_i$  wins the contract. The winning firm produces  $h(I(c_i)/\theta)$  units, which is the buyer's optimal quantity level defined in (11). For the details of the argument see Tan (1994a).

Proposition 3 shows that, if the buyer can precommit to her price-quantity schedule with the reserve price, it is always in her interest to do so. Through the precommitment, the buyer reveals the terms of the procurement contract as well as her private information  $\theta$  to all bidders. The equilibrium mechanism is then equivalent to the optimal mechanism that is used when the buyer's information about the demand is public (see Dasgupta and Spulber 1990). In other words, the buyer should select the same competitive bidding scheme as she would when  $\theta$  is publicly known.

Disclosure of demand information does not imply that the buyer's private information is not valuable to her. In fact, the buyer strictly benefits from being informed. With  $\theta$  unknown, the buyer chooses a quantity vector  $q(c)$  to maximize her expected pay-off, and the solution is independent of  $\theta$ . When the buyer is informed about  $\theta$ , she must be better off, since she has the option of offering a mechanism independent of  $\theta$  and chooses not to do so, as proposition 2 has shown. The buyer is not better off, however, if she conceals her information. This is an important point in our analysis.

In the rest of the section we discuss whether the equilibrium mechanism can be implemented using a second-price sealed-bid auction (SPA, or Vickrey auction).

In an SPA, the buyer announces a price-quantity schedule,  $Q = Q(b)$ , combined with a reserve price,  $r$ , and each firm submits a unit price bid. The firm with the lowest bid,  $b_l$ , wins provided that  $b_l$  is less than or equal to  $r$ . The winner receives the contract  $[b_{sl}Q(b_{sl}), Q(b_{sl})]$  if the second lowest bid  $b_{sl}$  does not exceed  $r$ , and  $[rQ(r), Q(r)]$  otherwise. It can be verified that, in an SPA, it is a dominant strategy for each firm to submit its true unit cost of production.

The question is whether there exists a price-quantity schedule and a reserve price such that an SPA provides the buyer with the same expected pay-off as in

the FPA described earlier. Suppose that the buyer chooses  $[Q(b), r]$  to maximize her expected pay-off in an SPA. The following proposition shows that when the buyer has a strictly concave benefit function, the optimal SPA yields less expected pay-off for the buyer than the optimal FPA.

**PROPOSITION 4.** *If  $B(Q)$  is strictly concave, then the buyer strictly prefers the optimal FPA to any SPA.*

The proof of proposition 4 is given in the appendix and the argument can be described as follows. First, in an SPA, since truth-telling is a dominant strategy for each firm, the buyer's expected pay-off can be decomposed into two parts. The first part is the buyer's expected pay-off when the second lowest bid is no less than  $r$  and is equal to  $\theta B(Q(r)) - rQ(r)$  multiplied by the probability of  $b_{sl}$ 's being no less than  $r$ . The second part is the expected value of  $\theta B(Q(b_{sl})) - b_{sl}Q(b_{sl})$  conditional on  $b_{sl}$ 's being below the reserve price  $r$ . The buyer chooses a price-quantity schedule as well as a reserve price to maximize her expected pay-off. The optimal quantity level is set so that the marginal benefit equals the marginal cost, which is the first-best quantity  $Q^*(b)$ , given by equation (8). The optimal reserve price  $r_S$  is different from the first-best cut-off level  $r^*(\theta)$ , however, since the buyer behaves as a monopsonist. Second, suppose that the buyer offers  $(Q^*(b), r_S)$  in an FPA. Then the FPA yields a higher expected pay-off for the buyer than the SPA does. This is because  $Q^*(b)$  is strictly decreasing in  $Q$ , owing to the strict concavity of the benefit function. Compared with the case where only one unit is procured (inelastic demand), the bidder in an FPA with downward-sloping demand has additional incentives to reduce his bid, which causes the quantity procured to rise. In an SPA, the bidder submits his true cost of production independent of the elasticity of demand. As is well known, two auction schemes yield the same expected pay-off for the buyer when there is only one unit being auctioned. Therefore, with downward-sloping demand, the buyer expects to pay less and receive more benefit in the FPA than in the SPA. Third, in an FPA, the buyer is able to choose optimally the price-quantity schedule and reserve price, which will increase her expected pay-off.

A similar comparison is made by Hansen (1988) in an auction model with variable quantities and an exogenous market demand function. He shows that when the demand function is negatively sloped, a first-price auction yields a lower expected price and a higher expected total surplus than an open auction does (which is strategically equivalent to a second-price auction). However, expected consumer's surplus may not be higher in the FPA than in the open auction. In our model the buyer, as a monopsonist, endogenously determines a price-quantity schedule combined with a reserve price and offers different schedules and reserve prices across auction schemes. We show that the optimal first-price auction yields a higher expected consumer's surplus than the optimal second-price auction does. Because most public and private procurement involves open-ended quantity contracts, our analysis provides an additional explanation for why first-price auctions are widely used in procurement.

In the case of a linear benefit function or of unit demand, the FPA does not have such an advantage. As in the case of propositions 2 and 3, it can be shown that both the FPA and the SPA with an announced reserve price are optimal for the buyer even if the buyer has private information about her demand. Therefore, to the buyer a policy of concealing the reserve price cannot be better than a policy of announcing the reserve price.<sup>7</sup>

#### V. CONCLUDING REMARKS

We have analysed a procurement model with independent private values and risk neutral buyer and suppliers. The main feature of the model is that the buyer is privately informed. We have examined the equilibrium mechanism that all types of buyer will agree upon. This is a deviation from the previous analysis on the buyer's ex ante commitment solution. We have obtained two major results. The first one is that the informed buyer should reveal her private information through the announcement of the optimal procurement contract and should use a first-price sealed-bid auction procedure to award the procurement contract. One specific implication of this result is that the buyer should make her reserve price public. The second is that the buyer with decreasing marginal willingness to pay strictly prefers a first-price auction with the optimal choice of the price-quantity schedule combined with a reserve price to any form of second-price auctions. This finding explains why first-price auctions, not second-price auctions, are widely used in procurement practices.

It should be noticed that, when suppliers are risk averse, a first-price sealed-bid auction with an announced reserve price may not be an optimal mechanism. The buyer may use a policy of concealing reserve price. This policy induces the suppliers to bid aggressively. In a separate paper (Tan 1994b), we compare two reservation price policies: announcing the reserve price in advance and concealing the reserve price. In the case of one supplier, we show that the buyer ex ante prefers concealing to announcing the reserve price when the supplier's utility function exhibits a high degree of constant relative risk aversion. The implication of this result is that the buyer's take-it-or-leave-it offer scheme is not her best bargaining strategy when the seller is very risk averse. When there is more than one risk averse supplier, we obtain similar results for certain classes of distribution functions. Comparing different reservation price policies in a general procurement environment remains for further research.

#### APPENDIX: PROOF OF PROPOSITION 4

We prove proposition 4 in two steps. First, it follows from proposition 3 that the buyer's expected pay-off in the FPA can be written as

<sup>7</sup> See Elyakime et al. (1994) for more discussion on secret reservation prices. They have calculated the seller's expected gain of moving from the optimal secret reservation price rule to the optimal publicly announced reservation price rule.

$$EV_F = \int_{\underline{c}}^{\hat{r}(\theta)} [\theta B(\hat{Q}(t)) - I(t)\hat{Q}(t)][1 - F(t)]^{n-1} dF(t),$$

where  $\hat{Q}(t) = h(I(t)/\theta)$ .

Second, in an SPA with a price-quantity schedule  $Q = Q(b)$  and a reserve price  $r$ , truth-telling is a dominant strategy for each firm. The firm with the lowest cost  $t_1$  wins the contract and is paid at the second-lowest bid  $t_2$ . The joint density function of  $t_1$  and  $t_2$  is

$$\psi(t_1, t_2) = \begin{cases} n(n-1)f(t_1)[1 - F(t_2)]^{n-2}f(t_2), & \text{if } t_1 \leq t_2, \\ 0, & \text{if } t_1 > t_2. \end{cases}$$

The expected benefit for the buyer is then equal to

$$\begin{aligned} & \int_{\underline{c}}^r \int_{\underline{c}}^{t_2} \theta B(Q(t_2))\psi(t_1, t_2)dt_1 dt_2 + \int_{\underline{c}}^r \int_r^{\bar{c}} B(Q(r))\psi(t_1, t_2)dt_1 dt_2 \\ & = n\theta B(Q(r))F(r)[1 - F(r)]^{n-1} + n(n-1) \int_{\underline{c}}^r \theta B(Q(t))F(t)[1 - F(t)]^{n-2}dF(t), \end{aligned}$$

where the equality follows from integration by parts. Similarly, the buyer's total expected payment is given by

$$\begin{aligned} & \int_{\underline{c}}^r \int_{\underline{c}}^{t_2} t_2 Q(t_2)\psi(t_1, t_2)dt_1 dt_2 + \int_{\underline{c}}^r \int_r^{\bar{c}} rQ(r)\psi(t_1, t_2)dt_1 dt_2 \\ & = nQ(r)F(r)[1 - F(r)]^{n-1}r + n(n-1) \int_{\underline{c}}^r tQ(t)F(t)[1 - F(t)]^{n-2}dF(t). \end{aligned}$$

It follows that the buyer's expected pay-off in the SPA is given by

$$\begin{aligned} EV_S(Q(\cdot), r) &= n[\theta B(Q(r)) - rQ(r)]F(r)[1 - F(r)]^{n-1} \\ & \quad + n(n-1) \int_{\underline{c}}^r [\theta B(Q(t)) - tQ(t)]F(t)[1 - F(t)]^{n-2}dF(t). \end{aligned}$$

$EV_S(Q(\cdot), r)$  is maximized by choosing  $Q(t)$  and  $r$  such that the following first-order conditions are satisfied:

$$\theta B'(Q(t)) = t$$

for  $t \in [\underline{c}, r]$  and

$$\theta B(Q(r)) = I(r)Q(r).$$

Using the first-order conditions and integrating by parts, we obtain the buyer's maximum expected pay-off from the best SPA as follows

$$EV_S = n \int_{\underline{c}}^r [\theta B(Q(t)) - I(t)Q(t)][1 - F(t)]^{n-1} dF(t).$$

Since  $B(Q)$  is strictly concave, it follows that  $EV_F > EV_S$ . ■

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