# Cost-Reducing Investment, Optimal Procurement and Implementation by Auctions

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# COST-REDUCING INVESTMENT, OPTIMAL PROCUREMENT AND IMPLEMENTATION BY AUCTIONS*

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We analyze a model in which potential suppliers invest in research and development (R&D) and then compete for a procurement contract from a buyer. If the buyer is able to commit to a procurement mechanism before the investment stage, the full-information solution can be uniquely implemented by first-price and second-price sealed-bid auction mechanisms when the R&D technology exhibits decreasing returns to scale. If the procurement mechanisms and the levels of investment are chosen simultaneously, the full-information solution cannot be implemented. We discuss how the second-best equilibrium contract varies with the number of suppliers.

## 1. INTRODUCTION

This paper examines a model in which firms invest in research and development (R&D) and compete for a procurement contract. Our objective is to investigate the relationship between the structure of the R&D technology and the implementation of the optimal mechanism by auctions. We generalize previous results and obtain new ones in a framework which classifies technologies according to their returns to scale.

In our model, a buyer purchases one unit of an indivisible product, choosing among several potential suppliers whose production processes consist of two stages. In the first stage, the suppliers invest in R&D and search among alternative methods of production. The outcome of this stage is knowledge about the cost of production. In the second stage, production is carried out and additional effort can be exerted to further reduce costs. R&D investment and production costs are assumed to be private information.

The form of the R&D technology that we adopt is quite general. Each level of investment in R&D determines a probability distribution of the unit cost. We assume that, given any two investment levels, the probability distribution associated with the higher level first-order stochastically dominates the probability distribution associated with the lower one.

We provide a classification which distinguishes among R&D technologies by their returns to scale. Suppose that all firms adopt the same technology and invest identical amounts. The technology exhibits decreasing returns to scale if, fixing the

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aggregate expenditure in R&D and a unit cost of production, $y$, the probability of finding a method of production with unit cost below $y$ by at least one firm increases with the number of firms. Namely, dispersing the aggregate R&D expenditure over a large number of firms improves the chances of observing a low unit cost. Increasing returns to scale are defined analogously. We also provide conditions for the R&D technology under which firms can be aggregated into a representative firm. These conditions allow us to obtain unambiguous comparative statics results.

As to the timing of moves, we consider two frameworks. The first framework is a sequential move game in which the buyer announces a procurement mechanism prior to investment. Guler (1990) and Tan (1992) show that first-price and second-price auctions yield the same expected revenue for the buyer. Laffont and Tirole (1993, Chapter 1) and Bag (1993) examine the implementation of the full-information solution in two specific settings. Laffont and Tirole consider the case of one firm and establish that an optimal mechanism implements the full-information solution. Bag considers the case of two firms and discrete investment and shows that the full information solution can be weakly implemented by a first-price and second-price auction. We study the issue of *unique* implementation by simple auction mechanisms for the case of many firms and a general specification of the R&D technology. We show that, if the technology exhibits decreasing returns to scale, the full-information solution requires all suppliers to invest identical amounts and is *uniquely* implemented by first-price and second-price auction mechanisms.

The second framework we consider is a game in which the buyer's choice of a procurement mechanism is simultaneous with the investment decisions by the suppliers. This framework has two interpretations. One is that the suppliers, anticipating a future contract offer, decide to invest before the buyer announces a procurement mechanism. An alternative interpretation is that the buyer is unable to commit to a mechanism before the firms invest in R&D. Since the investment levels are private information, the procurement and investment decisions can be assumed to be simultaneous. In this case, distortions due to information asymmetries cannot be avoided. The full-information solution cannot be implemented and the suppliers earn positive information rents. Dasgupta (1990) assumes that investment levels are symmetric and establishes that firms under-invest relatively to the ex-ante socially optimal levels. We relax the assumption of symmetric investment levels and show that, if the technology exhibits decreasing returns to scale, there exists a unique equilibrium in which all firms invest identical amounts and under-invest relatively to the full-information solution. We also examine the effects of competition among suppliers on the equilibrium mechanism, total R&D expenditure, and the expected production cost in the industry.

Information rents and allocation distortions caused by information asymmetries have been shown to occur in the special case of suppliers being endowed with private information exogenously. In this setting, Laffont and Tirole (1987, 1993, Chapter 7) also demonstrate that the problems of selecting the efficient supplier and providing incentives in production can be separated. The buyer can use an auction to select the efficient supplier, and the optimal levels of quantity and effort in production are independent of the number of competing suppliers. Analogous results have been obtained by McAfee and McMillan (1987), Riordan and Sappington (1987), and Dasgupta and Spulber (1990) in different environments. By contrast, we show that if the suppliers' private information is endogenously determined simultaneously with the buyer's offer, the separation property does not hold. Under the assumption of decreasing returns, as the number of competing suppliers increases, the equilibrium contract exhibits higher incentives for the winner to exert effort in production.

We conclude our paper by illustrating some of the problems that arise when the assumption of decreasing returns to scale is violated. In particular, we examine the implications for the implementation of the full-information solution by simple auctions and for the direction of allocation distortions.

## 2. THE MODEL

We consider a model in which a single buyer (e.g., a government agency) intends to purchase a single indivisible unit of a public project from $n$ firms, where $n$ is exogenously given. The value of the project for the buyer is denoted by $V$. The production process consists of two periods. In the first period, research and development (R&D) are conducted by the firms, but no production is carried out. The outcome of this period is knowledge about the costs of completing the project, which is assumed to be firm-specific. In the second period, the project is produced. During the production, the chosen firm may continue to exert some effort in order to reduce production cost further.[2]

In the initial period, each firm $i$ invests $x_i \in [0,\infty)$ in the R&D process. The outcome of the R&D process is the level of production cost, which is denoted by $y_i$. We assume that firm $i$ observes $y_i$ only after investing. Prior to the investment decision, the cost of production is represented by a stochastic process, $\{Y_i(\cdot, x_i); x_i \in [0,\infty)\}$, which is independent of other firms' R&D processes. Let $H(y_i, x_i)$ be the cumulative distribution function of $Y_i$ and $h(y_i, x_i)$ be the associated density function. The support of $H(y_i, x_i)$ is assumed to be independent of $x_i$. Let $[\underline{a}, \bar{a}]$ denote the support of $H(y_i, x_i)$, where $0 < \underline{a} < \bar{a}$. For simplicity, we assume that $\bar{a} \leq V$. The cost of investment is normalized to be $cx_i$, where $c > 0$, and fixed costs are assumed to be equal to zero.

In the second period, firm $i$ is able to produce the project at the following cost

$$C_i = y_i - e_i,$$

where $e_i$ is the effort level exerted by firm $i$ in production, $e_i \in [0, \bar{e}]$, and $\bar{e} \leq \underline{a}$. If firm $i$ exerts effort $e_i$, it decreases the cost of providing the project by $e_i$ and incurs a disutility of $\psi(e_i)$. Here both $e_i$ and $\psi(e_i)$ are measured in monetary units. All the firms have identical disutility functions. We assume $\psi(0) = 0$, $\psi'(e_i) > 0$ and $\psi''(e_i) > 0$ for $e_i > 0$, and $\psi'(0) < 1$ and $\psi'(\bar{e}) > 1$; $(x_i, y_i, e_i)$ are firm $i$'s private information.

As to the timing of the game, we consider two alternative specifications. In the first setting, the buyer offers a procurement mechanism before firms make investment decisions and commits to the same mechanism after investment is completed. This timing structure can be described as follows: In the first stage, the buyer announces a procurement mechanism and commits herself to implement the same

[2] For discussion of R&D in defense procurements, see Lichtenberg (1988) and Rogerson (1994).

mechanism at the subsequent stages. In the second stage, firms decide whether or not to participate. The participating firms simultaneously choose their levels of R&D investment and observe their own costs of production. In the third stage, a firm is selected to produce the good. Finally, the selected firm chooses a level of effort and produces the good.

In the second setting, firms invest before the buyer announces her mechanism. In the first stage, the firms choose levels of R&D investment simultaneously and observe their cost of production. In the second stage, the buyer offers a procurement mechanism. In the third stage, each firm decides whether to accept or reject the offer. Finally, the firm that is selected by the mechanism chooses the level of effort and produces the good. This specification is the same as the first setting except that the order of the first two stages is reversed.

Both the buyer and firms are assumed to be risk neutral. Each firm's objective is to maximize its expected profits. The buyer maximizes her expected surplus. In particular, the buyer designs a mechanism to achieve three objectives: (i) induce an appropriate level of investment before the contractor is selected, (ii) select the most efficient firm, and (iii) provide incentives for the selected firm to produce efficiently.

## 3. R&D TECHNOLOGY

In this section, we state our main assumptions on the R&D technology and provide some preliminary results. We make the following assumptions:

(A1) $H(y_i, x_i)$ is twice continuously differentiable in $(\underline{a}, \bar{a}) \times (0, \infty)$.

(A2) $H(y_i, x_i)$ is strictly increasing in $x_i$ for all $(y_i, x_i) \in (\underline{a}, \bar{a}) \times (0, \infty)$ and $H(y_i, 0) = 0$ for all $y_i < \bar{a}$.

(A3) $\int_{\underline{a}}^{\bar{a}} H_{x_i}(t, 0)\, dt > c$.

Assumption (A2) implies that an increase in $x_i$ has a nontrivial effect and modifies the distribution of $y_i$ in a way consistent with first-order stochastic dominance. Assumption (A3) rules out the trivial corner solution.

Given the stochastic nature of the production function, we classify the R&D technology by the project failure rate with respect to the level of investment, $r(y_i, x_i)$, for $(y_i, x_i) \in (\underline{a}, \bar{a}) \times (0, \infty)$, where

$$r(y_i, x_i) = \frac{H_{x_i}(y_i, x_i)}{1 - H(y_i, x_i)}.$$

The meaning of $r(y_i, x_i)$ can be understood as follows. Fix a target level of unit cost, $y_i$, and call any realization of the unit cost above $y_i$ a failure. Then the probability of failure is $1 - H(y_i, x_i)$. An increase in the level of investment reduces the probability of failure and $r(y_i, x_i)$ represents the rate of such reduction.

If $r(y_i, x_i)$ is strictly decreasing in $x_i$ for any $y_i \in (\underline{a}, \bar{a})$, we say that the R&D technology exhibits decreasing returns to scale. Conversely, if $r(y_i, x_i)$ is strictly increasing in $x_i$, the R&D technology exhibits increasing returns to scale. To clarify this, consider the following exercise. Suppose that $k$ firms invest a total aggregate amount $M$ evenly. The realized costs of production, $(y^1, \ldots, y^k)$, are i.i.d. with a cumulative distribution function $H(y_i, M/k)$. Then the minimum cost, $t$, is the realization of the minimum-order statistic with distribution $1-[1-H(t, M/k)]^k$. The following lemma shows that, if $r(y_i, x_i)$ is decreasing in $x_i$, then an increase in $k$ increases $1-[1-H(t, M/k)]^k$ uniformly.

LEMMA 1. *Given* (A1) *and* (A2), *for any* $t \in (\underline{a}, \bar{a})$ *and* $M > 0$, *the following hold:*
(i) *if* $r(t, x_i)$ *is decreasing in* $x_i$, *then* $1-[1-H(t, M/k)]^k$ *increases with* $k$; *and*
(ii) *if* $r(t, x_i)$ *is increasing in* $x_i$, *then* $1-[1-H(t, M/k)]^k$ *decreases with* $k$.

Lemma 1(i) implies that, when the project failure rate is decreasing in the level of investment, spreading the total investment expenditure over a greater number of firms increases the probability of observing a low cost. The converse is true when the project failure rate is increasing.

REMARK. In our specification of R&D technology, we assume that the support of $H(y_i, x_i)$ is fixed. The model can be modified to include the case of moving support by assuming that the lower bound, $\underline{a}(x_i)$, decreases with $x_i$ and by defining $\underline{a}$ as $\inf_{x_i}\{\underline{a}(x_i)\}$. It should be noted that in this case (A2) cannot hold and $r(y_i, x_i)$ cannot be decreasing in $x_i$ for all $y_i$ in $(\underline{a}, \bar{a}]$. Increasing returns to scale must exist for low production costs. Fixing aggregate investment expenditure and decreasing number of firms induce a positive probability for low production costs which would be unattainable with a higher number of firms.

In the analysis below, we will also be interested in the minimum cost across all firms, $t = \text{Min}\{y_1, \ldots, y_n\}$. Given a vector of expenditure, $x = (x_1, \ldots, x_n)$, the distribution of the minimum cost is given by $1-\prod_{i=1}^{n}[1-H(t, x_i)]$. When each firm invests the same amount, $\sigma$, this distribution can be written as $1-[1-H(t, \sigma)]^n$ and is denoted by $H_0(t; \sigma, n)$. In this case, two factors affect the distribution of the minimum cost. An increase in $\sigma$ uniformly shifts $H_0(t; \sigma, n)$ in the sense of first-order stochastic dominance. An increase in $n$ has a similar effect. The following proposition identifies a class of distributions for which a uniform comparison of $H_0(t; \sigma, n)$ for different $\sigma$ and $n$ is possible. We first provide the following definition:

DEFINITION. A distribution $H(y_i, x_i)$ satisfies the *complete stochastic ordering property* (CSO) if, for any $\sigma, \sigma' \in [0, \infty)$, and any integers $n, n'$, $H_0(t; \sigma, n) \geq H_0(t; \sigma', n')$ for some $t \in (\underline{a}, \bar{a})$ implies that $H_0(t; \sigma, n) \geq H_0(t; \sigma', n')$ for all $t \in (\underline{a}, \bar{a})$.[3]

The following proposition shows that the distributions with CSO property have a special functional form.

[3] Notice that when $n$ is any real number, this definition is equivalent to the definition of the strict separability of the variables $\sigma$ and $n$ from the variable $t$ in the theory of duality and aggregation. For a complete discussion, see Blackorby, Primont, and Russell (1978). It can easily be shown that the CSO in the definition implies that it also holds for any real number $n$.

PROPOSITION 1. *Given* (A1) *and* (A2), $H(y_i, x_i)$ *has CSO property if and only if there exists two functions* $F(y_i)$ *and* $\alpha(x_i)$ *such that*

$$H(y_i, x_i) = 1 - [1 - F(y_i)]^{\alpha(x_i)},$$

*where both* $F(y_i)$ *and* $\alpha(x_i)$ *are increasingfunctions,* $F(\underline{a}) = 0$, $F(\bar{a}) = 1$, *and* $\alpha(0) = 0$.[4]

One consequence of the (CSO) property is that

$$1 - \prod_{i=1}^{n} [1 - H(t, x_i)] = [1 - F(t)]^{\sum_{i=1}^{n} \alpha(x_i)},$$

which implies that individual investments can be aggregated through $\sum_{i=1}^{n} \alpha(x_i)$. Also,

$$r(y_i, x_i) = -\alpha'(x_i) \log[1 - F(y_i)].$$

It follows that the R&D technology exhibits decreasing returns to scale when $\alpha(x_i)$ is concave and increasing returns to scale when $\alpha(x_i)$ is convex.[5]

## 4. THE FULL-INFORMATION SOLUTION

In this section, we analyze the case of full information. We assume that the investment vector $(x_1, \ldots, x_n)$, the vector of R&D outcomes $(y_1, \ldots, y_n)$, and the effort vector $(e_1, \ldots, e_n)$ of the $n$ firms are observable and contractible, and that the buyer moves first by offering contracts to the firms. Let $x$ denote $(x_1, \ldots, x_n)$ and $y$ denote $(y_1, \ldots, y_n)$.

A contract for firm $i$ consists of $[q_i(y), p_i(y), e_i(y), x_i]$, where $q_i(y)$ is the probability of receiving the contract as a function of $y$, $p_i(y)$ is the payment, and $e_i(y)$ is the effort level. Given this contract, the ex ante expected profit for firm $i$ is

$$E\Pi_i = E_y\{p_i(y) - [y_i - e_i(y) + \psi(e_i(y))]q_i(y)\} - cx_i,$$

where the expectation is taken with respect to $y$. A full-information contract is obtained by maximizing the buyer's expected surplus

$$EW \equiv E_y \sum_{i=1}^{n} \{Vq_i(y) - p_i(y)\}$$

[4] This type of R&D process has been analyzed by Tan (1992) in the context of procurement contracting. In a special case where $F(y_i)$ is an exponential distribution function, $H(y_i, x_i)$ is also an exponential distribution function. This functional form is widely adopted in the literature on stochastic R&D races, where $y_i$ represents the waiting time of the innovation by firm $i$. See Reinganum (1989) for a survey on this literature.

[5] An example of the distribution that does not satisfy the CSO property is $H(y_i, x_i) = \gamma(x_i)F_1(y_i) + (1 - \gamma(x_i))F_2(y_i)$, where $F_1(y_i)$ and $F_2(y_i)$ are cumulative distributions such that $F_1(y_i) \geq F_2(y_i)$ for all $y_i \in [\underline{a}, \bar{a}]$, $\gamma(x_i)$ is increasing in $x_i$, and $0 \leq \gamma(x_i) \leq 1$.

subject to each firm's participation constraint,

$$E\Pi_i \geq \bar{\pi}_i,$$

for $i = 1, \ldots, n$, where $\bar{\pi}_i$ is the ex ante reservation profit for firm $i$. For simplicity, we assume $\bar{\pi}_i = 0$.

At the full-information solution, the participation constraint must be binding for each firm. The buyer maximizes the total expected surplus function

$$EW = E_y \sum_{i=1}^{n} [V - y_i + e_i(y) - \psi(e_i(y))] q_i(y) - c \sum_{i=1}^{n} x_i$$

with respect to $(q_1(y), \ldots, q_n(y))$, $(e_1(y), \ldots, e_n(y))$, and $x$. Using the first-order conditions with respect to $e_1(y), \ldots, e_n(y)$, we have

$$1 - \psi'(e_i^*(y)) = 0 \tag{1}$$

for $i$ and $y$ such that $q_i(y) > 0$. Thus, if a firm is awarded the contract with a positive probability, then the optimal level of effort, $e^*$, is independent of $y$ and $i$. Notice that the assumptions in our model imply $e^* > \psi(e^*)$. Define $z(y)$ as the second lowest cost in $y$. The optimal probability of procurement is given by

$$q_i^*(y) = \begin{cases} 1, & \text{if } y_i < z(y), \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

for $i = 1, \ldots, n$. By (1) and (2), the expected surplus can be rewritten as

$$EW(x) = V - \left[ \int_{\underline{a}}^{\bar{a}} t \, d\tilde{H}(t, x) - e^* + \psi(e^*) + c \sum_{i=1}^{n} x_i \right], \tag{3}$$

where $\tilde{H}(t, x) = 1 - \prod_{j=1}^{n} [1 - H(t, x_j)]$ is the distribution of the lowest-order statistic of $(y_1, \ldots, y_n)$. The expected surplus consists of the value of the project minus the expected total costs in R&D and production. The levels of the full-information investment, denoted by $x^*$, are then obtained by maximizing $EW(x)$. The following proposition provides a characterization of the full-information solution.

PROPOSITION 2. *Given* (A1)–(A3), *the full-information solution is characterized as follows*:

(i) *The procurement contract is awarded to the firm with the lowest production cost.*

(ii) *The optimal level of effort is determined by* (1).

(iiia) *If $r(y_i, x_i)$ is strictly decreasing in $x_i$, then $x^*$ is unique and symmetric*; (b) *if $r(y_i, x_i)$ is strictly increasing in $x_i$, then $x_j^* > 0$ for only one firm $j$*; *and* (c) *if $r(y_i, x_i)$ is independent of $x_i$, then only $\sum_{i=1}^{n} x_i^*$ is uniquely determined.*[6]

[6] It can be shown that, given (A1) and (A2), $r(y_i, x_i)$ is independent of $x_i$ if and only if there exists a function $F(y_i)$ such that $H(y_i, x_i) = 1 - [1 - F(y_i)]^{x_i}$.

Proposition 2 shows that full-information investment depends upon the type of R&D technology. When the failure rate is strictly increasing, the technology exhibits increasing returns to scale and the full-information solution requires only one firm to invest in R&D. This implies a natural monopoly for procurement. When the failure rate is independent of $x_i$, only aggregate investment is determined. When the project failure rate strictly decreases with the level of investment, the technology exhibits decreasing returns to scale. Spreading the total investment expenditure across firms increases the overall probability of observing a low cost. The optimal level of investment by each firm for each firm, $\sigma^*(n)$, is determined by the first-order condition

$$\int_{\underline{a}}^{\overline{a}} r(t,\sigma^*)[1-H(t,\sigma^*)]^n \, dt - c = 0. \tag{4}$$

## 5. CONTRACTING PRIOR TO INVESTMENT

When the levels of investment, R&D outcomes, and efforts are not observable and noncontractible, the buyer must design incentive contracts to induce firms to take appropriate levels of investment. In this section, we show that when the R&D technology exhibits decreasing returns to scale, standard auction mechanisms can be used to uniquely implement the full-information solution. Therefore, the noncontractibility of investment does not impose any efficiency loss. The buyer can then use participation fees prior to contracting to maximize her expected surplus.

5.1. *Implementation of the Full-Information Solution.* Suppose that the project failure rate is strictly decreasing in the level of investment. It follows from Proposition 2 that at the full-information solution, all firms participate in R&D and the procurement contract is awarded to the firm with the lowest cost of production. The following proposition shows that a second-price auction uniquely implements the full-information solution.

PROPOSITION 3. *Suppose that* (A1)–(A3) *hold and that* $r(y_i, x_i)$ *is strictly decreasing in* $x_i$. *Then a second-price sealed-bid auction uniquely implements the full-information solution.*

The intuition behind Proposition 3 is as follows. Given a second-price auction, submitting the true cost of production is a dominant strategy for each firm. It follows that it is optimal for each firm to choose the full-information level of effort, $e^*$. Since all the firms report their true costs, the marginal gross profit for firm $i$ is the same as the buyer's marginal gain of investing $x_i$. Thus, the full-information solution must be an equilibrium in the second-price auction game. When the project failure rate is strictly decreasing, the equilibrium levels of investment in the auction game are unique and symmetric. Therefore, unique implementation is obtained.

We now consider the implementation of the full-information solution by a first-price sealed-bid auction. Let

$$I(y_i, x_i) = \frac{h(y_i, x_i)}{1 - H(y_i, x_i)}$$

denote the hazard rate with respect to $y_i$. We assume that $I(y_i, x_i)$ is strictly increasing in $x_i$. Notice that for the distributions characterized in Proposition 1 this assumption is always satisfied since $I(y_i, x_i) = \alpha(x_i)f(y_i)/[1 - F(y_i)]$.

PROPOSITION 4. *Suppose that* (A1)–(A3) *hold and that* $I(y_i, x_i)$ *is strictly increasing in* $x_i$. *If* $r(y_i, x_i)$ *is strictly decreasing in* $x_i$, *then a first-price sealed-bid auction uniquely implements the full-information solution. If* $r(y_i, x_i)$ *is independent of* $x_i$, *then a first-price sealed-bid auction uniquely implements the symmetric full-information solution.*

In the first-price auction game, it is always optimal for a firm to choose the full-information level of effort upon winning. If firm $j$ invests more in R&D than firm $i$, then the former has a higher probability of observing a low production cost. The regularity condition on $I(\cdot, \cdot)$ ensures that if firms $i$ and $j$ observe the same cost, firm $j$ chooses a higher bid. Since the probability of winning decreases with the bid and the project failure rate strictly decreases with the level of investment, the marginal gross profit of investment for firm $j$ is less than the marginal gross profit of investment for firm $i$. This cannot be an equilibrium as the marginal costs of investment are identical. Hence, the equilibrium investment levels must be symmetric. It is well known that if the distribution of the production costs are symmetric, the first-price and the second-price auctions yield the same expected profits for each firm. Thus, the two auction games have identical equilibrium investment levels. Since $r(t, x_i)$ is strictly decreasing in $x_i$, unique implementation follows from Proposition 3.

5.2. *Properties of the Symmetric Equilibrium Investment.* In this section, we investigate the comparative static properties of the symmetric equilibrium in the auction games. In particular, we focus on the effects of a change in the number of firms on the equilibrium levels of investment and on the expected minimum cost of production in the industry. Let $EC^*(n)$ be the expected minimum cost of production across all firms, that is,

$$EC^*(n) = \int_{\underline{a}}^{\bar{a}} (t - e^*)\, dH_0(t; \sigma^*(n), n),$$

where $H_0(t; \sigma^*(n), n) = 1 - [1 - H(t, \sigma^*(n))]^n$. In the literature on stochastic R&D races, $EC^*(n)$ is often referred to as the measure of the pace of innovation in an industry.

Consider first the case in which the project failure rate is independent of the level of investment. Proposition 4 shows that a first-price auction implements the symmetric level of the full-information investment. In equilibrium, both aggregate investment and $EC^*(n)$ are independent of the number of firms. Sah and Stiglitz (1987) obtain a similar result in the context of R&D races for the case of constant returns to scale.

When the R&D process exhibits decreasing returns to scale, the above invariance result does not hold.

COROLLARY 1. *Suppose $r(y_i, x_i)$ is strictly decreasing in $x_i$. Then*
i) *the full-information level of investment, $\sigma^*(n)$, decreases strictly with $n$ and $\lim_{n \to +\infty} \sigma^*(n) = 0$; and*
ii) *if $H(y_i, x_i)$ satisfies the CSO property, $EC^*(n)$ decreases strictly with $n$.*

As the number of firms increases, two opposing forces affect the expected minimum cost. On the one side, given each firm's investment in R&D, an increase in the number of firms increases the probability of finding a low minimum cost. On the other side, individual investment levels decrease with the number of firms. Corollary 1 identifies a class of distributions under which the first effect outweighs the second.[7]

## 6. INVESTING PRIOR TO CONTRACTING

In the previous section, we have assumed that firms observe a procurement mechanism prior to investing. In many cases of procurement, however, firms may anticipate the buyer's decisions and invest in R&D before the buyer commits to a procurement mechanism. This allows firms to earn information rents since they possess private information when signing a procurement contract. In this section, we analyze how these potential information rents affect the buyer' choice of procurement mechanisms and the firms' incentives to invest. In particular, we provide conditions for unique equilibrium and show that as the number of firms increase it is optimal for the buyer to offer a high-powered incentive contract.

6.1. *Equilibrium.* Since the investment outcomes cannot be observed by the buyer, this situation can be described as a game in which the buyer and firms choose procurement mechanisms and investment amounts simultaneously. At the time the contract is accepted, firms have private information about their production costs. We assume that the final cost of production is ex post observable and that the buyer can offer contracts conditional on the realization of the final costs. By the Revelation Principle, we restrict the choices of the buyer to Bayesian incentive compatible (IC) allocations,[8] $[q_i(y), p_i(y), C_i(y)]$, $i = 1, \ldots, n$, where, conditional on $y$, $q_i(y)$ is the probability that firm $i$ is awarded production, $p_i(y)$ is the expected transfer to firm $i$, and $C_i(y)$ is the target level of final production costs.

[7] Notice that, for a general distribution, if $\lim_{x_i \to 0} H_{x_i}(y_i, x_i) = +\infty$ for all $y_i \in (\underline{a}, \bar{a})$, then the expected minimum cost always approaches the lowest bound $\underline{a} - e^*$ as $n \to \infty$.

[8] The issue of unique implementation of such allocations will be discussed later in this section.

To characterize the equilibria of the game, we first examine the buyer's best replies given an arbitrary $n$-tuple of investment levels by the firms. The following lemma is a simple extension of Theorem 1 in Laffont and Tirole (1987) to the case of asymmetric distributions.

LEMMA 2. *Suppose* $H(y_i, x_i)/h(y_i, x_i)$ *is strictly increasing in* $y_i$ *and* $\psi'''(e) \geq 0$. *Then for any* $x$, *the best reply IC allocation* $[\bar{q}_i(y), \bar{p}_i(y), \overline{C}_i(y)]$ *for the buyer is determined by*

$$\overline{C}_i(y) = y_i - g_1(H(y_i, x_i)/h(y_i, x_i)) \tag{5}$$

*where* $e = g_1(z)$ *solves* $\psi'(e) + z\psi''(e) = 1$;

$$\bar{q}_i(y) = \begin{cases} 1, & \text{if } J(y_i, x_i) < \min_{j \neq i} J(y_j, x_j) \text{ and } J(y_i, x_i) < V, \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

*where* $J(y_i, x_i) = y_i + g_2(H(y_i, x_i)/h(y_i, x_i))$ *is the adjusted cost of production and* $g_2(z) = -g_1(z) + \psi(g_1(z)) + z\psi'(g_1(z))$; *and*

$$E_{y_{-i}}\Big\{\bar{p}_i(y) - \big[\overline{C}_i(y) + \psi\big(y_i - \overline{C}_i(y)\big)\big]\bar{q}_i(y) + \int_{y_i}^{\bar{a}} \psi'\big(t - \overline{C}_i(t, y_{-i})\big)\bar{q}_i(t, y_{-i})\,dt\Big\} = 0. \tag{7}$$

Lemma 2 shows that, for each $x$, the best reply allocation for the buyer is completely determined by the inverse hazard rate, $H(y_i, x_i)/h(y_i, x_i)$. In particular, the best reply of effort depends only on the firm's own hazard rate. As the inverse hazard rate goes to zero, the optimal level of effort converges to the full-information level. Furthermore, the procurement contract is awarded to the firm with the lowest level of the adjusted costs provided that this cost is also below $V$. If the firms invest identical amounts, then the contract is awarded to the firm with the lowest level of the true production costs.

The following proposition provides sufficient conditions for the existence and uniqueness of the equilibrium. One of the conditions states that the inverse hazard rate strictly increases with the level of investment. It implies that, as firms invest more in R&D, the buyer is more uncertain about production costs. It is straightforward to verify that this condition is satisfied when the CSO property holds.

PROPOSITION 5. *Suppose that* (A1)–(A3) *hold,* $\psi'''(e) \geq 0$, *and* $H(y_i, x_i)/h(y_i, x_i)$ *is strictly increasing in both* $y_i$ *and* $x_i$. *If* $r(y_i, x_i)$ *is nonincreasing in* $x_i$ *then there exists a unique equilibrium in which all firms invest identical amounts.*

REMARK. Proposition 5 assumes that the buyer chooses a direct revelation mechanism as a best response. If the effort choice is absent and if the buyer is restricted to choose among particular auction mechanisms such as first-price or second-price auctions, decreasing returns to scale still imply that there exists a unique equilibrium level of investment. By suitably modifying the proofs of Propositions 3 and 4, one can easily show that, for any given reserve price, if all firms choose best replies, then investment levels are identical across the firms. Thus, in equilibrium, since the firms have symmetric distributions, the buyer implements her best reply allocation uniquely. Uniqueness of the investment level then follows by Proposition 5.

6.2. *Properties of the Equilibrium.* We now discuss some comparative-static properties of the equilibrium under the assumptions in Proposition 5. Let $\hat{\sigma}$ be the level of investment of each firm in the equilibrium and $[\hat{q}_i(y), \hat{p}_i(y), \hat{C}_i(y)]$ be the equilibrium allocation. Notice that $\hat{C}_i(y)$ is independent of $i$ and $y_{-i}$, and is denoted by $\hat{C}(y_i)$. Also, let $\hat{e}(y_i) = y_i - \hat{C}(y_i)$ be the effort level. The first-order condition with respect to investment yields

$$\int_{\underline{a}}^{\hat{R}} \psi'(t - \hat{C}(t)) r(t, \hat{\sigma})[1 - H(t, \hat{\sigma})]^n \, dt - c = 0, \tag{8}$$

where the equilibrium reserve price, $\hat{R}$, is equal to $\mathrm{Min}\{\bar{a}, J^{-1}(V, \hat{\sigma})\}$. $\hat{C}(y_i)$ and $\hat{\sigma}$ are simultaneously determined by (5) and (8).

The following corollaries illustrate some features of the symmetric equilibrium. Recall that $\sigma^*$ and $e^*$ are the full-information levels of investment and effort, respectively. Inspecting (1), (4), (5), and (8), we obtain

COROLLARY 2. *Suppose that the assumptions in Proposition 5 hold. Then $\hat{\sigma} \leq \sigma^*$ and $\hat{e}(y_i) \leq e^*$ for any $y_i \in [\underline{a}, \bar{a}]$.*

Corollary 2 is a standard result in adverse selection models. It shows that the firms underinvest relatively to the full-information solution. A similar result is obtained by Dasgupta (1990) and Lewis and Sappington (1993). Since firms invest before the buyer commits to a procurement contract, they possess private information when accepting the contract. Due to incentive compatibility restrictions, the winning firm earns positive information rents which decrease with the target level of final costs. To mitigate this effect, the buyer chooses a higher target level as compared to the full-information solution. Consequently, the incentive structure provides the firms with less incentives to invest in R&D.

COROLLARY 3. *Suppose that all the assumptions in Proposition 5 hold. Then*
i) *$\hat{\sigma}$ decreases with $n$ and $\lim_{n \to +\infty} \hat{\sigma} = 0$;*
ii) *$\hat{R}$ increases with $n$ and $\lim_{n \to +\infty} \hat{R} = \bar{a}$; and*
iii) *$\hat{e}(y_i)$ increases with $n$ and $\lim_{n \to =\infty} \hat{e}(y_i) = e^*$ for any $y_i \in [\underline{a}, \bar{a}]$.*

Corollary 3 shows that the equilibrium effort schedule varies with the number of competing firms. This result is in contrast with the *separation property* in Laffont and Tirole (1987), McAfee and McMillan (1987), Riordan and Sappington (1987), and Dasgupta and Spulber (1990), where the winner's effort is independent of the number of competing firms.[9] In our model, the inverse hazard rates depend on the investment levels. As the number of firms becomes larger, the individual level of investment decreases. If the inverse hazard rate increases with the level of investment, then the equilibrium effort level increases and the adjusted cost decreases. In the limit, the full-information level of production cost is achieved.

The failure of the separation property can be further illustrated using linear contracts. Similar to Laffont and Tirole (1987), the buyer can implement *uniquely* the best reply allocation given a vector of investment levels by offering an auction in which the firms submit their unit costs. Firm $i$ is awarded the contract if its reported adjusted cost, $J(\tilde{y}_i, x_i)$, is the lowest among all firms provided that $J(\tilde{y}_i, x_i)$ is smaller than $V$, and is paid according to the payment schedule

$$P_i(\tilde{y};C) = T_i(\tilde{y}) + \beta_i(\tilde{y})\left[C - \overline{C}_i(\tilde{y})\right] \tag{9}$$

where $\tilde{y}$ is the vector of reported unit costs, $C$ is the realization of final production cost, $\beta(\tilde{y}) = 1 - \psi'(\tilde{y}_i - \overline{C}_i(\tilde{y}))$,

$$T_i(\tilde{y}) = \overline{C}_i(\tilde{y}) + \psi\left(\tilde{y}_i - \overline{C}_i(\tilde{y})\right) + \int_{\tilde{y}_i}^{z_i(\tilde{y}_{-i})} \psi'\left(t - \overline{C}_i(t, \tilde{y}_{-i})\right) dt,$$

and $z_i(\tilde{y}_{-i})$ is the highest level of firm $i$'s reported unit cost for which firm $i$ is awarded the contract.

By simple adaptation of the proof in Laffont and Tirole (1987, Appendix D), one can show that reporting the true cost is the unique equilibrium. Therefore, the above auction scheme uniquely implements the buyer's best reply allocation.

Since in equilibrium firms choose identical levels of investment, the contract is awarded to the firm with the lowest unit cost and the equilibrium transfer becomes

$$P_i(y_i, z(y); C) = \hat{T}(y_i, z(y)) + \beta(y_i)\left[C - \hat{C}(y_i)\right] \tag{10}$$

where $z(y)$ is the second lowest unit cost, $\hat{C}(y_i) = y_i - g_1(H(y_i, \hat{\sigma}))/h(y_i, \hat{\sigma}))$, $\beta(y_i) = 1 - \psi'(y_i - \hat{C}(y_i))$, and

$$\hat{T}(y_i, z(y)) = \hat{C}(y_i) + \psi\left(y_i - \hat{C}(y_i)\right) + \int_{y_i}^{\mathrm{Min}\{\hat{R}, z(y)\}} \psi'\left(t - \hat{C}(t)\right) dt,$$

if $y_i < \mathrm{Min}\{\hat{R}, z(y)\}$ and is equal to zero otherwise. Notice that $\hat{T}(y_i, z(y))$ is the same for all firms and depends on the lowest and second-lowest levels of costs. It increases with the winner's bid $y_i$ and the second lowest bid $z(y)$.

The target level of production cost, $\hat{C}$, and the buyer's share of cost overrun, $\beta$, depend only upon the lowest bid in the auction. Because of information asymmetry, the equilibrium $\beta$ is between 0 and 1. A low value of $\beta$ induces high-powered incentives for the winner to exert effort in production.

[9] For further discussions on the separation property, see Laffont and Tirole (1993, Chapter 7).

If information asymmetry is exogenously given as in Laffont and Tirole (1987) and McAfee and McMillan (1987), then the buyer's optimal share $\beta$ and the target cost $\hat{C}$ are independent of the number of competing bidders. In our model, however, private information is endogenously determined. As the number of firms increases, each firm's level of investment falls. Hence, $\beta$ and $\hat{C}$ decrease since the inverse hazard rate increases with the level of investment. Therefore, as the degree of competition increases, the buyer offers a high-powered incentive contract with a low level of the target cost.[10] This property provides a testable hypothesis.

Finally, we identify sufficient conditions under which the pace of innovation and the buyer's equilibrium welfare increase as the number of firms increases. Let $E\hat{C}(n)$ be the expected cost of production and $E\hat{W}(n)$ the buyer's welfare in the symmetric equilibrium.

PROPOSITION 6. *Suppose $H(y_i, x_i)/h(y_i, x_i)$ is strictly increasing in $y_i$, $r(y_i, x_i)$ is nonincreasing in $x_i$, and the CSO property holds. Then,*

i) *$E\hat{C}(n)$ strictly decreases with $n$; and*

ii) *$E\hat{W}$ strictly increases with $n$.*

## 7. DISCUSSION

The results in Sections 5 and 6 depend on the assumption of decreasing returns to scale. In this section, we provide two examples which illustrate some of the consequences of relaxing this assumption.

Suppose that the technology exhibits increasing returns to scale. Let $n = 2$, $c = 1/38$, and $H(y_i, x_i) = 1 - [1 - y_i]^{\alpha(x_i)}$ for $y_i \in [0,1]$, where $\alpha(x_i) = x_i^2/8 + x_i$ for $x_i \geq 0$. The full-information levels of investment are $(x_1^*, x_2^*) = (0, 5.07)$ or $(5.07, 0)$. However, when a first-price or second-price auction is used, there exists a symmetric equilibrium in which each firm invests $x_i = 2.61$. Thus, unique implementation is not achieved. Of course, this problem can be avoided using a random draw to select a single supplier. The selected firm is then awarded a take-it-or-leave-it fixed-price contract. Since the contract is signed when the buyer and the firm have symmetric information, the supplier behaves as a residual claimant and chooses the full-information levels of investment and effort.

Auction mechanisms or a random draw may fail to implement uniquely the full-information solution when the project failure rate is not monotonic in the level of investment. Consider the case in which the technology exhibits increasing returns to scale for an initial range of investment levels and decreasing returns eventually.[11]

[10] Risk aversion is another factor that affects the incentive contract. McAfee and McMillan (1986) have considered a procurement model in which firms are risk averse and information asymmetry is exogenous. Assuming linearity of the contract and additive separability of the cost function with exponential distribution, they show that the greater competition in bidding the smaller is the cost-sharing parameter.

[11] Loury (1979) considers this type of R&D technology.

Suppose $n = 2$, $c = 1/38$, and $H(y_i, x_i) = 1 - [1 - y_i]^{\alpha(x_i)}$ for $y_i \in [0, 1]$, where

$$\alpha(x_i) = \begin{cases} x_i^2/8 + x_i, & \text{if } 0 \le x_i \le 3, \\ -7x_i^2/8 + 7x_i - 9, & \text{if } 3 < x_i \le 4, \\ 5, & \text{if } 4 < x_i. \end{cases}$$

This technology exhibits increasing returns to scale for $0 \le x_i \le 3$ and decreasing returns for $x_i > 3$. The full-information levels of investment are $(x_1^*, x_2^*) = (1.48, 3.22)$ which yield an expected minimum cost of production equal to 0.14. If only one firm is selected at outset, it will invest $x_i = 3.5$. The expected minimum cost of production is 0.17 and hence under-investment occurs. If a second-price auction is used, there exists an equilibrium in which each firm invests the full-information level. However, as in the previous example, both a first-price and second-price auction admit a symmetric equilibrium in which each firm invests $\sigma = 2.61$. The expected minimum cost of production is equal to 0.13, which is lower than the expected minimum cost in the full-information solution. Over-investment occurs in this case. Notice also that a first-price auction cannot implement the full-information solution. Since the hazard rate $I(y_i, x_i)$ is increasing in both $y_i$ and $x_i$, the more a firm invests, the higher it bids. If the investment levels are asymmetric across the firms, the equilibrium bidding strategies are asymmetric as well. This implies that the auction outcome is not efficient since the firm with the lowest production cost may not be awarded a contract.

The problem of existence of a symmetric equilibrium for both a first-price and second-price auction when the full-information levels of investment are asymmetric is not specific to the above example. This can be illustrated for the case of two firms. Suppose that the full-information investment levels are such that $0 < x_1^* < x_2^*$. The first-order conditions imply

$$\int_{\underline{a}}^{\overline{a}} H_{x_1}(t, x_1^*)[1 - H(t, x_2^*)]\, dt = c = \int_{\underline{a}}^{\overline{a}} H_{x_2}(t, x_2^*)[1 - H(t, x_1^*)]\, dt.$$

If both firms invest $x_2^*$, then the marginal benefit is less than the marginal cost. Similarly, if both firms invest $x_1^*$, then the marginal benefit is higher than the marginal cost. Thus, there is a level of investment $\sigma$ such that $x_1^* < \sigma < x_2^*$ and

$$\int_{\underline{a}}^{\overline{a}} H_{x_1}(t, \sigma)[1 - H(t, \sigma)]\, dt = c = \int_{\underline{a}}^{\overline{a}} H_{x_2}(t, \sigma)[1 - H(t, \sigma)]\, dt.$$

The above equations are the first-order conditions for a symmetric equilibrium in a first-price and second-price auction. If the second-order conditions are satisfied, a symmetric equilibrium is obtained.

If the buyer's choice of a procurement mechanism is simultaneous with the investment decisions by the suppliers, the assumption of decreasing returns to scale guarantees that there exists a unique equilibrium in which all firms invest identical amounts and under-invest relatively to the full-information solution. If the assumption of decreasing returns to scale is violated, the issue of under- or over-investment is ambiguous. Consider the second example in this section and, for simplicity, assume that the firms cannot exert effort to further reduce cost during production. Also, suppose that $V = 2$. In this case, there exists an equilibrium in which both firms invest $\hat{\sigma} = 2.57$ and the buyer awards the contract to the firm with the lowest cost provided that this is smaller than the reserve price $R = 0.51$. The buyer's equilibrium mechanism can be implemented by a first-price or second-price sealed-bid auction with the same reserve price. The expected minimum cost is equal to 0.13, which is lower than the expected minimum cost at the full-information solution. Thus, over-investment relative to the full-information solution occurs.[12]

## 8. CONCLUSIONS

In this paper, we analyze a procurement model with R&D investment. We show that if the buyer announces a contract prior to investment and the R&D technology exhibits decreasing returns to scale, the full-information solution can be uniquely implemented via standard auction mechanisms. We discuss the importance of the assumption of decreasing returns for unique implementation. Another important assumption is the absence of interim or ex post participation constraints: suppliers cannot reject the contract if a high cost of production is realized. Laffont and Tirole (1993, Chapter 1) show that if these constraints are imposed, the full-information solution cannot be implemented.

We also analyze the case in which suppliers invest before the buyer announces a procurement mechanism. We provide conditions for the existence of a unique equilibrium in which the firms underinvest relatively to the full-information solution and discuss the validity of this result under different conditions. We show that in the symmetric equilibrium the buyer offers a high-powered incentive contract as the number of potential suppliers increases.

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## APPENDIX

PROOF OF LEMMA 1. For any $t \in (\underline{a}, \bar{a})$ and $M > 0$, let $\tilde{H} = 1 - [1 - H(t, M/k)]^k$. Taking the derivative of $\tilde{H}$ with respect to $k$ yields

$$d\tilde{H}/dk = [1 - H(t, M/k)]^k \{\log[1 - H(t, M/k)] + r(t, M/k)M/k\}.$$

Let $x_i = M/k$ and denote $-\log[1 - H(t, x_i)]$ by $\delta(x_i)$. It follows that $d\delta(x_i)/dx_i = r(t, x_i)$, and that, by (A2), $\delta(0) = 0$. Suppose that $r(t, x_i)$ is decreasing in $x_i$. Then $\delta(x_i)$ is concave in $x_i$. Thus, $\delta(x_i) - \delta(0) \leq x_i \, d\delta(x_i)/dx_i$, which implies $d\tilde{H}/dk \geq 0$. Claim (i) follows. The proof of claim (ii) is analogous. Q.E.D.

[12] Dasgupta (1990) assumes that both the full-information and the equilibrium levels of investment are symmetric. In Propositions 2 and 5, we provide sufficient conditions for this assumption to be valid.

PROOF OF PROPOSITION 1. Sufficiency follows from simple calculations. To prove necessity, we first show that the CSO property is satisfied for any positive real numbers $n$ and $n'$. Suppose not. Then there exist $(\sigma, n), (\sigma', n') \in [0, \infty) \times R^1_+$, and $t, t' \in [\underline{a}, \bar{a}]$ such that

$$n \log[1 - H(t, \sigma)] > n' \log[1 - H(t, \sigma')]$$

$$n \log[1 - H(t', \sigma)] < n' \log[1 - H(t', \sigma')].$$

As $n' \neq 0$, there exists a rational number $r = n/n'$ such that

$$r \log[1 - H(t, \sigma)] > \log[1 - H(t, \sigma')]$$

$$r \log[1 - H(t', \sigma)] < \log[1 - H(t', \sigma')].$$

Since a rational number is the ratio of two integers, the above equations contradict with the definition of the CSO property. The claim follows.

Fix $\sigma_0 \in (0, \infty)$. Let $\sigma$ and $n$ satisfy

$$n \log[1 - H(t, \sigma)] = \log[1 - H(t, \sigma_0)]$$

for some $t \in (\underline{a}, \bar{a})$. Then since $H(t, \sigma)$ is continuous and strictly increasing in $\sigma$, it follows from CSO that the above equality holds for all $t \in [\underline{a}, \bar{a})$. Define a function of $\sigma$ as

$$n(\sigma) = \frac{\log[1 - H(t, \sigma_0)]}{\log[1 - H(t, \sigma)]}$$

for all $\sigma \in [0, \infty)$. Clearly, $n(\sigma)$ is independent of $t$. The claim then follows by setting $F(t) = H(t, \sigma_0)$ and $\alpha(\sigma) = 1/n(\sigma)$. Q.E.D.

PROOF OF PROPOSITION 2. First notice that (A1) and (A3) imply that a solution exists and $x_j^* > 0$ for some $j$. To prove part (iiia), suppose that $0 \leq x_i^* < x_j^*$ for some $i \neq j$ and consider a vector of differentials $dx = (dx_1, \ldots, dx_n)$ such that $dx_k = 0$ for $k \neq i, j$, $dx_i + dx_j = 0$, and $dx_i = 0$. Then

$$dEW = \int_{\underline{a}}^{\bar{a}} \left[1 - \tilde{H}(t, x)\right] \left[r(t, x_i^*)\, dx_i + r(t, x_j^*)\, dx_j\right] dt.$$

Since $r(y_i, x_i)$ is strictly decreasing in $x_i$ for any $y_i \in (\underline{a}, \bar{a})$ and $0 \leq x_i^* < x_j^*$, letting $x = x^* + dx$ yields $dEW > 0$. Thus, the optimal investment must be symmetric.

Let $x = (\sigma, \ldots, \sigma)$. We can rewrite the buyer's expected surplus as

$$EW = V - \bar{a} + e^* - \psi(e^*) + \int_{\underline{a}}^{\bar{a}} \left(1 - [1 - H(t, \sigma)]^n\right) dt - cn\sigma.$$

Since $EW$ is strictly concave in $\sigma$, the optimal investment must be unique.

The proofs of parts (iiib) and (iiic) are analogous and are omitted. Q.E.D.

PROOF OF PROPOSITION 3. Let $(x_i, B_i(y_i), e_i(y_i))$ be firm $i$'s strategy when a second-price sealed-bid auction is used. It is well known that a dominant strategy for firm $i$ is to submit its true production cost, that is, $B_i(y_i) = y_i - e_i(y_i) + \psi(e_i(y_i))$. The profit for firm $i$ of type $y_i$ from bidding is

$$\pi_i(y_i, e_i) = E_{y_{-i}}\{\text{Min}_{j \neq i} B_j(y_j) - y_i + e_i - \psi(e_i) | y_i - e_i + \psi(e_i) < B_j(y_j)\ \forall j \neq i\},$$

which is increasing with $e_i - \psi(e_i)$ and independent of $x_i$. Thus, it is optimal for firm $i$ to choose the full-information level of effort, $e^*$. The ex ante expected profit of firm $i$ is then given by

$$E\Pi_i(x) = E_{y_i}\pi_i(y_i, e^*) - cx_i.$$

The first-order condition with respect to $x_i$ yields:

$$\int_{\underline{a}}^{\bar{a}} \pi_i(y_i, e^*)\, dH_{x_i}(y_i, x_i) - c = 0.$$

In the equilibrium, all the firms submit their true costs of production and choose the full-information level of effort. It follows that

$$\begin{aligned}\pi_i(y_i, e^*) &= E_{y_{-i}}\{\text{Min}_{j \neq i}\, y_j - y_i | y_i < y_j\ \forall j \neq i\} \\ &= -\int_{y_i}^{\bar{a}} (t - y_i)\, d\prod_{j \neq i}\left[1 - H(t, x_j)\right]\end{aligned}$$

It follows from integration by parts that the first-order condition becomes

$$\int_{\underline{a}}^{\bar{a}} r(t, x_i) \prod_{j=1}^{n} \left[1 - H(t, x_j)\right] dt - c = 0. \tag{11}$$

Since $r(t, x_i)$ is strictly decreasing in $x_i$, there exists a unique equilibrium, which is the full-information solution. Q.E.D.

PROOF OF PROPOSITION 4. Let $(x_i, B_i(y_i), e_i(y_i))$ denote firm $i$'s equilibrium strategy in a first-price sealed-bid auction game. Lebrun (1994a, b) establishes that (i) $B_i(\underline{a}) = B_j(\underline{a})$ and $B_i(\bar{a}) = B_j(\bar{a})$ for all $i, j = 1, \ldots, n$; and (ii) $B_i(y_i)$ is continuous, strictly increasing in $y_i$, and its inverse function is differentiable over $(B_i(\underline{a}), B_i(\bar{a}))$. Moreover, he shows that if $I(y_i, x_i)$ is strictly increasing in $x_i$ for every $i$, then $x_i < x_j$ implies $B_i(t) < B_j(t)$ for all $t \in (\underline{a}, \bar{a})$. This last result is crucial for this Proposition and we provide its proof for convenience.

Let $\phi_i(b) = B_i^{-1}(b)$. If firm $i$ observes $y_i$, submits a bid $b$, and exerts an effort level $e$, its expected profits can be written as

$$\pi_i(b, e, y_i) = (b - y_i + e - \psi(e)) \prod_{k \neq i} \left[1 - H(\phi_k(b), x_k)\right]$$

Clearly, it is optimal for $i$ to choose the full-information level of effort, $e^*$. The first-order conditions with respect to $b$ yield the following system of differential equations

$$\sum_{k \neq i} I(\phi_k(b), x_k)\phi_k'(b) = \frac{1}{b - \phi_i(b) + e^* - \psi(e^*)},$$

for $i = 1, \ldots, n$. Combining the first-order conditions for firms $i$ and $j$ yields

$$(12) \qquad I(\phi_i(b), x_i)\phi_i'(b) - I(\phi_j(b), x_j)\phi_j'(b)$$
$$= \frac{1}{b - \phi_j(b) + e^* - \psi(e^*)} - \frac{1}{b - \phi_i(b) + e^* - \psi(e^*)}.$$

Suppose that $x_i < x_j$. We show that $B_i(t) < B_j(t)$ for all $t \in (\underline{a}, \bar{a})$. Suppose that $B_i(t) > B_j(t)$ for some $t \in (\underline{a}, \bar{a})$. Then there exists an interval $(\gamma, \delta)$ such that $\phi_i(\gamma) = \phi_j(\gamma)$ and $\phi_i(b) < \phi_j(b)$ for $b \in (\gamma, \delta)$. It then follows from (12) that $I(\phi_i(b), x_i)\phi_i'(b) > I(\phi_j(b), x_j)\phi_j'(b)$ for $b \in (\gamma, \delta)$. Since $I(y_i, x_i)$ is strictly increasing in $x_i$ and $\phi_i(\gamma) = \phi_j(\gamma)$, it follows that there exists $\delta' > \gamma$ such that $\phi_i'(b) > \phi_j'(b)$ for $b \in (\gamma, \delta')$. Since $\phi_k(b) = \int_\gamma^b \phi_k'(t)\,dt + \phi_k(\gamma)$, $k = i, j$, a contradiction is obtained. Thus, $B_i(t) \leq B_j(t)$ for $t \in (\underline{a}, \bar{a})$. Suppose now $B_i(t) = B_j(t) = b$ for some $t$. It follows from (12) that $\phi_i'(b) > \phi'(b)$. Then there exists $t_0$ such that $B_i(t_0) > B_j(t_0)$. A contradiction.

We now show that $x_i = x_j$ for all $i, j$. It follows from the envelope theorem and integration by parts that the ex ante expected profits for firm $i$ can be written as

$$E\Pi_i(x) = \int_{\underline{a}}^{\bar{a}} \prod_{k \neq i} [1 - H(\phi_k(B_i(t)), x_k)] H(t, x_i)\,dt - cx_i.$$

The first-order condition with respect to $x_i$ implies

$$\int_{\underline{a}}^{\bar{a}} r(t, x_i) \prod_{k=1}^{n} [1 - H(\phi_k(B_i(t)), x_k)]\,dt - c = 0.$$

A similar equation follows from firm $j$'s first-order condition. Suppose that $r(t, x_i)$ is nonincreasing in $x_i$. Since $x_i < x_j$, $B_i(t) < B_j(t)$, and $\prod_{k=1}^n [1 - H(\phi_k(b)), x_k)]$ is strictly decreasing in $b$, a contradiction is obtained. The claim follows.

Finally, we establish that the full-information solution is the unique symmetric equilibrium. Given a symmetric level of investment, $\sigma$, there exists symmetric equilibrium bidding strategies by standard arguments. Since the optimal bidding strategy of firm $i$ is independent of $x_i$, $\sigma$ solves

$$\int_{\underline{a}}^{\bar{a}} r(t, \sigma)[1 - H(t, \sigma)]^n\,dt - c = 0.$$

Since $r(y_i, x_i)$ is nonincreasing in $x_i$, the full-information investment, $\sigma^*$, is the only symmetric solution. Uniqueness of the equilibrium bidding strategies follows from Maskin and Riley (1986). Q.E.D.

PROOF OF COROLLARY 1. Part (i) follows directly from (4). To prove (ii), notice that by Proposition 1 one can rewrite (4) as

$$-\alpha'(\sigma^*)\int_{\underline{a}}^{\bar{a}}[1-F(t)]^{n\alpha(\sigma^*)}\,dt-c=0$$

and $EC^*(n)$ as

$$EC^*(n)=e^*-\underline{a}+\int_{\underline{a}}^{\bar{a}}[1-F(t)]^{n\alpha(\sigma^*)}\,dt.$$

Since $r(y_i, x_i)$ is strictly decreasing in $x_i$, $\alpha(x_i)$ is strictly concave. It follows from (i) that $\alpha'(\sigma^*)$ strictly increases with $n$. Thus, $n\alpha(\sigma^*)$ increases strictly with $n$ and the claim follows. Q.E.D.

PROOF OF LEMMA 2. Given any allocation $[p_i(y), q_i(y), C_i(y)]$, the expected profit for firm $i$ of type $y_i$, when it submits $\tilde{y}_i$ and other firms submit their true types $y_{-i}$, is given by

$$\Pi_i(y_i,\tilde{y}_i)=E_{y_{-i}}\{p_i(\tilde{y}_i,y_{-i})-[C_i(\tilde{y}_i,y_{-i})+\psi(y_i-C_i(\tilde{y}_i,y_{-i}))]q_i(\tilde{y}_i,y_{-i})\}.$$

Bayesian incentive compatibility requires that, for all $i$ and $y_i, \tilde{y}_i \in [\underline{a}, \bar{a}]$,

$$\Pi_i(y_i,y_i)\geq\Pi_i(y_i,\tilde{y}_i).$$

Interim participation constraint requires $\Pi_i(y_i, y_i) \geq 0$ for all $y_i, \tilde{y}_i \in [\underline{a}, \bar{a}]$. Applying the Envelope Theorem, we obtain

$$d\Pi_i(y_i,y_i)/dy_i=-E_{y_{-i}}\psi'(y_i-C_i(y))q_i(y).$$

Thus, the participation constraint is reduced to $\Pi_i(\bar{a}, \bar{a}) \geq 0$. Without loss of generality, we assume $\Pi_i(\bar{a}, \bar{a}) = 0$. Integrating the above differential equation yields

$$\Pi_i(y_i,y_i)=\int_{y_i}^{\bar{a}}E_{y_{-i}}\psi'(t-C_i(t,y_{-i}))q_i(t,y_{-i})\,dt.$$

Using integration by parts, we can write the expected payment as

$$E_y p_i(y)=E_y\{[C_i(y)+\psi(y_i-C_i(y))+\psi'(y_i-C_i(y))H(y_i,x_i)/h(y_i,x_i)]q_i(y)\}.$$

The expected surplus for the buyer can be written as

$$EW=E_y\{[V-C_i(y)-\psi(y_i-C_i(y))-\psi'(y_i-C_i(y))H(y_i,x_i)/h(y_i,x_i)]q_i(y)\}.$$

The buyer then chooses $C_i(y)$ and $q_i(y)$ to maximize $EW$. The first-order condition with respect to $C_i(y)$ yields

$$-1+\psi'(y_i-C_i(y))+\psi''(y_i-C_i(y))H(y_i,x_i)/h(y_i,x_i)=0.$$

Let $e=g_1(z)$ solves $\psi'(e)+\psi''(e)z=1$. Then (5) follows. Given the definition of $J(y_i,x_i)$ in the Lemma, the optimal $q_i(y)$ is determined by (6), and (7) follows from the definition of $\Pi_i(y_i,y_i)$. A sufficient condition for incentive compatibility is that, for all $i$ and $y_i$, $E_{y_{-i}}\psi'(y_i-\overline{C}_i(\tilde{y}_i,y_{-i}))\overline{q}_i(\tilde{y}_i,y_{-i})$ decreases with $\tilde{y}_i$, which can be easily verified. Q.E.D.

PROOF OF PROPOSITION 5. Given any incentive-compatible allocation chosen by the buyer, firm $i$'s best reply is obtained by maximizing its expected profit

$$E\Pi_i=E_y\{p_i(y)-[C_i(y)+\psi(y_i-C_i(y))]q_i(y)\}-cx_i.$$

Taking the derivative of $E\Pi_i$ with respect to $x_i$ and evaluating it at the best reply mechanism by the buyer characterized in Lemma 2, we obtain the equilibrium conditions

$$\int_{\underline{a}}^{R(x_i)}\psi'(g_1(H(t,x_i)/h(t,x_i)))r(t,x_i)\phi(u_i,x)\,dt-c=0, \tag{13}$$

$i=1,\dots,n$, where $R(x_i)=\text{Min}\{\bar{a},J^{-1}(V,x_i)\}$, $\phi(u_i,x)=\prod_{j=1}^{n}[1-H(J^{-1}(u_i,x_j),x_j)]$, and $u_i=J(t,x_i)$. By setting $x_i=\sigma$ for $i=1,\dots,n$, the first-order conditions yield

$$\int_{\underline{a}}^{R(\sigma)}\psi'(g_1(H(t,\sigma)/h(t,\sigma)))r(t,\sigma)[1-H(t,\sigma)]^n\,dt-c=0.$$

Since the left-hand side is strictly decreasing in $\sigma$ and (A3) holds, it follows that there exists a unique symmetric equilibrium level of investment, $\sigma>0$. The buyer's equilibrium contract is then determined by Lemma 2.

Suppose that there exists an equilibrium in which $x_i<x_j$. It follows that: (i) $R(x_i)\geq R(x_j)$ since $J(y_i,x_i)$ is increasing in both $y_i$ and $x_i$; (ii)

$$\psi'(g_1(H(t,x_i)/h(t,x_i)))>\psi'\big(g_1\big(H(t,x_j)/h(t,x_j)\big)\big)$$

since the hazard rate is increasing in $x_i$; (iii) $r(t,x_i)\geq r(t,x_j)$; and (iv) $\phi(J(t,x_i),x)>\phi(J(t,x_j),x)$ since $\phi(u,x)$ is strictly decreasing in $u$. It follows that (13) cannot hold for both $i$ and $j$. Uniqueness of the equilibrium follows. Q.E.D.

PROOF OF COROLLARY 3. Notice that, by the assumptions, $\overline{C}(y_i)$ in (5) increases with $\sigma$ and $J^{-1}(V,\sigma)$ decreases with $\sigma$. Then the left-hand side of (8) decreases with $\sigma$ and $n$. Thus, $\hat{\sigma}$ decreases with $n$. Clearly, $\lim_{\to+\infty}\hat{\sigma}=0$. Parts ii) and iii) follow immediately. Q.E.D.

PROOF OF PROPOSITION 6. Given the distribution function in Proposition 1, the first-order condition (8) can be written as

$$-\int_{\underline{a}}^{\bar{a}} \psi'(\hat{e}(t))[1-F(t)]^{z_n} \log[1-F(t)]\, dt = c/\alpha'(\hat{\sigma}),$$

where $z_n = n\alpha(\hat{\sigma})$. By Corollary 3, $\hat{\sigma}$ decreases with $n$ and $\psi'(\hat{e}(t))$ increases with $n$. The concavity of $\alpha(\sigma)$ then implies the right-hand side of the above equation is nonincreasing in $n$. Thus, $z_n$ must increase with $n$. Part (i) then follows from the definition of $E\hat{C}(n)$.

To prove part (ii), we write the buyer's expected payoff as

$$E\hat{W} = \int_{\underline{a}}^{\bar{a}} [V - J(t,\hat{\sigma})]\, dH_0(t;\hat{\sigma},n)$$

where $H_0(t;\hat{\sigma},n) = 1-[1-F(t)]^{z_n}$. By the envelope theorem,

$$\frac{dE\hat{W}}{dn} = -\frac{\partial\hat{\sigma}}{\partial n}\int_{\underline{a}}^{\bar{a}} \frac{\partial J}{\partial \sigma}\, dH_0 - \frac{\partial z_n}{\partial n}\int_{\underline{a}}^{\bar{a}} \frac{\partial J}{\partial t}[1-F(t)]^{z_n} \log[1-F(t)]\, dt.$$

Since $J(t,\sigma)$ is increasing in both $t$ and $\sigma$, $\sigma$ decreases with $n$, and $z_n$ increases with $n$, we obtain $dE\hat{W}/dn > 0$. Q.E.D.

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