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ORIGINAL PAPER

Coalition formation in the presence of continuing conflict

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Abstract This paper studies endogenous coalition formation in a rivalry environment where continuing conflict exists. A group of heterogeneous players compete for a prize with the probability of winning for a player depending on his strength as well as the distribution of strengths among his rivals. Players can pool their strengths together to increase their probabilities of winning as a group through coalition formation. The players in the winning coalition will compete further until one individual winner is left. We show that in any equilibrium there are only two coalitions in the initial stage of the contest. In the case of three players, the equilibrium often has a coalition of the two weaker players against the strongest. The equilibrium coalition structure with four players mainly takes one of the two forms: a coalition of the three weaker players against the strongest or a coalition of the weakest and strongest players against a coalition of the remaining two. Our findings imply that the rivalry with the possibility of coalition formation in our model exhibits a pattern of two-sidedness and a balance of power. We further study the impact of binding agreements by coalition members on equilibrium coalition structures. Our analysis sheds some light on problems of temporary cooperation among individuals who are rivals by nature.

Keywords Coalition formation · Rivalry · Continuing conflict · Binding agreements

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1 Introduction

There are numerous examples of real-world rivalry situations in which individual players form transitory coalitions (or alliances), often followed by a spiral of continuing conflict.[1] Allies in a coalition benefit from coalition formation, probably at the expense of other players, but may not be able to eliminate completely their rivalry within a coalition. In this paper, we study endogenous coalition formation among a group of *heterogeneous* players in a rivalry environment where both external effects of coalition formation and *continuing conflict* within a coalition are present. We characterize equilibrium coalition structures and show that the rivalry with the possibility of coalition formation in our setting exhibits a pattern of two-sidedness and a balance of power.

To analyze possible external effects generated from coalition formation and continuing conflict within coalitions, we adopt the following stylized contest model. A group of players with differential strengths (or certain endowments) compete for an indivisible prize and the probability of winning the prize for a player depends on his own strength and on the distribution of strengths among other players. When a subset of players form a coalition, they are able to increase their joint probability of winning the prize as a group by simply pooling their strengths together and generating synergy in the contest, where the extent of synergy is determined by a synergy function mapping strengths into effective fighting strengths. Coalition formation thus alters the number of effective players in the contest as well as the corresponding distribution of strengths. Moreover, the contest is of a sequential nature. If a coalition wins, the allies within the coalition are unable to commit to certain sharing rules and hence in turn compete among themselves to decide who should have the prize, leading to further conflicts until one individual winner is left. We model coalition formation in the presence of continuing conflict as a sequential-move game with multiple stages and determine subgame-perfect equilibrium outcomes of the game.

We have characterized equilibrium coalition structures in our setting which are shown to depend on the number of players, the distribution of players' strengths and the property of the synergy function. Specifically, our findings can be briefly summarized as follows. In our contest game with three players, there always exists a unique equilibrium coalition structure and, under mild restrictions on the synergy function, the equilibrium has a coalition of the two weaker players in an attempt to balance the power of the strongest. In the case of four players, an equilibrium coalition structure exists and mainly takes one of the following two forms: (i) a coalition of the three weaker players against the strongest or (ii) a coalition of the weakest and strongest players against a coalition of the remaining two. The latter coalition structure results when the players' strengths are relatively close to each other, while the former arises when their strengths exhibit a wide disparity. In our contest game with more than

[1] For some examples of temporary alliances and sequencing of conflict, see Hirshleifer (1988), Niou and Tan (1997), Esteban and Sakovics (2003), and Wagner (2004).

four players, we show that in any equilibrium there are only two coalitions competing against each other in the initial contest, resulting in a "bipolar" system. However, completely characterizing equilibrium sub-coalition structures of this game turns out to be a challenging task.

We further consider a setting in which any subset of players are able to make binding agreements and study how such agreements may change equilibrium coalition structures. We assume that allies in a coalition are able to avoid further conflict by agreeing to share the prize proportionally according to their strengths. Given this commitment, we show that when the synergy of forming a coalition is strong, the equilibrium coalition structure involves two coalitions of asymmetric sizes, where the sizes are determined by the extent of synergy in forming coalitions and the distribution of strengths. We illustrate in the case of three players that as compared with the presence of continuing conflict (or no commitment in a coalition), the commitment within a coalition to share the prize proportionally changes the players' incentives of forming coalitions and hence the equilibrium coalition structure.

The contemporary economics literature on alliance formation in the context of international politics has mainly focused on the collective action problem within an alliance. In their pioneering study of the budget-sharing problem within an international organization, Olson and Zeckhauser (1966) provide a model of one alliance in which each member of the alliance makes a contribution to collective defense as a collective good and external threats to the alliance are assumed to be constant or nonexistent.[2] Their analysis does not take into account two important factors that may affect alliance formation: externalities generated from alliance formation and continuing conflict among the allies within an alliance.

There is a large economics literature on conflict and rent-seeking behavior among several independent individuals (or states).[3] Early studies in this literature are mostly concerned with how a contestant chooses between productive use (or consumption) of his current resources and spending part of these to either defend his resources or acquire resources from others. In those studies, contestants are allowed to make a one-time investment decision so that conflict is resolved within one period. More recent literature starts to address coalition formation and continuing conflict. With the exception of Esteban and Sakovics (2003), however, this literature analyzes these two important issues separately.

In the following, we first discuss briefly the literature on coalition formation without continuing conflict. We begin with the two papers with general approaches to coalition formation. Bloch (1996) studies sequential formation of coalitions in a model where coalitional worths depend on the whole coalition structure and are distributed among

2 Further developments from this line of research can be found in a survey by Sandler (1993). In a recent paper, Niou and Tan (2005) develop a collective good model that endogenizes external threat, and study how inter-group competition and intra-group collective action interact. Alesina and Spolaore (2006) examine the size of countries and their defense spending, taking into consideration the extent of external threat. The issue of how the size of an alliance affects the collective action problem within an alliance has also been studied by Esteban and Ray (2001).

3 See, e.g., Hirshleifer (1988, 1991, 1995), Skaperdas (1992, 1996), Perez-Castrillo and Verdier (1992), Grossman and Kim (1995), Neary (1997a,b), Esteban and Ray (1999), and references therein. For an overview of the recent literature on the economic analysis of conflict, see Garfinkel and Skaperdas (2007).

coalition partners according to a fixed sharing rule. Ray and Vohra (1999) provide a theory of endogenous coalition structure where coalitional worths are endogenously distributed. In both papers, the authors explicitly model the formation of coalitions as a non-cooperative sequential bargaining process, characterize stationary subgame perfect equilibria in the spirit of Rubinstein (1982) bargaining game, and further develop procedures that generate the equilibrium coalition structure for a class of symmetric games. The main difference between our model and theirs is that, in our stylized model, a group of heterogeneous players compete sequentially and thus the total payoff of a winning coalition in any stage of the contest is distributed through subsequent contest stages, and so forth.[4]

Several authors recently apply the approach of Bloch (1996) and Ray and Vohra (1999) to study coalition formation in the context of conflict in which players invest in appropriation activities (see, for instance, Noh 2002; Bloch et al. 2006; Sanchez-Pages 2007a,b). Common features of these models include a specific contest success function to describe the nature of conflict and a fixed sharing rule among coalition members. For instance, in a symmetric rent-seeking model where the prize is equally shared among coalition members, Bloch et al. (2006) show that the grand coalition emerges as the unique subgame perfect equilibrium outcome. This finding depends crucially on two features of the model: Players in the grand coalition equally share the prize, and if a player deviates from the grand coalition then the coalition breaks up into singletons. Since there is no effective synergy in the contest success function, a profile of singletons induces the highest level of investment into fighting and hence players have the lowest payoff level, leading to the stability of the grand coalition. In our setting, players in the grand coalition are unable to commit to sharing the prize in any specific way and there exists synergy from forming sub-coalitions, and consequently two sub-coalitions emerge in the initial contest.

Sanchez-Pages (2007a) studies coalition formation in a symmetric model of contests in which coalitional payoffs are exogenously specified and shared equally among members. He derives conditions on the stability of the grand coalition and other types of coalition structures, which depend crucially on the properties of the coalitional payoff structure. In a related paper, Sanchez-Pages (2007b) analyzes coalition formation in a symmetric model of contests in which members of a coalition commit to sharing the winning prize according to a linear combination of equal and proportional sharing rules. The contest success function takes the power-ratio form so that synergy exists when a coalition pools resources together into effective fighting. When the winning prize is produced using the remaining resources in the coalition and the production technology exhibits constant returns to scale, he shows that the grand coalition is an equilibrium outcome. In our model with a more general synergy function, the winning prize is fixed, all resources are allocated to the contest, and conflict may continue, which lead to a bipolar coalition structure instead of the grand coalition as an equilibrium outcome.

[4] Yi (1997) analyzes a model of stable coalition structures under positive or negative externalities and different rules of coalition formation. The idea that rivals may react to the formation of coalitions by other players is further explored and developed in a series of papers by Yi (1998, 1999), and is fruitfully applied to the study of customs unions and trading blocs (See Yi 1996).

Noh (2002) analyzes alliance formation in a model of conflict among three players. A unique feature of his model is that the formation of a coalition of two players allows them to pool resources together and is more effective proportionally in appropriative activities than those of the outside adversary. It is shown that in equilibrium two stronger players form a coalition against the weakest. We allow for a more general form of synergy function from forming coalitions. In our model with three players and proportional sharing rule, his result can arise for some distributions of strengths, but an equilibrium involving a coalition of the strongest and the weakest can also arise for other distributions of strengths. When considering continuing conflict explicitly, we find that the equilibrium involves a coalition of the two weaker players against the strongest, which is opposite to Noh's finding. This feature of balancing power through coalition formation holds generally in our setting with continuing conflict and more than three players.

Hirshleifer (1988, 1995) addresses the issue of the continuing struggle for resources, but does not allow players to form coalitions. Esteban and Sakovics (2003) explicitly allow for continuing conflict in a model of coalition formation with three players. The contest success technology in their setting exhibits a linear-ratio form with asymmetric strengths and identical quadratic cost function, implying that there is no extra synergy for a coalition in transforming coalitional resources into effective fighting except for aggregating individual members' contributions. This assumption could help explain their finding that players do not have incentives to form a coalition. In our setting, it is the synergy in the contest success function that induces players to form coalitions despite that allies in the winning coalition may engage in further conflict. Our finding on sequencing of conflict as an equilibrium outcome appears consistent with the real-world examples discussed in their paper, although our model is stylized.

The rest of the paper is organized as follows. In the next section, a formal model is outlined in detail. We characterize equilibrium coalition structures with continuing conflict in Sect. 3 and with binding agreements in Sect. 4, respectively. Some concluding remarks are given in Sect. 5 and all formal proofs are relegated to an appendix.

2 The model

Consider a model in which n players compete (or fight) for a prize. The value of this prize is the same for all players and is normalized to 1. Each player is endowed with a strength (or skill, resource, etc.). Let a_i represent player i's strength, where $a_i > 0$ for $i = 1, \ldots, n$, and let $N = \{1, \ldots, n\}$ be the set of all players.

The rule of competing is the winner-takes-all: one final winner gets the prize and all other players receive nothing. Therefore, a player maximizes the winning probability of eventually winning the prize. The winning probability for a player depends on a synergy function $h(\cdot)$, the players' strengths, and the coalitions they have formed. For example, when they do not form any coalition, so that all players in N compete individually, the probability of winning for player i is equal to

$$P_i(N) = \frac{h(a_i)}{\sum_{j=1}^{n} h(a_j)}.$$

We assume that h is continuously differentiable, and $h(x) > 0$ and $h'(x) > 0$ for all $x > 0$. This functional form of winning probabilities (or called contest success function) has been axiomatized by Skaperdas (1996).[5] In the literature on conflict and rent-seeking, this contest success functional form with some specific h functions has been widely used. In later sections, we will impose further restrictions on h to determine the pattern of equilibrium coalition structures.

Players form coalitions strategically. In reality, a coalition may be formed at one stage and then broken up later (sometimes after incurring some costs). In this paper, we do not consider any explicit cost of forming a coalition. We therefore need to impose some restrictions so that the coalition formation process will converge. We assume that once a coalition (say, S) is formed at some point in the coalition formation process, it cannot be broken up in the remaining process. It can form a larger coalition with other coalitions/players, but the structure in S remains unchanged.

For example, suppose that S consists of players 1 and 3. So $S = \{1, 3\}$. Now S forms a larger coalition with individual players 2 and 5 to form a larger coalition S', that is, $S' = \{\{1, 3\}, \{2\}, \{5\}\}$. If the members in S' continue to fight after S' wins, players 1 and 3 will fight together as a coalition against individual players 2 and 5. Now suppose that players 2 and 5 already formed a coalition $\{2, 5\}$. When it forms a larger coalition with S, it becomes $S'' = \{\{1, 3\}, \{2, 5\}\}$. If S'' wins and the fight continues, players 1 and 3 (as a coalition) will fight players 2 and 5 (as a coalition).

Given the coalition formation process that we will discuss below, the final coalition structure can be defined as a partition of N, which is denoted by π. (Note that inside each partition, there is a sub-partition of that partition.) Let $\pi = [S_1, S_2, \ldots, S_m]$, where $1 < m \leq n$. Some of these S's could be singletons where a player does not form a coalition with any other players. We call such a coalition an individual coalition. As we noted above, some of these S's could have very complicated sub-structures, such as S' and S'' in the example above.

Given the coalition structure $\pi = [S_1, S_2, \ldots, S_m]$, these m coalitions compete against each other. The probability that coalition S_k wins is assumed to be equal to

$$P_{S_k}(\pi) = \frac{h(\sum_{j \in S_k} a_j)}{\sum_{t=1}^{m} h(\sum_{j \in S_t} a_j)},$$

where $k = 1, \ldots, m$. Here, we abuse the notation a bit by using $j \in S_t$ to denote a player belonging to coalition S_t, regardless of the sub-structure of S_t.

We will consider two different rules regarding what happens after a coalition wins the initial fight. The first case is that the players in the winning coalition continue to fight until there is only one individual player left, and this individual becomes the final winner of the prize. In this case, the sub-structure in the winning coalition matters, and the same probability of winning function applies in all fights. We shall discuss

[5] Skaperdas (1996) shows that the crucial property of the functional form is the independence from irrelevant alternatives; namely, if player i participates in a contest among a subset of players, then his probability of winning is independent of the strengths of players not included in the subset. The other property that characterizes the functional form is the consistency: the contests among a subset of players are qualitatively similar to the one among all players.

these rules in greater detail later. The second one is that the players in the winning coalition share the prize proportionally according to their strengths. In this case, the sub-structure in the winning coalition does not matter. This case is equivalent to the case of continuing fighting within the coalition, but in subsequent contests there is no synergy from forming of coalitions.

We assume that there is no discounting in the game and that no transfer payment between players is feasible. Players form coalitions strategically to maximize their payoffs, which is the probability of finally winning the prize.

There are three effects of forming a coalition. The first is that the allied players aggregate their strengths and become stronger. In our model, synergy exists when players form a coalition. The magnitude of the synergy depends on the properties of the h function. The second effect of forming a coalition is that it imposes a negative externality on those players outside the coalition since they will have lower probability of winning and will likely react to the coalition in a particular way in the coalition formation process.[6] The third effect of forming a coalition is that, similarly to Skaperdas (1998), it changes the probability of eventually winning the prize by a player when the coalition he belongs to wins the initial fight. This is because once a coalition wins a fight, the sub-coalitions in that coalition may fight against each other. While forming a coalition with a strong player increases a player's probability of winning the first battle, he has to compete against the strong player to win the prize eventually. Given these effects, players must act strategically in the coalition formation process.

2.1 Coalition formation process

We now formally describe the coalition formation process. Coalitions are formed in a sequential order. In stage 1, all players are individual coalitions. Suppose that $S_1, S_2, \ldots, S_m$ are the list of all coalitions (including individual coalitions) in stage i. Then in stage $i + 1$, nature proposes all possible strictly larger coalitions (as a subset of $\{S_1, S_2, \ldots, S_m\}$) in a random, non-repetitive, and exhaustive order. Coalitions, once formed, do not break up. Given that a new coalition is proposed, the players in that coalition vote sequentially. The new coalition is formed if and only if all of its members voted "yes" , and the original structures of the member coalitions that form the new coalition are preserved. As soon as a new coalition is formed, Nature starts a new sequence, in the next stage. The formation process is complete when no new coalition is formed by the end of a sequence.

The no-break-up assumption ensures convergence of the coalition formation process. Even with this assumption, our process is still flexible enough to allow for the formation of any particular coalition, as long as all members in this coalition wait for that proposal to appear. The delay involved is not significant, since there is no discounting in this game. On the other hand, coalitions can form larger coalitions, as long as this augmentation improves the payoffs of every player involved and does not break up any coalition already formed.

[6] Coalition formation may also generate positive spillover and this has been discussed in Yi (1997) and Sanchez-Pages (2007a).

Let us illustrate the coalition formation process with an example of 4 players. In stage 1, by default, the coalition structure is $[\{1\}, \{2\}, \{3\}, \{4\}]$. In stage 2, all possible larger coalitions are proposed. These larger coalitions are $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2, 3\}$, $\{2, 4\}$, $\{3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, and $\{1, 2, 3, 4\}$. (For simplicity, we sometimes do not use brackets for individual coalitions.) These larger coalitions are proposed in a random order. Suppose that $\{2, 4\}$ is formed in stage 2. Let $S_1 = \{1\}$, $S_2 = \{2, 4\}$, and $S_3 = \{3\}$. Then in stage 3, larger coalitions of $\{S_1, S_2, S_3\}$ are proposed. These coalitions are $\{S_1, S_2\}$, $\{S_1, S_3\}$, $\{S_2, S_3\}$, and $\{S_1, S_2, S_3\}$. Suppose that $\{S_1, S_2\}$ is formed, and the coalition structure becomes $[\{S_1, S_2\}, \{S_3\}] = [\{1, \{2, 4\}\}, \{3\}]$. If there is no further (and larger) coalition forming, this becomes the final coalition structure.

In the next two sections, we shall examine two cases. In one, fighting continues until a (final) individual coalition is left. In the other, fighting stops after the initial phase. In either case, we shall look for subgame-perfect equilibria of the extensive-form game. Note that a grand coalition is not a sensible concept when there is continuing conflict, as the subcoalitions inside the grand coalition will fight against each other anyway. Note also that in our setting, the outcomes are independent of the actual order of coalitions proposed. Two equilibria are regarded as equivalent in our model if the resulting coalition structures are exactly the same, even though the timing of actual coalition formation may be different. A coalition structure is called an *equilibrium coalition structure* in the game if it is a subgame-perfect equilibrium given any order of proposals presented.

In general, the coalition structure in a subgame-perfect equilibrium in the game is not independent of the order of partitions proposed. For example, let $n = 3$. Suppose that i wants to form a coalition with j, that j wants to form a coalition with k, but that k wants to form a coalition with i, and that all players prefer a two-player coalition to an individual coalition. If $\{i, j\}$ is proposed first, then with equal probability $\{i, k\}$ or $\{j, k\}$ will be proposed next. Suppose that $\{i, k\}$ is proposed next, then it must be that $\{j, k\}$ will be proposed thereafter. If no coalition is formed by the process and $\{j, k\}$ is proposed, players j and k will vote "yes" and form a coalition. This is because the end of the proposal process has arrived; if no coalition is formed, all three players will then fight individually. Given that a two-player coalition is better than an individual coalition, j and k will form a coalition. Anticipating this, when $\{i, k\}$ is proposed a step earlier, i and k will both vote "yes" , since both i and k prefer $[\{i, k\}, \{j\}]$ to $[\{j, k\}, \{i\}]$. Anticipating this, when $\{i, j\}$ is proposed still a step earlier, i and j will both vote "yes". On the other hand, if the order turns out to be $\{i, j\} \longrightarrow \{j, k\} \longrightarrow \{i, k\}$, then i and k will form a coalition when the last proposal comes up, given that no coalition has been formed previously. Anticipating that outcome, k will vote "no" when the second proposal comes up, since k prefers $\{i, k\}$ to $\{j, k\}$. Anticipating this, i and j will form a coalition when the first proposal comes up. Therefore, $\{i, j\}$ will be formed in the game. Since i, j, k are "symmetric" , we conclude that the first proposal will be accepted by the players involved regardless of which players are in the proposed coalition. Therefore, in this game, the equilibrium outcome depends on the actual order of proposals. This kind of player's preference (and thus the outcomes) will not occur in our setting.

Note that there are a few procedural similarities and differences between our coalition formation process and the ones in the literature, notably by Bloch (1996). First, in both models, once a coalition is formed it cannot be broken. Second, in our model a coalition is allowed to join other coalitions and form a large coalition, while this possibility is not permitted in Bloch's model. More importantly, in our setting there are possibly different layers of coalition structures to represent coalitions in continuing conflicts and such layers are not necessary in Bloch's model of one-shot conflict.

In what follows, we investigate the existence of equilibria and characterize the equilibrium outcomes. Under some additional assumptions, such an equilibrium exists. In general, equilibrium coalition structures will depend on the number of players, the distribution of the players' strengths, and the properties of the h function.

2.2 Synergy of forming a coalition

Recall that in our model, h is the synergy function representing the players' incentives to form a coalition in the game. In addition to our earlier assumptions that $h(x) > 0$ and $h'(x) > 0$, for all $x > 0$, we assume the following super-additivity property throughout the paper.

(R1) For any $x > 0$ and $y > 0$, $h(x + y) > h(x) + h(y)$.

Restriction (R1) implies that there exists strictly positive synergy between any two players or two coalitions. If two players form a coalition, then their joint winning probability will be increased. This is the main reason why players form a coalition in our game. Note also that (R1) implies $h(0) = 0$. This can be seen by letting x go to zero in the above inequality.

Examples of h that satisfy (R1) include $h(x) = x^\alpha$ for $\alpha > 1$ and $h(x) = e^{\gamma x} - 1$ for $\gamma > 0$. Both functional forms have been axiomatically derived in Skaperdas (1996). The power function has the property that winning probabilities depend only on the ratios of players' strengths. The parameter α measures the decisiveness of strengths in the contest (see Hirshleifer 1995). This functional form is widely used in the literature on conflict and rent-seeking.[7] This literature also adopts an exponential functional form, $h(x) = e^{\gamma x}$ with $\gamma > 0$, which has the property that winning probabilities depend only on the differences in strengths. However, such an exponential function does not satisfy our synergy property. We have made a minor modification by requiring $h(0) = 0$ and hence considered $h(x) = e^{\gamma x} - 1$ instead for an example. Furthermore, the following lemma shows that (R1) is satisfied if h is convex.

Lemma 1 *If h is strictly convex and $h(0) = 0$, then (R1) holds.*

Note that Lemma 1 has been previously observed by Skaperdas (1998, p. 31). On the other hand, (R1) does not necessarily imply the convexity of h.[8]

[7] For instance, see Perez-Castrillo and Verdier (1992), Skaperdas (1992), Hirshleifer (1995), Sanchez-Pages (2007b), and references therein.

[8] For example, consider $h(x) = x^4 - 20x^3 + 148.5x^2$, which is concave between $x = 4.5$ and $x = 5.5$ and convex everywhere else. It is easy to check that $h(0) = 0$, $h(x) > 0$ and $h'(x) > 0$ for all $x > 0$. Notice

3 Continuing conflict and coalition formation

As we discussed in the Introduction, a coalition (or an alliance) may face dissolution as it wins the fight against others. They may disagree on how to share the prize, and thus further fights can occur. In this section, we assume that fighting continues within the winning coalition until a final individual winner is determined, and examine how this continuing conflict may affect coalition formation structures. The following lemma, which is a direct implication of (R1), will be useful in characterizing the equilibrium coalition structures. Recall that the value of the final prize is normalized to 1 so that the payoff to each player is simply the probability of winning the prize.

Lemma 2 *Two players prefer forming a coalition to competing individually, assuming that the rest of the coalition structure does not change.*

It should be noted that players, say i and j, may still not vote "yes" when $\{i, j\}$ is proposed in the process, since forming (or not forming) a coalition has strategic effects on what coalitions will be formed later in the process. Likewise, two coalitions may not form a larger coalition when that option is proposed.

3.1 Three rivals

We start by characterizing equilibrium coalition structures in the case of three players. Our model rules out the possibility of a grand coalition. Lemma 2 implies that $[\{1\}, \{2\}, \{3\}]$ cannot be an equilibrium coalition structure. Thus, there are only three possibilities to consider: $[\{1, 2\}, \{3\}]$, $[\{2, 3\}, \{1\}]$, and $[\{3, 1\}, \{2\}]$, denoted by π_1, π_2 and π_3 respectively. We denote the winning probabilities (or payoffs) for each player in these coalition structures by $u_i(\pi_t)$, where $i = 1, 2, 3$ and $t = 1, 2, 3$. Each player ranks his payoff across the three coalition structures and prefers the one with the highest payoff. Such ranking depends only on a player's modified payoff, $u_i(\pi_t)/h(a_i)$.

Proposition 1 *Suppose that* $n = 3$. *Then a unique equilibrium coalition structure exists and is in the form of a coalition of two players against an individual coalition.*

Proposition 1 is essentially the existence result in Skaperdas (1998). Naturally, the equilibrium coalition structure depends on the distribution of the players' strengths and the properties of the h function.

One interesting case is when the two weaker players form a coalition against the strongest player. The following restriction on h provides a sufficient condition:

(R2) For any $y > 0$, $[h(x + y) - h(y)]/h(x)$ is weakly decreasing in x, for $x > 0$.

According to (R1), the incremental gain in effective strength to a player (or a coalition) with strength y from forming a coalition with another player (or a coalition) with

Footnote 8 continued
that $h(x + y) - h(x) - h(y) = 2xyg(x, y)$, where $g(x, y) = 2x^2 + 3xy + 2y^2 - 30x - 30y + 148.5$. It can be easily verified that $g(x, y)$ reaches its minimum at $x = y = 30/7$, where it is positive. Therefore, $g(x, y)$ is positive everywhere and hence (R1) is satisfied.

strength x, $h(x+y)-h(y)$, exceeds the effective strength of the second player. As x increases, both terms increase. It is reasonable to assume that the incremental gain in effective strengths to the first player should not increase faster than the effective strength of the second player when the strength of the second player increases. In particular, if we use the ratio of the two terms to measure the relative effectiveness of strengths from forming a coalition between the two players (or coalitions), then (R2) simply states that this relative effectiveness decreases with the strength of the second player in comparison. Therefore, a player benefits more from adding a weaker player than from adding a stronger player.

The restriction (R2) is satisfied by the power function, $h(x)=x^{\alpha}$, where $\alpha>1$, and by the exponential function, $h(x)=e^{\gamma x}-1$, where $\gamma>0$. The following lemma provides a sufficient condition which is easy to verify.

Lemma 3 *If $h''(x)/h'(x)$ is weakly decreasing in x, then (R2) holds.*

The condition in Lemma 3 means that as x increases, the rate of increase in the slope of h falls. This restriction puts a limit on the rate of the effective strength increase when strengths are becoming larger. This decreasing rate of increase in effective strengths is enough to guarantee the validity of (R2).

One implication of (R2), together with the synergy assumption (R1), is stated in the following Proposition 2.

Proposition 2 *Suppose that (R2) holds, $n=3$, and $a_1>\max\{a_2,a_3\}$. Then $[\{1\},\{2,3\}]$ is the unique equilibrium coalition structure.*

Proposition 2 implies that, when an increase in a player's strength benefits himself along relatively more than benefiting his coalition partner, the two weaker players will initially form a coalition in an attempt to balance the power of the strongest. And if they win, they will then fight between themselves for the final victory. This result extends a similar observation by Skaperdas (1998) (Proposition 2 and Corollary). One difference is that we have provided a non-cooperative justification for the stability concept that he used. This justification proves useful when we extend our analysis to the game with more than three players. The other difference is that Skaperdas (1998)'s sufficient condition is more complicated than ours.

It is interesting to note that in equilibrium the strongest player may not have the highest probability of winning, because the weak players can increase their probabilities of winning by forming a coalition. For example, let $h(x)=x^2$, $a_1=0.6$, $a_2=0.5$, and $a_3=0.2$. In this case, $[\{1\},\{2,3\}]$ is the equilibrium coalition structure. It can be computed that the final probabilities of winning for players 1, 2 and 3 are 0.42, 0.50, and 0.08, respectively. It is player 2 who has the highest probability of winning. This result raises the interesting issue of *disarmament*. In the example, player 1 has incentives to reduce his strength to a level below a_2 so that he will form a coalition with player 3, provided that such a reduction is credible. For instance, if a_1 is reduced to $a_1'=0.49$ and a_2 and a_3 are the same as before, then the new equilibrium coalition structure is $[\{1,3\},\{2\}]$ where the final probabilities of winning for players 1, 2 and 3 are 0.56, 0.35, and 0.09, respectively. Clearly, player 1 is better off. Then, player 2 may want to do the same, followed by further disarmaments by other players. To

Table 1 Players' probabilities of winning

	Player 1	Player 2	Player 3
[{1, 2}, {3}]	0.5897	0.4095	0.0007
[{2, 3}, {1}]	0.2851	0.3949	0.3199
[{3, 1}, {2}]	**0.5985**	0.0648	**0.3367**

Bold values indicate equilibrium configuration

understand this type of contest, we need a more general model in which players first invest to increase or reduce their strengths (or military capabilities) and then form coalitions and fight. This is, however, beyond the scope of the present paper.

One might conjecture that the strongest player always has a higher probability of winning than the weakest player. However, this conjecture is false when the players have similar strengths. Let $h(x) = x^2$ again, but now $a_1 = 1.2, a_2 = 1.1$, and $a_3 = 1.0$. The probabilities of winning for players 1, 2 and 3 are 0.25, 0.41, and 0.34, respectively. This and the earlier examples clearly illustrate the gain from forming a coalition for the coalition members.

Proposition 2 provides a sufficient condition on h under which the contest exhibits a balance of power through coalition formation. We should note that this result may not hold when (R2) is violated. Here we offer two examples. In the first example, $a_1 = 2a, a_2 = a_3 = a$. A balance of power implies that players 2 and 3 should form a coalition, in which case each of them has a winning probability of $1/4$ while player 1 has a winning probability of $1/2$. Suppose instead players 1 and 2 form a coalition, in which case the winning probability for player 1 exceeds $1/2$, for player 3 is less than $1/4$, and for player 2 is

$$\frac{h(3a)}{h(3a)+h(a)} \cdot \frac{h(a)}{h(2a)+h(a)}.$$

Suppose further $h(x) = x^{1.01} + x^4$ and $a = 0.25$. Then the above probability for player 2 is equal to 0.250459. Thus, in equilibrium, players 1 and 2 or 1 and 3 form a coalition. It can be verified that this h function violates (R2).

Consider another example of $h(x)$ which violates (R2).

$$h(x) = \begin{cases} \frac{1}{2}x^2, & \text{if } x \le 1; \\ \frac{1}{100}x^{100} + \frac{49}{100}, & \text{if } x > 1. \end{cases}$$

It is easy to check that h is everywhere increasing and convex, and $h(0) = 0$. Thus, by Lemma 1, (R1) is satisfied. Let $a_1 = 0.6$, $a_2 = 0.5$, and $a_3 = 0.45$. Table 1 presents the payoffs for all three players in different coalition structures. Observe that the unique equilibrium coalition structure is [{3, 1}, {2}], which is different from the one predicted by Proposition 2. In this case, the weakest player joins the strongest.

We further note that given the h function in this example, the two strongest players may form a coalition. Take the following values: $a_1 = 0.6, a_2 = 0.5$, and $a_3 = 0.43$. It can be computed that the equilibrium requires players 1 and 2 to form a coalition. This outcome implies that coalition formation is not necessarily to balance power, but to increase the probability of winning for the allied partners. The equilibrium here depends on who makes player 1 happier. If player i makes 1 happier then 1 will join i and i clearly gains. The strongest player can pick whoever he wants to form a coalition.

Both examples illustrate the possibility that one of the weaker players joining the strongest player is the equilibrium outcome. This is related to the so-called "bandwagoning" phenomenon in forming alliances in the context of international relations.[9] In the context of our contest model, balancing or bandwagoning in alliances is determined by the distribution of individual strengths and the nature of rivalry described by the contest success function along with the synergy property of transforming individual strengths into effective strengths in contests. We next extend our analysis and findings to the case of more than three players.

3.2 Four rivals

Suppose now that four players compete in the contest. This results in several different types of coalition structures in the initial stage of coalition formation as follows: $[\{i, \{j, k\}\}, \{l\}]$ (12 possibilities), $[\{i, j, k\}, \{l\}]$ (3 possibilities), $[\{i, j\}, \{k, l\}]$ (3 possibilities), $[\{i, j\}, \{k\}, \{l\}]$ (6 possibilities), and $[\{i\}, \{j\}, \{k\}, \{l\}]$ (1 possibility). When a coalition wins, the members within that winning coalition fight again according to the sub-coalition structure. For example, in $[\{i, \{j, k\}\}, \{l\}]$, when $\{i, \{j, k\}\}$ wins, i will then fight against $\{j, k\}$; if $\{j, k\}$ wins, j will then fight against k. As we can see, the situation is now much more complex than the contest of three players or the case of just one round of fighting.

Lemma 2 implies that $[\{i, j\}, \{k\}, \{l\}]$ dominates $[\{i\}, \{j\}, \{k\}, \{l\}]$ for players i and j, $[\{i, \{j, k\}\}, \{l\}]$ dominates $[\{i, j, k\}, \{l\}]$ for j and k, and $[\{i, j\}, \{k, l\}]$ dominates $[\{i, j\}, \{k\}, \{l\}]$ for players k and l. Therefore, the only possible equilibrium coalition structures in the initial stage are of the following two types: $[\{i, \{j, k\}\}, \{l\}]$ and $[\{i, j\}, \{k, l\}]$. Moreover, if h satisfies (R2), then Proposition 2 implies that the weakest two coalitions/players form a coalition when there are three coalitions left. This helps determining the equilibrium coalition structures.

Proposition 3 below provides a characterization of equilibrium coalition structures for the case of four players.

Proposition 3 *Suppose (R2) holds, $n = 4$, and $a_1 > a_2 > a_3 > a_4$.*

(i) *If $a_1 > a_3 + a_4$, then the unique equilibrium coalition structure is* $[\{2, \{3, 4\}\}, \{1\}]$.
(ii) *If $a_1 < a_3 + a_4$, then the equilibrium coalition structure is unique and is in the form of* $[\{i, j\}, \{k, l\}]$.

Note that $[\{2, \{3, 4\}\}, \{1\}]$ gives players 3 and 4 the highest probability of winning, since they collude with as many players as possible in the most favorable way. This coalition structure can be achieved only if $a_3 + a_4 < a_1$, since players 3 and 4 can form an early coalition and induce player 2 to join. If $a_3 + a_4 > a_1$, however, player 2 refuses to join the coalition of 3 and 4. Anticipating this, players 3 and 4 may not form a coalition. In this case, it is hard to pin down the exact equilibrium coalition configuration for a general h function, even though the equilibrium coalition structure

[9] For a general discussion on bandwagoning and balancing in alliances in the context of international conflict, see Wagner (2004). He uses a bargaining model with three players to illustrate the trade-offs that players face in choosing sides in an international conflict.

is unique and must be in the form of two coalitions with two players per coalition. The following proposition shows that, for some special h function, the unique equilibrium coalition structure can be explicitly characterized.

Proposition 4 *Suppose* $h(x) = x^2, a_1 > a_2 > a_3 > a_4$ *, and* $a_1 < a_3 + a_4$*. Then* $[\{2, 3\}, \{1, 4\}]$ *is the unique equilibrium coalition structure.*

The equilibrium structures in Propositions 3 and 4 imply a rule of balance. The equilibrium coalition in Proposition 3(i) is the most evenly balanced among all 3-and-1 coalition structures. Likewise, the equilibrium coalition in part (ii) in the case of $h(x) = x^2$ is also the most evenly balanced among all 2-and-2 coalition structures. As a principle, the combined strength of each side is likely to be close. Of course, a word of caution is in order, since strategic considerations also have to be taken into account. For example, when $a_1 = 10, a_2 = 9, a_3 = 4$, and $a_4 = 3$, the equilibrium structure is $[\{1\}, \{2, \{3, 4\}\}]$, instead of the more evenly balanced $[\{1, 4\}, \{2, 3\}]$. In this case, 3 and 4 form a coalition first, and then induce 2 to join them.

It should be noted that $[\{2, 3\}, \{1, 4\}]$ may not be the equilibrium coalition for other h functions. For example, for $h(x) = e^x - 1$, if a_2, a_3, and a_4 are close, then player 2 would prefer to form a coalition with player 1, not with player 3. By forming a coalition with player 1, player 2 will have a greater probability of winning against the coalition $\{3, 4\}$, but a lesser probability of winning against player 1. Given the exponential functional form of h, the gain for player 2 by forming a coalition with 1 can outweigh the loss. What will be the equilibrium coalition structure? In the case of $a_1 = 1, a_2 = 0.52, a_3 = 0.51$, and $a_4 = 0.50$, $[\{1, 2\}, \{3, 4\}]$ offers the highest modified payoffs for players 1 and 2 and hence is the unique equilibrium coalition structure. This equilibrium structure does not imply a balance of power.

3.3 More rivals

When there are more than four rivals, the exact characterization of the equilibrium coalition structure is very difficult. However, the general conclusion in the previous section remains valid. That is, there are only two coalitions in any equilibrium of the coalition formation game. This is mainly due to the fact that the coalition formation game in our model is a constant-sum game. Given that there is synergy from forming a coalition, if there are still more than two coalitions left, some of the coalitions will have incentive to form a larger coalition. We have the following proposition.

Proposition 5 *Any equilibrium of the coalition formation game will have only two coalitions.*

The intuition for Proposition 5 is as follows. Suppose that coalitions $S_1, S_2, \ldots, S_m$ have been formed, where $m \geq 3$. This cannot be the final coalition structure because $S_2, \ldots, S_m$ have incentive to form a larger coalition S' and every player in $S' \equiv \{S_2, \ldots, S_m\}$ is better off. This is because S' preserves the relationships between each coalition in S'. Because of the synergy effect, a coalition in S', say S_m, has a higher probability of winning in $[S_1, \{S_2, \ldots, S_m\}]$ than in $[S_1, S_2, \ldots, S_m]$. Therefore, every player in S_m is better off.

The conclusion of Proposition 5 depends crucially on the assumption that a larger coalition preserves the original coalitional structures in its members when it is formed. For example, when $S' \equiv \{S_2, \ldots, S_m\}$ is formed, members in S_m remain as a coalition when S' wins, and they fight together against other coalitions in S'. If forming S' causes the members in S' reshuffle and regroup, then some of the members in S' may oppose the forming of S' as they may be at disadvantage when regroup occurs in S'. In this case, there could be more than two coalitions in the equilibrium.

4 Equilibrium coalition structure under proportional sharing agreements

In the analysis above, we have used the contest probability function along with synergy function h as the primitive to describe the rivalry situation faced by n players. If the players are unable to make binding commitments on how to resolve their rivalry among themselves or among a subset of them, they have to compete following this primitive contest probability function. That is, in the presence of continuing conflict, contest success technologies among a subset of players are consistent with the contest success technology among the whole set of the players (see footnote 11). If a subset of players are able to make binding agreements, which may arise in some situations, we would like to study how such agreements may change equilibrium coalition structures. We adopt the proportional sharing rule as an example of such binding agreements. While most studies in the literature on coalition formation assume various binding agreements, our purpose in this section is to illustrate the difference in equilibrium coalition structures with and without binding agreements, and hence the importance of considering continuing conflict explicitly.

We consider a situation in which members in the winning coalition are able to commit to dividing the prize proportionally according to their strengths. Suppose that coalition S wins and player i in S will receive the following share of the prize that is normalized to 1,

$$\frac{a_i}{\sum_{j \in S} a_j},$$

where $j \in S$ denotes that j is a member of S. This is the payoff of player i, and it does not depend on the sub-structure of S. For example, the payoffs for players 1, 3 and 4 are exactly the same in $S = \{\{1, 3\}, \{4\}\}$ and in $S = \{\{1\}, \{3, 4\}\}$, given other things unchanged. Thus, the payoff to each player i in S is simply the product of the above share and the probability of S winning the contest.

This proportional sharing rule can be a result of an agreement negotiated among the members in a coalition. The members in a coalition agree that no further fights will occur between them, and when they win, they will share the prize according to their strength.[10] An alternative interpretation of using the proportional sharing rule is that it is a result of further fighting to determine the final winner, where there is no

[10] The proportional sharing rule, and more generally a linear combination of equal and proportional sharing rules, has been justified axiomatically by Moulin (1987). One of the key axioms states that any subset of the players will not be able to gain by transfering their strengths (or endowments) among themselves.

synergy from further forming coalitions or equivalently $h(a) = a$ in the subsequent contests and hence forming sub-coalitions has no effect on the members' payoffs.[11] This can be seen as follows. Suppose that player 1 is in coalition C_1, and C_1 is in a larger coalition C_2, and C_2 in $C_3, \ldots$, and finally C_{n-1} in C_n. Let A_i denotes the total strength of coalition C_i, A be the strength of the grand coalition, and a_1 be the strength of player 1. Then without synergy, player 1's payoff (i.e. probability of winning) is given by

$$\frac{A_n}{A} \cdot \frac{A_{n-1}}{A_n} \cdots \frac{A_2}{A_3} \cdot \frac{A_1}{A_2} \cdot \frac{a_1}{A_1} = \frac{a_1}{A}.$$

In the special case of identical players, the proportional sharing rule is equivalent to an equal sharing rule. In fact, any anonymous sharing rule would yield the same outcome.

The following lemma is useful in characterizing the equilibrium coalition structure.

Lemma 4 *Suppose $h(x)/x$ is convex. Then for any x, y, and C such that $h(x) < h(y) + C$,*

$$\frac{h(x+y)}{h(x+y)+C}\left(\frac{1}{x+y}\right) > \frac{h(x)}{h(x)+h(y)+C}\left(\frac{1}{x}\right). \tag{1}$$

Note that $h(x)/x$ represents the average effective strength of an individual player or a coalition. The convexity of $h(x)/x$ means that the average effective strength increases with an increase in strength x at an increasing speed, implying a strong form of synergy from coalition formation. This assumption is satisfied with the exponential function $h(x) = e^{\gamma x} - 1$ for $\gamma > 0$ and with the power function $h(x) = x^{\alpha}$ for $\alpha \geq 2$.

Lemma 4 implies that when the synergy of forming coalitions is sufficiently strong, members in any coalition always have incentives to merge with another coalition if the effective strength of the first coalition, $h(x)$, is small relatively to the total effective strength, $h(y)+C$, of the other coalitions. Intuitively, this should lead to the conclusion that at most two coalitions exist by the end of the coalition formation process.

Let S be a coalition of players. Assume that all remaining players form another coalition. Under the proportional sharing rule, each player in S has the same payoff multiples, since the payoff for player $i \in S$ can be written as

$$\frac{h\left(\sum_{j\in S} a_j\right)}{h\left(\sum_{j\in S} a_j\right) + h\left(\sum_{j\notin S} a_j\right)}\left(\frac{1}{\sum_{j\in S} a_j}\right) a_i \equiv g(S) a_i.$$

[11] We recognize that this alternative interpretation suffers from an inconsistency between the initial stage and subsequent stages of the contest in the sense that the contests among a smaller number of players are qualitatively different from the intial contest with the presence of all players (see axiom A4 in Skaperdas 1996). Perhaps a smaller group of the players has the ability to change the nature of the rivalry in some situations. Nevertheless, the proportional sharing rule satisfies the axiom of independence from irrelevant alternatives in Skaperdas (1996).

That is, the payoff of each player in S is equal to the same $g(S)$ multiplied by his own strength. Let S^* be the coalition of players that maximizes $g(S)$, i.e.,

$$S^* = \arg\max_{S \subset N} g(S). \tag{2}$$

We have the following proposition.

Proposition 6 *Suppose $h(x)/x$ is convex and $n \geq 3$. Under the proportional sharing rule, in any equilibrium of the coalition formation game there are only two coalitions given by S^* and N/S^* with $\sum_{j \in S^*} a_j \geq \sum_{j \notin S^*} a_j$.*

Proposition 6 implies that when the synergy of forming a coalition is strong in the form of the average effective strength being increasing rapidly with coalitional strength, the equilibrium structure is a bipolar system of asymmetric sizes. The intuition is as follows. Suppose first that only two coalitions are allowed to form. When forming the first coalition, players face the following trade-off: a larger coalition implies a higher probability of winning for the coalition, but the spoils have to be shared by more partners. This trade-off determines the optimal composition of the first coalition. Thus, there is a first-mover advantage. Moreover, as the synergy effect becomes stronger, the positive effect of an increase in size becomes less significant compared to the negative effect of sharing among a large number of players. When more coalitions can be formed, according to Lemma 4, the smaller coalitions always have incentives to merge. This force effectively leads to two coalitions in any equilibrium.

Note that S being empty is not well defined in (2), and that S^* will not be equal to the grand coalition N. This is because the payoffs of all players sum up to 1. If any coalition S can induce some payoffs that are *different* from those in the grand coalition for some players, it means that some players will earn payoffs that are *higher* than those from the grand coalition. (We can always find such an S unless when there are only two players with equal strengths. In this case, both players are neutral to forming a grand coalition, and either individual coalitions or a grand coalition are the solutions in (2).) Therefore, the S^* in (2) can be anything in the case of two players with equal strengths and must not be N in all other cases.

The result in Proposition 6 is similar in spirit to that in Proposition 2 of Bloch (1995). In his model, identical firms play a two-stage game in which firms first form associations and then compete noncooperatively (Cournot or Bertrand) in the product market. An association of firms allow them to reduce their costs of production. Thus, a larger association offers more benefits to its members than a smaller coalition. However, a larger association means more firms with lower production costs will compete in the product market. In equilibrium the firms balance this trade-off. Given the assumptions of linear demand and cost functions in his setting, two smaller associations always have incentives to merge, and as a result there are only two associations in equilibrium.[12] In our setting, a larger coalition also offers more benefits to

[12] Bloch (1995) discusses a number of industry examples in which firms indeed form two groups in research and development. Similar results are obtained by Yi (1996, 1998) in different contexts.

its members than a smaller coalition, but the fixed pie has to be shared among more players. When synergy is strong, two smaller coalitions have incentives to merge. In equilibrium only two coalitions exist.

In the case of identical players, without loss of generality assuming $a_i = 1$ for all i, Proposition 6 implies that the size of the first coalition, k^*, is the integer that maximizes

$$\frac{h(k)}{h(k) + h(n-k)} \left(\frac{1}{k}\right).$$

The following corollary describes equilibrium coalition sizes for a special class of h functions.

Corollary *Suppose all players have identical strength and $h(x) = x^\alpha$ with $\alpha \geq 2$ and coalition members commit to the proportional sharing rule. Then the equilibrium size of the first coalition k^* is an integer close to k_1 determined by*

$$\left(\frac{n}{k_1} - 1\right)^{\alpha-1} \left((\alpha - 1)\frac{n}{k_1} + 1\right) = 1.$$

Note that k_1 is decreasing in α. As the synergy effect becomes stronger (a larger α), the positive effect of an increase in size becomes less significant as compared to the negative effect of equal sharing, so that the optimal size of the first coalition decreases and approaches $n/2$ as α goes to infinity (which corresponds to an auction-like contest).[13] The above findings in the case of identical players and $h(x) = x^\alpha$ with $\alpha \geq 2$ have been discussed by Sanchez-Pages (2007b) in relation to his model with endogenous determination of investment levels. In his Proposition 5(i), he shows that if labor resources are productive with constant returns to scale, the grand coalition is the equilibrium outcome.

To illustrate the difference between the equilibrium coalition structure under the commitment with the proportional sharing rule and the one involving continuing conflict, we consider the case of three players with a specific h function. The equilibrium coalition structure is summarized in the following proposition.

Proposition 7 *Suppose $h(x) = x^2, n = 3$, and $a_1 > a_2 > a_3$. Then the equilibrium coalition structure under the proportional sharing rule is $[\{1, 2\}, 3]$ if $a_3^2 + a_2^2 > a_1^2$ and $[\{1, 3\}, 2]$ if $a_3^2 + a_2^2 < a_1^2$.*

Under the assumptions in Proposition 7, each of the two weaker players has incentives to ally himself with the strongest. This is another example of the "Bandwagoning" phenomenon that we discussed following Proposition 2. On the other hand, the strongest player prefers to ally himself with the weakest if his own strength is sufficiently large, and with the middle player otherwise.

Note that $h(x) = x^2$ satisfies (R2) and hence, according to Proposition 2, the unique equilibrium coalition structure under continuing conflict is $[1, \{2, 3\}]$, i.e., the

[13] It should be noted that if α is small, say close to 1, $h(x)/x$ is no longer convex and there may be more than two coalitions in equilibrium.

two weaker players join forces to balance the strongest. Therefore, when coalitions could be formed with certain binding agreements, such as agreeing to share the spoils with a proportional rule, the equilibrium coalition structure tends to differ from the one when such agreements could not be reached. While most of the studies in the economics literature on conflict ignore continuing conflict or focus on symmetric environments, our analysis above indicates that the nature of coalition formation can be sensitive to the possibility of continuing conflict and the heterogeneity of rivals, in addition to the synergy of forming a coalition discussed in Sect. 3.1.

5 Concluding remarks

This paper analyzes a model of endogenous coalition formation in a rivalry situation where players can pool their resources/strengths together to compete for a fixed prize. We look for coalition structures that do not depend on the order of proposals in the coalition formation process. Therefore, the resulting coalition structures are stable. We consider two cases regarding how the players would share the prize when their coalition wins the initial competition. In the case of proportional sharing rule, we find that there are exactly two coalitions of asymmetric sizes and completely characterize these two equilibrium coalitions. In the case of continuing conflict, we show that there are still exactly two coalitions in the equilibrium, although a general characterization of the equilibrium structure is rather difficult. When there are only three players, we find that the equilibrium is in the form of two weaker players forming a coalition to fight against the strongest. Similar results are obtained when there are four players. The three weaker players form a coalition and compete against the strongest, so long as the strongest is significantly stronger than each of the remaining three players. When the players' strengths are comparable, however, the equilibrium coalition structure takes the form of two-against-two, where the weakest and strongest form one of the two coalitions. A general implication of our analysis is that when the rivalry in our setting exhibits strong synergy from forming coalitions, coalition formation generates two-sided conflicts with a balance of strengths (or power) and the structure of the coalitions depends on the distribution of strengths and on the ability of players making binding commitments.

This analysis is intended to shed some light on the problem of temporary cooperation between players who are rivals by nature. Examples include war among different states, political party members fighting for leadership, and firms competing for monopoly positions. Our analysis may have implications regarding market structures for certain industries, such as legal, consulting and accounting services, and regarding political party formation and country formation. The model provided in the paper can also be used to analyze long-term cooperations, as it proposes a credible mechanism for how the surplus generated from cooperation will be shared among those players.

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Appendix

Proof of Lemma 1 Notice that $\forall x > 0, \forall y > 0$,

$$h(x+y) - h(x) = \int_0^y h'(t+x)dt > \int_0^y h'(t)dt = h(y) - h(0),$$

where the inequality holds because the convexity of h implies that h' is increasing, and thus $h'(t+x) > h'(t), \forall x > 0$. The claim then follows from $h(0) = 0$. □

Proof of Lemma 2 Consider a coalition structure $\pi = [C_1, C_2, \ldots, C_m]$ in which $C_1 = \{i\}$ and $C_2 = \{j\}$. Player i's winning probability is given by

$$P_i = \frac{h(a_i)}{h(a_i) + h(a_j) + \theta},$$

where $\theta = \sum_{s=3}^{m} h(\sum_{k \in C_s} a_k) > 0$. By forming a coalition with j, i has a final winning probability

$$\begin{aligned}\tilde{P}_i &= \frac{h(a_i + a_j)}{h(a_i + a_j) + \theta} \cdot \frac{h(a_i)}{h(a_i) + h(a_j)} \\ &> \frac{h(a_i) + h(a_j)}{h(a_i) + h(a_j) + \theta} \cdot \frac{h(a_i)}{h(a_i) + h(a_j)} = \frac{h(a_i)}{h(a_i) + h(a_j) + \theta},\end{aligned}$$

where the inequality follows from (R1) and the monotonicity of $x/(x+\theta)$. Therefore, joining with j increases i's probability of winning. Symmetrically, joining with i increases j's probability of winning. □

Proof of Proposition 1 A player prefers to be in a coalition structure that gives him the highest payoff, or equivalently the highest "modified" payoff. Let us select the highest number among all "modified" payoffs from all three possible coalition structures for all players. Since there are only finite numbers, a highest number always exists.

Observe that the highest number cannot be the modified payoff for a player who is standing alone in the coalition structure since

$$\begin{aligned}\frac{1}{h(a_i) + h(a_j + a_k)} &< \frac{1}{h(a_i) + h(a_j) + h(a_k)} \\ &< \frac{h(a_i + a_j)}{h(a_i + a_j) + h(a_k)} \cdot \frac{1}{h(a_i) + h(a_j)},\end{aligned}$$

where the first inequality makes use of (R1) and the second inequality follows from Lemma 2. Let the highest number be the "modified" payoff of i from the coalition structure $[\{i, j\}, \{k\}]$. It is also the modified payoff of j. By forming a coalition, i and j receive the highest "modified" payoff and thus the highest payoff from $[\{i, j\}, \{k\}]$.

Therefore, an equilibrium coalition structure exists. It is unique except in the degenerate case where more than two of the highest "modified" payoffs are the same, such as in the case when $a_i = a_j = a_k$. (We ignore these degenerate cases.) □

Proof of Lemma 3 Note that

$$h(y+a) - h(y) = \int_{y}^{y+a} h'(x)dx$$

$$h(a) = h(a) - h(0) = \int_{0}^{a} h'(t)dt$$

To show that $\frac{h(y+a)-h(y)}{h(a)}$ is weakly decreasing in a, we need to show that for $b > a$,

$$\frac{\int_y^{y+a} h'(x)dx}{\int_0^a h'(t)dt} \geq \frac{\int_y^{y+b} h'(x)dx}{\int_0^b h'(t)dt}$$

or

$$\int_{y}^{y+a} h'(x)dx \left(\int_{0}^{a} h'(t)dt + \int_{a}^{b} h'(t)dt \right) \geq \left(\int_{y}^{y+a} h'(x)dx + \int_{y+a}^{y+b} h'(x)dx \right) \times \int_{0}^{a} h'(t)dt$$

which becomes

$$\int_{y}^{y+a} h'(x)dx \int_{a}^{b} h'(t)dt \geq \int_{y+a}^{y+b} h'(x)dx \int_{0}^{a} h'(t)dt.$$

On the left-hand side of the inequality, change variables by letting $x = y + z$, and $t = k$ (the new variables are z and k). On the right-hand side, change variables by letting $x = y+k$, and $t = z$ (the new variables are again z and k). Note that $z \in [0, a]$ and $k \in [a, b]$. It follows that

$$\int_{0}^{a} h'(y+z)dz \int_{a}^{b} h'(k)dk \geq \int_{a}^{b} h'(y+k)dk \int_{0}^{a} h'(z)dz$$

or equivalently,

$$\int_0^a \int_a^b \left[h'(y+z)h'(k) - h'(y+k)h'(z) \right] dz dk \geq 0 \tag{3}$$

If $h''(x)/h'(x)$ is weakly decreasing in x, then for $y > 0$,

$$\frac{h''(y+x)}{h'(y+x)} \leq \frac{h''(x)}{h'(x)}$$

This implies that $h'(y+x)/h'(x)$ is weakly decreasing in x. Thus, for $z < k$, we obtain

$$h'(y+z)h'(k) \geq h'(y+k)h'(z)$$

implying that (3) holds. The claim follows. □

Proof of Proposition 2 Let x, y, and z be the strengths for players 1,2, and 3 respectively, where $x > y, z$. Given Proposition 1, we only need to show that, by forming a coalition, players 2 and 3 receive the highest "modified" payoffs, i.e.,

$$\frac{h(y+z)}{h(x)+h(y+z)} \cdot \frac{1}{h(y)+h(z)} > \frac{h(x+y)}{h(x+y)+h(z)} \cdot \frac{1}{h(x)+h(y)}.$$

The inequality can be rewritten as

$$\frac{h(y+z)-h(y)}{h(z)-h(0)} > \frac{h(x+y)-h(y+z)}{h(x)-h(z)} \cdot \frac{h(x)+h(y)}{h(x+y)}.$$

Given the synergy assumption (R1), it suffices to show

$$\frac{h(y+z)-h(y)}{h(z)} \geq \frac{h(x+y)-h(y+z)}{h(x)-h(z)}$$

which is equivalent to

$$\frac{h(y+z)-h(y)}{h(z)} \geq \frac{h(x+y)-h(y)}{h(x)}.$$

The claim follows from the assumption. □

Proof of Proposition 3 The proof of part (i) consists of three steps. The first step is to show that $[\{2, \{3, 4\}\}, \{1\}]$ is the most favorable coalition structure for players 3 and 4 among all possible equilibrium coalition structures. Suppose that $\{i, j\}$ is formed first. Then from Proposition 3, the weakest two of $\{i, j\}$, $\{k\}$, and $\{l\}$ will form a coalition. If i or j is player 1 then k and l will form a coalition. If neither i nor j is player 1 and l is player 1, then k will form a coalition with $\{i, j\}$ if the combined strength of i and j is smaller than the strength of player 1, and k forms a coalition with player 1 otherwise.

The following lists all possibilities: $\pi_1 = [\{\{3, 4\}, 2\}, \{1\}]$, $\pi_2 = [\{1, j\}, \{k, l\}]$, $\pi_3 = [\{\{i, j\}, k\}, \{1\}]$. We shall show that players 3 and 4 prefer π_1 to both π_2 and π_3.

Players 3 and 4 prefer π_1 to π_3 when $k \neq 2$. Indeed, Proposition 3 implies that players 3 and 4 strictly prefer to form a coalition with each other when only players 2, 3 and 4 exist. Since their probabilities of winning against player 1 are the same, the claim follows.

Players 3 and 4 prefer π_1 to π_2. There are three cases. First, if $j = 2$, then both j and $\{k, l\}$ prefer π_1 to π_2 , since $a_1 > a_k + a_l$. Second, if $j = 3$, then $\pi_2 = [\{1, 3\}, \{2, 4\}]$. Player 4 obviously prefers π_1 to $[\{1\}, \{\{3\}, \{2, 4\}\}]$ since he prefers $[\{2\}, \{3, 4\}]$ to $[\{3\}, \{2, 4\}]$. Therefore, player 4 must prefer π_1 to π_2. We can also conclude that player 3 prefers π_1 to π_2. This is because player 3 prefers π_1 to $[\{1\}, \{2\}, \{3, 4\}\}]$, which is then preferred by player 3 to $[\{1, 3\}, \{2\}, \{4\}\}]$, which is then preferred by player 3 to π_2. Finally, if $j = 4$, the analysis is similar to the case when $j = 3$.

The second step is to show that players 3 and 4 can achieve the maximum probability of winning from $[\{2, \{3, 4\}\}, \{1\}]$ in the following way. They can first join a coalition when $\{3, 4\}$ is proposed. Given this coalition, it follows from Proposition 2 that 2 will join them when $\{2, \{3, 4\}\}$ is proposed, since $a_1 > a_3 + a_4$. Players 3 and 4 welcome 2, because $a_1 > a_2$.

The final step is to show that there are no profitable deviations. We have established earlier that 3 and 4 prefer π_1 to any other feasible coalition structure. Moreover, $\{1, 2\}$ is the only coalition players can form without the participation of 3 and 4. Suppose that $\{1, 2\}$ is first formed. Consequently, 3 and 4 will form the coalition $\{3, 4\}$, since 3 and 4 are the weakest two among the three coalitions. The resulting coalition structure $[\{1, 2\}, \{3, 4\}]$ is not as good as $[\{2, \{3, 4\}\}, \{1\}]$ for player 2, since $a_1 > a_3 + a_4$. Therefore, 2 will not form a coalition with 1. This means that there are no profitable deviations.

We now prove part (ii). The condition $a_1 < a_3 + a_4$ ensures that the combined strength of any two players exceeds the strength of any individual player. It follows from Proposition 2 that $[\{\{i, j\}, k\}, \{l\}]$ can never be an equilibrium coalition structure, since k would deviate and join l. Hence, the possible remaining equilibrium structures are in the form of 2-against-2. Once i and j form a coalition, $\{k\}$ and $\{l\}$ become the weakest two coalitions among the three coalitions. Therefore, they will form a coalition, and the resulting coalition structure becomes $[\{i, j\}, \{k, l\}]$.

Look for the highest number in the modified payoffs across all 2-against-2 coalition structures among all players. This payoff is shared by two players, say i and j. The equilibrium coalition structure is then $[\{i, j\}, \{k, l\}]$. It is easy to check that there is no profitable deviation for every deviator involved. For instance, if $[\{i, k\}, \{j, l\}]$ generates the highest modified payoff, say, for player k, then player i gets the same highest number. This violates the fact that player i's highest number is obtained in $[\{i, j\}, \{k, l\}]$. Therefore, $[\{i, j\}, \{k, l\}]$ must be the equilibrium coalition structure. □

Proof of Proposition 4 First, it follows from the proof of Proposition 3(ii) that the 2-against-2 coalition structure that yields the highest modified payoff across all 2-against-2 coalition structures is the unique equilibrium structure. We want to show that [{1, 4}, {2, 3}] maximizes the modified payoffs for both players 2 and 3. Equivalently, we need to prove the following two inequalities for player 2 and two similar ones for player 3:

$$\frac{(a_2+a_3)^2}{(a_2+a_3)^2+(a_1+a_4)^2}\cdot\frac{1}{a_2^2+a_3^2} > \frac{(a_1+a_2)^2}{(a_1+a_2)^2+(a_3+a_4)^2}\cdot\frac{1}{a_1^2+a_2^2}, \quad (4)$$

and

$$\frac{(a_2+a_3)^2}{(a_2+a_3)^2+(a_1+a_4)^2}\cdot\frac{1}{a_2^2+a_3^2} > \frac{(a_2+a_4)^2}{(a_2+a_4)^2+(a_1+a_3)^2}\cdot\frac{1}{a_2^2+a_4^2}. \quad (5)$$

Inequality (4) holds because

$$\begin{aligned}&(a_2+a_3)^2[(a_1+a_2)^2+(a_3+a_4)^2][a_1^2+a_2^2]\\&\quad-[(a_2+a_3)^2+(a_1+a_4)^2][a_2^2+a_3^2](a_1+a_2)^2\\&=2(a_1-a_3)(a_1a_3-a_2a_4)\\&\quad\cdot[(a_1+a_2)a_2(a_1+a_4)+a_2(a_2+a_3)^2+a_1(a_2+a_3)(a_2-a_4)]>0. \quad (6)\end{aligned}$$

Inequality (5) can be obtained by switching subscripts 1 and 4 in (4), and it holds because (6) is also true when we switch subscripts 1 and 4:

$$\begin{aligned}&(a_2+a_3)^2[(a_4+a_2)^2+(a_3+a_1)^2][a_4^2+a_2^2]\\&\quad-[(a_2+a_3)^2+(a_4+a_1)^2][a_2^2+a_3^2](a_4+a_2)^2\\&=2(a_4-a_3)(a_4a_3-a_2a_1)\\&\quad\cdot[(a_4+a_2)a_2(a_4+a_1)+a_2(a_2+a_3)^2+a_4(a_2+a_3)(a_2-a_1)]\\&=2(a_3-a_4)(a_1a_2-a_3a_4)\\&\quad\cdot[a_2a_4(2a_2+a_3+a_4)+a_2(a_2+a_3)^2+a_1(a_2^2-a_3a_4)]>0. \quad (7)\end{aligned}$$

For player 3, we can just switch 2 and 3 in the above two inequalities. Obviously, (6) continues to hold. Equation (7) holds if $a_3a_4(2a_3+a_2+a_4)+a_3(a_2+a_3)^2+a_1(a_3^2-a_2a_4)>0$, which is implied by assumption $a_1<a_3+a_4$. Therefore, the proposed structure maximizes players 2 and 3's probabilities of winning and thus is the unique equilibrium coalition structure. □

Proof of Proposition 5 Suppose that coalitions $S_1, S_2, \ldots, S_m$ have been formed, where $m \geq 3$. We will show that $S_2, \ldots, S_m$ have incentive to form a larger coalition S' and every player in $S' \equiv \{S_2, \ldots, S_m\}$ is better off. Let A_i be the total strength in S_i. Then we have

$$\begin{aligned}&\frac{h(A_2+\cdots+A_m)}{h(A_1)+h(A_2+\cdots+A_m)}\cdot\frac{h(A_j)}{h(A_2)+\cdots+h(A_m)}\\&\quad>\frac{h(A_j)}{h(A_1)+h(A_2)+\cdots+h(A_m)},\end{aligned}$$

for $j = 2, \ldots, m$. This is true because $h(A_2 + \cdots + A_m) > h(A_2) + \cdots + h(A_m)$ from our synergy assumption and $x/(h(A_1) + x)$ is an increasing function of x. The probability of winning of a player in S_j conditional on S_j wins is the same before and after S' is formed. Therefore, $S_2, \ldots, S_m$ have incentive to form a larger coalition S', and $[S_1, S_2, \ldots, S_m]$ cannot be an equilibrium coalition structure. □

Proof of Lemma 4 Letting $\phi(x) = h(x)/x$, (1) can be rewritten as

$$\frac{\phi(x+y)}{(x+y)\phi(x+y)+C} > \frac{\phi(x)}{x\phi(x)+y\phi(y)+C}.$$

or equivalently,

$$\phi(x+y) - \phi(x) > \frac{y\phi(x)[\phi(x)-\phi(y)]}{C + y[\phi(y)-\phi(x)]}. \tag{8}$$

The convexity of ϕ and synergy assumption (R1) imply that ϕ is increasing. If $y \geq x$, (8) obviously holds. Suppose $y < x$. The convexity of ϕ implies $\phi(x+y) - \phi(x) \geq \phi'(x)y$ and $\phi(x) - \phi(y) \leq \phi'(x)(x-y)$. Furthermore, by assumption, $\phi(x)x < y\phi(y) + C$, implying that $\phi(x)(x-y) < C + y[\phi(y) - \phi(x)]$, or equivalently,

$$\frac{\phi(x)(x-y)}{C + y[\phi(y)-\phi(x)]} < 1.$$

It follows that (8) holds. The claim follows. □

Proof of Proposition 6 We first show that there are at most two coalitions in any equilibrium. Suppose that in equilibrium, $m \geq 3$ coalitions are formed, in the sequence of $1, 2, \ldots, m$. Let x_k denote the total strength in coalition k. Let us focus on the last three coalitions, $m-2$, $m-1$, and m. Since $m-1$ and m did not form a joint coalition, from Lemma 4, it must be that either

$$h(x_m) > h(x_{m-1}) + h(x_{m-2}) + \sum_{k=1}^{m-3} h(x_k) \tag{9}$$

or

$$h(x_{m-1}) > h(x_m) + h(x_{m-2}) + \sum_{k=1}^{m-3} h(x_i) \tag{10}$$

In the first case, Lemma 4 implies that coalitions $m-2$ and $m-1$ have incentive to form a joint coalition if coalition m remains a coalition. Coalitions $m-2$ and $m-1$ also have incentives to form a single coalition if the players in the original coalition m breaks up and form more than one coalition. Therefore, the player who proposed coalition $m-2$ will instead propose a coalition with strengths $x_{m-2} + x_{m-1}$ and be better off. Since this is a profitable deviation, it cannot happen in equilibrium.

In the second case, by Lemma 4, coalitions $m-2$ and m have incentives to form a joint coalition. It follows that instead of proposing a coalition with strength x_{m-2}, the proposer proposes a coalition with $x_{m-2}+x_m$, which consists of all of the players in coalitions $m-2$ and m. This makes everyone in the two coalitions better off, assuming that coalition $m-1$ remains a single coalition. If coalition $m-1$ breaks apart after the new coalition is formed, that is even better for the two merging coalitions. Since this is a profitable deviation, it cannot happen in equilibrium.

Therefore, in any equilibrium, there are only two coalitions. The coalition consisting players in the subset of S^* gives the highest payoff to everyone in the subset, assuming that the remaining players form one coalition. □

Proof of Proposition 7 Let the strengths of the three players be x, y and z, respectively. Given that $h(x)=x^2$, we compare the payoff for player of strength x to join player of y with the payoff for player of x to join player with z, i.e.,

$$u_x\{xy\} = \frac{(x+y)^2}{(x+y)^2+z^2}\cdot\frac{x}{x+y},$$

and

$$u_x\{xz\} = \frac{(x+z)^2}{(x+z)^2+y^2}\cdot\frac{x}{x+z}.$$

Note that $u_x\{xz\} > u_x\{xy\}$ if and only if

$$\frac{x+z}{(x+z)^2+y^2} > \frac{x+y}{(x+y)^2+z^2}$$

or equivalently,

$$(x+z)\left((x+y)^2+z^2\right) > (x+y)\left((x+z)^2+y^2\right),$$

which can be simplified as

$$(z-y)\left(y^2+z^2-x^2\right) > 0.$$

Now, given $a_1 > a_2 > a_3$, the above inequality implies that (i) player 2 prefers to join player 1 (setting $x=a_2$, $y=a_3$, and $z=a_1$), (ii) player 3 prefers to join player 1 (setting $x=a_3$, $y=a_2$, and $z=a_1$), (iii) player 1 prefers to join player 2 if $a_3^2+a_2^2-a_1^2>0$ (setting $x=a_1$, $y=a_3$, and $z=a_2$), and (iv) player 1 prefers to join player 3 if $a_3^2+a_2^2-a_1^2<0$ (setting $x=a_1$, $y=a_3$, and $z=a_2$). The claims follow. □

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