

# A Simple Model of Expert and Non-Expert Bidding in First-Price Auctions

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This paper studies the equilibrium bidding behavior in a first-price sealed-bid auction when the number of informed bidders is not common knowledge. Both the independent private values and the common value cases are analyzed, under the assumption that a “neutral” signal exists. In equilibrium, experts and non-experts draw their bids from distinct supports: experts bid in the upper and lower tail of the bidding distribution and non-experts randomize their bids in between. For common values, it is shown that the seller’s expected revenue always decreases with the probability of a bidder being informed when this probability is small. The opposite result is shown for the case of independent private values. *Journal of Economic Literature* Classification Numbers: D44, D82. © 1996 Academic Press, Inc.

## 1. INTRODUCTION

The objective of this paper is to investigate the properties of equilibrium bidding behavior in a first-price sealed-bid auction in which the number of informed bidders (or experts) is not common knowledge.

In auction theory, it is commonly assumed that potential bidders are endowed with private information and that each bidder knows whether other bidders have private information. In many applications, however, private information is endogenously acquired and a bidder may not know whether other bidders are privately informed. In OCS auctions, for example, bidders can conduct seismic surveys prior to the auction. Since surveys are costly, bidders may decide not to acquire information, and use the expected values of the oil tract as their estimate. In addition, several aspects of the process of acquiring and analyzing information are private

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knowledge of an individual bidder. It might be that each bidder does not know which rivals have acquired private information, whereas the number of potential bidders and the average value of the item to be auctioned might be common knowledge (see Hendricks and Porter [3] for a complete discussion on this issue). Used car and antique auctions may also exhibit these attributes.

Several factors determine the composition of expert and non-expert bidders in an auction. Entry costs in information acquisition and the lag between the announcement and the implementation of an auction affect the chances of participation of expert bidders. Some of these factors are under the control of the seller and can be used to modify the distribution of bidders. Such policies can have significant effects on bidding behavior and on the amount of revenue generated by an auction.

In this paper, we study the equilibrium bidding behavior in a first-price sealed-bid auction when the bidders are uncertain about how many rivals have acquired private information prior to the auction. We analyze a simple model which covers both the independent private values and the common value cases. In the case of common values, we assume that a neutral signal exists. This enables us to show the existence of equilibrium strategies with a very simple structure. In equilibrium, experts and non-experts draw their bids from distinct supports: experts bid in the upper and lower tail of the bidding distribution and non-experts randomize their bids in between.

We then analyze the effects of a change in the probability of expert participation on the seller's expected revenue. In the case of common values, we show that the seller's expected revenue always decreases with the probability of a bidder being informed when this probability is small. Consequently, the seller may prefer to adopt a policy which would reduce the changes of non-experts acquiring information. This phenomenon is caused primarily by the winner's curse and becomes more severe as the number of bidders increases. In the case of independent private values, an increase in the number of bidders causes this phenomenon to disappear. The negative effects on expected revenue are only due to the creation of information rents and are offset by a higher degree of competition.

Several papers are closely related to ours. Engelbrecht-Wiggans *et al.* [2] consider a common value environment where several uninformed bidders compete with a single informed bidder. The identity of the uninformed and informed bidders is common knowledge. Hendricks *et al.* [4] extend this model to include a random reservation price. In both models, uninformed bidders make zero expected profits in equilibrium. This result is not obtained in our model since it is not common knowledge which bidders are uninformed. McAfee and McMillan [5], Matthews [7], and McAfee and Vincent [6] study auctions in which all participating bidders have private

information and their number is stochastic. In our model, a bidder does not have to be informed to bid. The potential number of bidders is fixed, but the number of informed bidders is random.

Matthews [8] investigates the effects of a change in the informativeness of private signals on equilibrium bidding and the seller's expected revenue in a common value environment. In his model, every bidder is symmetrically endowed with private information. He shows that, if the bidders' signals are not very informative, an increase in their precision may decrease the seller's revenue. In our model, an analogous result is obtained when the probability of expert participation increases.

Baye *et al.* [1] analyze a common-values model bilateral sealed-bid auctions where informed and uninformed traders (buyers and sellers) submit bids to a computerized specialist. Similar to our model, a trader does not know whether the opponent knows the value of the item. They analyze the case of two buyers and two sellers under the assumption that the value of the object follows from a two-point distribution. In contrast, in our model  $n$ -buyers submit bids to one seller and the distribution of valuations and signals has greater generality. They obtain results that are analogous to ours: the support of the bidding distribution for an uninformed trader is distinct from the support for an informed one and the revenue for the computerized specialist decreases with the probability of a trader being informed when this probability is small.

## 2. THE MODEL

We consider a model in which a seller sells a single unit of an object to  $n$  potential buyers via a first-price sealed-bid auction,  $n \geq 2$ . The value of the object to buyer  $i$ ,  $v_i \in [\underline{v}, \bar{v}]$ , is uncertain.  $(v_1, \dots, v_n)$  is distributed according to a symmetric cumulative distribution  $Q(v_1, \dots, v_n)$  with density  $q(v_1, \dots, v_n)$ . Each buyer is either an expert or a non-expert. An expert observes a private signal,  $x_i$ , in the interval  $[a, z]$ . We assume that, conditional upon buyer  $i$ 's valuation, each  $x_i$  is independently and identically distributed with cumulative distribution  $H(x_i | v_i)$  and density  $h(x_i | v_i)$ .  $h$  is positive, bounded, and continuously differentiable in  $x_i$ , and satisfies the strict monotone likelihood ratio property (MLRP):

(A1). For any  $v_i > v'_i$ ,  $h(x_i | v_i)/h(x_i | v'_i)$  is strictly increasing in  $x_i$ .

A non-expert does not observe any private signals. Being an expert is private information and the seller and the other buyers assign a probability equal to  $p$  to buyer  $i$  being an expert.

We assume that  $q$  takes one of the following two forms:

- (i) independent private values (IPV):  $(v_1, \dots, v_n)$  are independently and identically distributed;
- (ii) common values in which  $v_1 = v_2 = \dots = v_n$ .

In the case of common values, we assume that there exists a neutral signal in the sense of Milgrom [9]. Let  $q_i(v_i)$  denote the prior density of  $v_i$  and  $q_i(v_i | x_i)$  denote the posterior density.

(A2). In the case of common values, there exists  $x^*$  such that, for any  $v_i \in [\underline{v}, \bar{v}]$ ,  $q_i(v_i) = q_i(v_i | x^*)$ ,  $i = 1, 2, \dots, n$ .

(A2) states that there exists a signal  $x^*$  which is uninformative in the sense that the posterior density of  $v_i$  conditional on  $x^*$  is equal to the prior density. This assumption is equivalent to assuming that there exists  $x^*$  for which  $h(x^* | v_i)$  is independent of  $v_i$ . The latter is straightforward to check.

Notice that (A2) is always satisfied when  $v_i$  takes only two values,  $\underline{v}$  or  $\bar{v}$ , where  $\underline{v} < \bar{v}$ , since strict MLRP implies that there exists a unique  $x^*$  such that  $h(x^* | \bar{v}) = h(x^* | \underline{v})$ . If  $v_i$  takes more than two values, (A2) imposes significant restrictions on the conditional distributions of signals. The following example provides a class of distributions which satisfies (A2). Consider

$$h(x_i | v_i) = (v_i - E[v_i])(x_i - 1/2) + 1$$

for  $x_i \in [0, 1]$ , where  $E[\cdot]$  denote the expectation operator. Clearly,  $h$  satisfies MLRP and  $x^* = 1/2$  is neutral signal.

### 3. EQUILIBRIUM

In this section, we analyze the equilibrium bidding strategies for the cases of IPV and common values. The assumption on neutral signals enables us to show the existence of equilibrium bidding strategies with a simple structure. We first introduce some additional notation. Consider bidder 1 and define

$$w_k(x, y) = E[v_1 | x_1 = x, \max\{x_2, x_3, \dots, x_{k+1}\} = y], \quad k = 1, \dots, n-1,$$

to be bidder 1's expected value of the object conditional on  $x_1$  being equal to  $x$  and the largest signal for  $k$  expert bidders being equal to  $y$ . We can define  $w_k$  as independent of the particular choice of bidder 1 and bidders  $\{2, 3, \dots, k+1\}$  because of the symmetry assumption. Also define

$$w_k(y) = E[v_1 | \max\{x_2, x_3, \dots, x_{k+1}\} = y], \quad k = 1, \dots, n-1,$$

to be bidder 1's expected value of the object conditional only on the largest signal for  $k$  expert bidders being equal to  $y$  and

$$\bar{w}(x) = E[v_1 | x_1 = x]$$

to be the expected value of the object conditional on  $x_1$  being equal to  $x$  only. Let  $F_k(y | x)$  denote the cumulative distribution function of the largest signal in  $\{x_2, x_3, \dots, x_{k+1}\}$  given that bidder 1's signal is  $x$ . The corresponding density function is denoted by  $f_k(y | x)$ .  $F_k(y)$  and  $f_k(y)$  denote the unconditional distribution and density function, respectively.

In the case of IPV,  $F_k(y | x) = F_k(y)$  and  $w_k(x, y) = \bar{w}(x)$  for any  $x, y \in [a, z]$  and  $k = 1, \dots, n - 1$ . Also notice that there exist a unique  $x^* \in (a, z)$  such that  $\bar{w}(x^*) = E[v_1]$ . These properties turn out to be very useful for the existence of a symmetric equilibrium with a simple structure. In the case of common values, (A2) ensures that similar properties are satisfied. We summarize these properties in the following simple lemma which we state without proof.

LEMMA 1. *There exists a unique  $x^* \in (a, z)$  such that  $F_k(y | x^*) = F_k(y)$  and  $w_k(x^*, y) = w_k(y)$  for  $y \in [a, z]$  and  $k = 1, \dots, n - 1$  and  $\bar{w}(x^*) = E[v_1]$ .*

We shall also use the following lemmas the proofs of which are provided in the Appendix. Let  $P_k = p^k(1 - p)^{n-1-k} (n - 1)!/[k!(n - k - 1)!]$  be the probability that a bidder faces  $k$  expert bidders, and define

$$F(y | x) = \sum_{k=1}^{n-1} P_k F_k(y | x) + P_0, \quad f(y | x) = \sum_{k=1}^{n-1} P_k f_k(y | x),$$

$$F(y) = \sum_{k=1}^{n-1} P_k F_k(y) + P_0, \quad f(y) = \sum_{k=1}^{n-1} P_k f_k(y),$$

and

$$W(x, y) = \sum_{k=1}^{n-1} P_k w_k(x, y) f_k(y | x)/f(y | x).$$

LEMMA 2. (i) *For any  $y \in (x^*, z]$ ,  $F(y | x)/f(y | x)$  is non-increasing in  $x$ ,  $x \in [a, z]$ ; and (ii)  $W(x, y)$  is strictly increasing in  $x$ ,  $x \in [a, z]$ , and non-decreasing in  $y$ ,  $y \in (x^*, z]$ .*

LEMMA 3. (i)  $\int_a^x w_k(x, y) f_k(y | x) dy/F_k(x | x)$  is non-increasing in  $k$  and strictly increasing in  $x$  for  $x \in [a, z]$ ; and (ii)

$$\left[ \sum_{k=1}^{n-1} P_k \int_a^{x^*} w_k(y) f_k(y) dy + P_0 \bar{w}(x^*) \right] / F(x^*) \leq W(x^*, x^*).$$

Consider now  $x^*$  in Lemma 1 and define a bidding function for expert bidders as follows. First, let

$$B(x) = w_{n-1}(x, x) - \int_a^x \theta(t) dw_{n-1}(t, t) / \theta(x) \tag{1}$$

for  $x \leq x^*$ , where  $\theta(x) = \exp\{\int_a^x f_{n-1}(t | t) / F_{n-1}(t | t) dt\}$ . Next, let  $b_1 = B(x^*)$  and  $b_2$  be the solution of

$$\begin{aligned} & \sum_{k=1}^{n-1} P_k \int_a^{x^*} [w_k(y) - b_2] f_k(y) dy + P_0 [\bar{w}(x^*) - b_2] \\ &= P_{n-1} \int_a^{x^*} [w_{n-1}(y) - b_1] f_{n-1}(y) dy. \end{aligned} \tag{2}$$

By Lemma 3,  $b_1 \leq b_2$  and  $b_1 < b_2$  if  $p < 1$ . Now define

$$B(x) = W(x, x) - \left[ \int_{x^*}^x \sigma(t) dW(t, t) - b_2 + W(x^*, x^*) \right] / \sigma(x) \tag{3}$$

for  $x \geq x^*$ , where  $\sigma(x) = \exp\{\int_{x^*}^x f(t | t) / F(t | t) dt\}$ .

A non-expert randomizes his bid according to a cumulative distribution,  $G(b)$ , which solves the polynomial equation

$$\begin{aligned} & \sum_{k=1}^{n-1} P_k G(b)^{n-1-k} \int_a^{x^*} [w_k(y) - b] f_k(y) dy + P_0 G(b)^{n-1} [\bar{w}(x^*) - b] \\ &= P_{n-1} \int_a^{x^*} [w_{n-1}(y) - b_1] f_{n-1}(y) dy \end{aligned} \tag{4}$$

for any  $b \in [b_1, b_2]$ . Since  $b_1 \leq b_2$ , a solution  $G(b)$  always exists. Lemma 3 guaranties uniqueness of the solution since the left-hand side of (4) is strictly increasing in  $G$  at the solution. The continuity and strict monotonicity of  $G(b)$  are straightforward.

The following proposition shows that these bidding functions are an equilibrium.

**PROPOSITION 1.** *There exists an equilibrium in which an expert bidder submits his bid according to  $B(x)$  and a non-expert bidder randomizes his bid according to the cumulative distribution  $G(b)$ .*

*Proof.* See Appendix.

In the case of common values, the existence of a neutral signal is crucial for the proof of Proposition 1. First, non-expert bidders can be treated as expert bidders observing signal  $x^*$ . This allows us to proceed as if all bidders are privately informed and the conditional distribution of signals has a mass point at  $x^*$ . Second, the monotonicity of  $F(y | x)/f(y | x)$  in  $x$  and  $W(y, y)$  in  $y$ , for  $y \geq x^*$ , ensures that the bidding functions defined in (1)–(4) are an equilibrium. This argument does not carry over to a more general model in which non-expert bidders observe a common signal  $\bar{x} \in [a, z]$  which is not necessarily neutral. In this case,  $F(y | x)/f(y | x)$  may be increasing in  $x$  and  $W(y, y)$  decreasing in  $y$  for some  $y \geq \bar{x}$  and the argument of Proposition 1 does not apply. Consider the following example. Let  $H(x | v) = x^v$  for  $x \in [0, 1]$ ,  $v \in \{1, 2\}$ ,  $q(1) = q(2) = 1/2$ , and let  $\bar{x} = 0$ . Notice that this is the case of an auction model where the number of participants is stochastic. It is easy to verify that  $H(x | v)$  satisfies MLRP. However,  $F(0.2 | x)/f(0.2 | x)$  is increasing in  $x$  for  $p = 0.5$  and  $n = 4$ . Thus,  $x_i$  is not affiliated with the maximum signal across other bidders.<sup>1</sup> One can also verify that  $W(y, y)$  is decreasing in  $y$  at  $y = 0.1$  for  $p = 0.9$  and  $n = 12$ .

#### 4. OPTIMAL RESERVE PRICES AND COMPARATIVE STATICS

In this section, we allow the seller to use a reserve price. The equilibrium structure is a straightforward extension of the one we provide in the previous section. We describe it briefly. Let

$$r_0 = \int_a^{x^*} w_{n-1}(y) f_{n-1}(y) dy / F_{n-1}(x^*)$$

be the expected valuation of buyer  $i$  conditional on the largest signal among  $n - 1$  expert buyers being smaller than  $x^*$  and

$$\bar{r} = \left\{ \sum_{k=1}^{n-1} P_k \int_a^{x^*} w_k(y) f_k(y) dy + P_0 w_0(x^*) \right\} / F(x^*)$$

be the expected valuation of buyer  $i$  conditional on the largest signal among other expert buyers being smaller than  $x^*$ . Lemma 3 implies that  $r_0 \leq \bar{r}$ . In the case of IPV,  $r_0 = \bar{r} = E[v_1]$ .

<sup>1</sup> This affiliation property is assumed by McAfee and Vincent [6] in a common values model with a stochastic number of bidders. Thus, their model is consistent with ours when only bidders with signals above  $x^*$  participate in the auction.

Consider first a reserve price  $r \in [w_{n-1}(a, a), r_0]$ . Define  $\alpha$  to be the signal in  $[a, x^*]$  for which  $\int_a^\alpha w_{n-1}(\alpha, y) f_{n-1}(y | \alpha) dy = r F_{n-1}(\alpha | \alpha)$ . Then, the symmetric equilibrium bidding function is given by

$$B_r(x) = w_{n-1}(x, x) - \left[ \int_\alpha^x \theta_r(t) dw_{n-1}(t, t) - r + w_{n-1}(\alpha, \alpha) \right] / \theta_r(x)$$

for  $\alpha \leq x \leq x^*$ , where  $\theta_r(x) = \exp\{\int_\alpha^x f_{n-1}(t | t) / F_{n-1}(t | t) dt\}$ .

$B_r(x)$ ,  $x > x^*$ , and the equilibrium strategy of the non-experts,  $G_r(b)$ , are determined by (2)–(4) with  $b_1 = B_r(x^*)$ .

Suppose that  $r \in [r_0, \bar{r}]$ . In the symmetric equilibrium, expert bidders submit no bid if  $x < x^*$  and bid according to  $B_{r_0}(x)$  if  $x \geq x^*$ . Non-expert bidders submit a bid with probability  $1 - G_{r_0}(r)$  according to the cumulative distribution  $[G_{r_0}(b) - G_{r_0}(r)] / [1 - G_{r_0}(r)]$  and submit no bid with probability  $G_{r_0}(r)$ . In this equilibrium, non-expert bidders make zero expected profits.

Suppose that  $r > \bar{r}$ . Define  $\beta$  to be the signal for which

$$\sum_{k=1}^{n-1} P_k \int_a^\beta w_k(\beta, y) f_k(y | \beta) dy + P_0 \bar{w}(\beta) = r F(\beta | \beta).$$

Then, only expert bidders with  $x \geq \beta$  submit a bid and according to

$$B_r(x) = W(x, x) - \left[ \int_\beta^x \sigma_r(t) dW(t, t) - r + W(\beta, \beta) \right] / \sigma_r(x),$$

where  $\sigma_r(x) = \exp\{\int_\beta^x f(t | t) / F(t | t) dt\}$ .

The seller chooses a reserve price to maximize her expected revenue. For a given  $p$ , let  $\Gamma(p)$  be the seller's expected revenue at the optimal reserve price. We are interested in how changes in  $p$  affect  $\Gamma(p)$ .

Consider first the IPV case. To simplify the notation, let the buyer's valuation be his signal  $x_i$  and assume that signals are drawn independently and identically from a cumulative distribution  $\Phi(x_i)$  with support  $[a, z]$  and density  $\phi$ . Thus,  $x^*$  is the expected value of  $x_i$ . Given a reserve price  $r \leq x^*$ , the seller's expected revenue is

$$\begin{aligned} & \sum_{k=0}^n C_k \Phi(x^*)^k \int_{b_1}^{b_2} b dG(b)^{n-k} \\ & + \sum_{k=0}^{n-1} C_k \int_{x^*}^z B_r(x) d\Phi(x)^k + p^n \int_r^{x^*} B_r(x) d\Phi(x)^n, \end{aligned}$$

where  $C_k = p^k(1-p)^{n-k} n!/[k!(n-k)!]$ . Tedious calculations show that the first term is equal to

$$x^*[1-p+p\Phi(x^*)]^n - x^*p^n\Phi(x^*)^n - (x^* - b_1)n(1-p)p^{n-1}\Phi(x^*)^{n-1},$$

the second term is equal to

$$\int_{x^*}^z I(x) d[1-p+p\Phi(x)]^n - (x^* - b_1)np^n\Phi(x^*)^{n-1}(1-\Phi(x^*)),$$

where  $I(x) = x - [1 - \Phi(x)]/\phi(x)$ , and the third term is equal to

$$p^n \int_r^{x^*} I_p(x) d\Phi(x)^n + (x^* - b_1)n(1-p\Phi(x^*))p^{n-1}\Phi(x^*)^{n-1},$$

where  $I_p(x) = x - [1 - p\Phi(x)]/(p\phi(x))$ . Thus, the seller's expected revenue is

$$\begin{aligned} &x^*[1-p+p\Phi(x^*)]^n - p^n x^* \Phi(x^*)^n \\ &+ p^n \int_r^{x^*} I_p(x) d\Phi(x)^n + \int_{x^*}^z I(x) d[1-p+p\Phi(x)]^n. \end{aligned}$$

If  $r > x^*$ , the seller's expected revenue is

$$\sum_{k=0}^n c_k \int_r^z B_r(x) d\Phi(x)^k = \int_r^z I(x) d[1-p+p\Phi(x)]^n.$$

The optimal reserve price is either in  $I_p^{-1}(0)$ , or in  $I^{-1}(0)$ , or equal to  $x^*$ . In particular, when  $p$  is small, the optimal reserve price is  $x^*$ . In this case, the non-expert bidders make zero expected payoffs and the seller extracts all the surplus from them. The ex ante expected payoff for an expert bidder is always positive.

Two factors affect the seller's expected revenue: information rents and competition. As  $n$  increases, competition among bidders reduces the information rents and increases expected revenue. As  $p$  increases, bidders become more privately informed and extract higher information rents from the seller. However, if  $n$  is large, a rise in  $p$  increases the chances that a high bid will be submitted. The following proposition shows that the competition effect dominates for  $p$  small.

**PROPOSITION 2.** *In the IPV model,  $\Gamma(p)$  is increasing in  $p$  for  $p$  sufficiently small and  $n \geq 3$ .*

*Proof.* The claim follows by noting that, for  $n \geq 3$ ,

$$\Gamma'(0) = -nx^*(1 - \Phi(x^*)) + n \int_{x^*}^z I(x) d\Phi(x) = 0,$$

and

$$\begin{aligned} \Gamma''(0) &= n(n-1)x^*(1 - \Phi(x^*))^2 - 2n(n-1) \int_{x^*}^z I(x)(1 - \Phi(x)) d\Phi(x) \\ &= n(n-1) \int_{x^*}^z [1 - \Phi(x)]^2 dx > 0. \end{aligned} \quad \text{Q.E.D.}$$

The intuition behind this result is as follows. Recall that for  $p$  small and positive the optimal reserve price is  $x^*$ . Non-expert bidders then provide a lower bound of  $x^*$  for the winning bid and, if at least one bidder is an expert, there is a positive probability that the winning bid is above  $x^*$ . Thus, the realization of the seller's revenue can be lower than  $x^*$  only if all bidders are experts. The probability of this event is  $p^n$ , which is of no more than a third order magnitude  $p^n$  when there are at least 3 bidders and  $p$  is arbitrarily small.

To see how  $\Gamma(p)$  changes with respect to  $p$  for any  $p$ , we consider the following example in which  $\Phi(x_i)$  is the uniform distribution on  $[0, 1]$ . It can be shown that the optimal reserve price is always  $x^*$  which is equal to  $1/2$ . Then, for any  $p$ ,  $\Gamma(p) = 0.5(1 - 0.5p)^n - (0.5p)^n + 1 - 2[1 - (1 - 0.5p)^{n+1}]/[p(n+1)]$ . Simulation shows that (i) when  $n=2$ ,  $\Gamma(p)$  is always decreasing in  $p$ ; (ii) when  $n=3$  and  $4$ ,  $\Gamma(p)$  increases with  $p$  initially and decreases with  $p$  for  $p$  large; and (iii) when  $n > 4$ ,  $\Gamma(p)$  always increases with  $p$ . In this case, a non-expert bidder makes zero expected payoff. The ex ante expected payoff for an expert bidder is  $[1 - (1 - 0.5p)^n(1 + np)]/[n(n+1)p^2]$ , which is positive and always decreases with  $p$ .

In the case of common values, an additional factor dominates the effects illustrated above. For small  $p$ , the winner's curse always causes the seller's expected revenue to be decreasing in  $p$ . This phenomenon becomes more severe as the degree of competition rises.

**PROPOSITION 3.** *In the case of common values,  $\Gamma(p)$  is decreasing in  $p$  for  $p$  sufficiently small.*

The intuition behind Proposition 3 is as follows. An increase in  $p$  causes non-expert bidders to decrease their bids since the chances that other bidders are privately informed are higher. The total expected revenue for the seller is decreasing in  $p$  since any gains from the more informed bidders are of smaller magnitudes. When  $n$  is large, the expected revenue declines faster with  $p$  since the winner's curse becomes more severe.

5. CONCLUSION

In this paper, we have analyzed a simple model of expert and non-expert bidding. The existence of a “neutral” signal,  $x^*$ , allows us to provide simple equilibrium bidding functions in which expert and non-expert bidders choose bids from distinct supports. We have shown that, for the case of common values, an increase in the likelihood of expert participation in the auction may decrease the revenue of the seller. We believe that this result will extend to more general environments. The opposite result has been shown for the case of independent private values.

In our model, we have assumed that the probability  $p$  of being informed is exogenous. Adding an unobservable move of costly information acquisition prior to the auction stage,  $p$  can be generated in a symmetric mixed strategy equilibrium by imposing the condition that the ex ante expected profits of the informed minus the cost of becoming informed are equal to the ex ante expected profits of the uninformed. We do not provide the details here.

APPENDIX

*Proof of Lemma 2.* In the case of IPV the claim is straightforward. For the case of common values, we first show that  $F(y|x)/f(y|x)$  is non-increasing in  $x$ ,  $x \in [a, z]$ , for  $y > x^*$ . For  $j = 1, 2, 3$ , define

$$\psi_j(v) = \frac{[1 - p + pH(y|v)]^{n-j} h(y|v)^{j-1} h(x|v) q_i(v)}{\int [1 - p + pH(y|v)]^{n-j} h(y|v)^{j-1} h(x|v) q_i(v) dv}.$$

Notice that  $F_k(y|x) = \int H(y|v)^k h(x|v) q_i(v) dv / \int h(x|v) q_i(v) dv$  and  $\sum_{k=0}^{n-1} P_k H(y|v)^k = [1 - p + pH(y|v)]^{n-1}$ . Thus,

$$F(y|x) = \int [1 - p + pH(y|v)]^{n-1} h(x|v) q_i(v) dv \bigg/ \int h(x|v) q_i(v) dv$$

and  $(n - 1)^{-1} \partial(F/f)/\partial x$  can be written as

$$\left\{ \int \psi_1(v) \frac{h_x(x|v)}{h(x|v)} dv \int \psi_1(v) \frac{ph(y|v)}{1 - p + pH(y|v)} dv - \int \psi_1(v) \frac{h_x(x|v)}{h(x|v)} \frac{ph(y|v)}{1 - p + pH(y|v)} dv \right\} \frac{F^2}{f^2}.$$

(A1) implies that  $h_x(x|v)/h(x|v)$  is increasing in  $v$  and that  $H(y|v)$  is non-increasing in  $v$ . Moreover, (A1) and (A2) imply that, for  $y > x^*$ ,

$h(y|v)$  is increasing in  $v$ . Thus, by the Tchebycheff inequality,  $F(y|x)/f(y|x)$  is non-increasing in  $x$  for any  $y > x^*$ .

To show that  $W(x, y)$  is strictly increasing in  $x$ , notice that

$$w_k(x, y) = \frac{\int vkH(y|v)^{k-1} h(y|v) h(x|v) q_i(v) dv}{\int kH(y|v)^{k-1} h(y|v) h(x|v) q_i(v) dv}$$

and  $\sum_{k=1}^{n-1} kP_k H(y|v)^k = [1 - p + pH(y|v)]^{n-2}$ . It follows that

$$W(x, y) = \frac{\int v[1 - p + pH(y|v)]^{n-2} h(y|v) h(x|v) q_i(v) dv}{\int [1 - p + pH(y|v)]^{n-2} h(y|v) h(x|v) q_i(v) dv}$$

and that  $\partial W(x, y)/\partial x$  can be written as

$$\int \psi_2(v) v \frac{h_x(x|v)}{h(x|v)} dv - \int \psi_2(v) v dv \int \psi_2(v) \frac{h_x(x|v)}{h(x|v)} dv.$$

The claim again follows by strict MLRP and the Tchebycheff inequality.

To show that  $W(x, y)$  is non-decreasing in  $y$  for  $y > x^*$ , given the previous argument, it suffices to show that

$$\frac{\int v[1 - p + pH(t|v)]^{n-2} h(y|v) h(x|v) q_i(v) dv}{\int [1 - p + pH(t|v)]^{n-2} h(y|v) h(x|v) q_i(v) dv}$$

is non-decreasing in  $t$  at  $t = y$ ,  $y > x^*$ . Thus, it is sufficient to show that

$$\int v\psi_3(v) dv \int \psi_3(v) \frac{1 - p + pH(y|v)}{h(y|v)} dv \geq \int v\psi_3(v) \frac{1 - p + pH(y|v)}{h(y|v)} dv.$$

Since  $h(y|v)$  is non-decreasing in  $v$  for  $y \geq x^*$  and  $H(y|v)$  is non-increasing in  $v$ , the claim follows by the Tchebycheff inequality. Q.E.D.

*Proof of Lemma 3.* The case of IPV is straightforward. For common values, (i) follows from [10, Theorem 5]. To prove (ii), notice that

$$W(x^*, x^*) = \frac{\int v[1 - p + pH(x^*|v)]^{n-2} q_i(v) dv}{\int [1 - p + pH(x^*|v)]^{n-2} q_i(v) dv}$$

and

$$\begin{aligned} & \frac{\sum_{k=1}^{n-1} P_k \int_a^{x^*} w_k(y) dF_k(y) + P_0 \bar{w}(x^*)}{F(x^*)} \\ &= \frac{\int v[1 - p + pH(x^*|v)]^{n-1} q_i(v) dv}{\int [1 - p + pH(x^*|v)]^{n-1} q_i(v) dv}. \end{aligned}$$

The claim then follows by applying Tchebycheff inequality as in the Proof of Lemma 2. Q.E.D.

*Proof of Proposition 1.* Consider the strategy defined in (1)–(4). First one can verify that the bidding function in (1) satisfies the first-order condition

$$B'(x) F_{n-1}(x | x) = [w_{n-1}(x, x) - B(x)] f_{n-1}(x | x) \tag{5}$$

with  $B(a) = w_{n-1}(a, a)$ , and the bidding function in (3) satisfies the first-order condition

$$B'(x) F(x | x) = [W(x, x) - B(x)] f(x | x) \tag{6}$$

with  $B(x^*) = b_2$ .

Since  $w_{n-1}(t, t)$  is strictly increasing in  $t$  by MLRP,  $B(x)$  is strictly increasing and continuous for  $x \leq x^*$ . By standard arguments, strict MLRP implies that the right-hand-side of (2) is positive. Hence,

$$b_2 < \left[ \sum_{k=1}^{n-1} P_k \int_a^{x^*} w_k(y) f_k(y) dy + P_0 \bar{w}(x^*) \right] / F(x^*) \leq W(x^*, x^*)$$

where the last inequality follows from Lemma 3. Since  $W(t, t)$  is strictly increasing in  $t$  for  $t > x^*$  by Lemma 2,  $B(x)$  is strictly increasing and continuous for  $x \geq x^*$  as well.

Now, let  $u(x, b)$  denote the expected payoff of bidder 1 given that bidder 1 is an expert, observes  $x$  and submits  $b$ , and all other bidders choose  $[G(b), B(x)]$ . By standard arguments, we only need to show that

$$u(x, B(x)) \geq u(x, b) \tag{7}$$

for  $x \in [a, z]$  and  $b \in [B(a), B(z)]$ . Notice that  $u(x, b)$  can be written as

$$P_{n-1} \int_a^{B^{-1}(b)} [w_{n-1}(x, y) - b] f_{n-1}(y | x) dy$$

if  $B(a) \leq b \leq b_1$ ,

$$\sum_{k=1}^{n-1} P_k G(b)^{n-1-k} \int_a^{x^*} [w_k(x, y) - b] f_k(y | x) dy + P_0 G(b)^{n-1} [\bar{w}(x) - b]$$

if  $b_1 < b < b_2$ , and

$$\sum_{k=1}^{n-1} P_k \int_a^{B^{-1}(b)} [w_k(x, y) - b] f_k(y | x) dy + P_0 [\bar{w}(x) - b]$$

if  $b_2 \leq b \leq B(z)$ .

By Lemma 1, the expected payoff of a non-expert from submitting  $b$  is  $u(x^*, b)$ .

Suppose that  $x \leq x^*$ . It follows from [10, Theorem 14] that  $u(x, b)$  is a single-peaked function in  $b$  for  $B(a) \leq b \leq b_1$  and reaches the maximum at  $b = B(x)$ . Thus, (7) is satisfied for  $B(a) \leq b \leq b_1$ .

If  $b_1 < b < b_2$ , consider first the case of private values. One can easily show that  $\partial u(x, b)/\partial b$  is non-decreasing in  $x$ . Since  $u(x^*, b)$  is constant for  $b \in [b_1, b_2]$  by (4), it follows that  $\partial u(x, b)/\partial b \leq \partial u(x^*, b)/\partial b = 0$ . Thus,  $u(x, b) \leq u(x, b_1) \leq u(x, B(x))$ . For the case of common values,  $u(x, b)$  can be written as

$$u(x, b) = \frac{\int (v - b)[(1 - p)G(b) + pH(x^* | v)]^{n-1} h(x | v) q_i(v) dv}{\int h(x | v) q_i(v) dv}.$$

By arguments similar to the Proof of Lemma 2, one can show that MLRP and the Tchebycheff inequality imply that  $\partial \log[u(x, b)]/\partial b$  is non-decreasing in  $x$ . Since  $\partial u(x^*, b)/\partial b = 0$ , it follows that  $u(x, b) \leq u(x, b_1) \leq u(x, B(x))$ .

If  $b_2 \leq b \leq B(z)$ , consider  $\hat{x} \geq x^*$  such that  $b = B(\hat{x})$ . Then

$$\begin{aligned} \frac{\partial u(x, b)}{\partial b} &= \frac{f(\hat{x} | x)}{B'(\hat{x})} \left\{ W(x, \hat{x}) - B(\hat{x}) - \frac{B'(\hat{x}) F(\hat{x} | x)}{f(\hat{x} | x)} \right\} \\ &\leq \frac{f(\hat{x} | x)}{B'(\hat{x})} \left\{ W(\hat{x}, \hat{x}) - B(\hat{x}) - \frac{B'(\hat{x}) F(\hat{x} | \hat{x})}{f(\hat{x} | \hat{x})} \right\} = 0 \end{aligned}$$

where the inequality follows from Lemma 2 and the last equality follows from (6) at  $x = \hat{x}$ . Since  $B(x^*_+) = b_2$ , (7) holds for all  $b$ .

For the case  $x > x^*$ , the proof is analogous and is not provided.

If the bidder is a non-expert, it follows by (4) that  $u(x^*, b) = u(x^*, b_1)$  for  $b \in [b_1, b_2]$ . Thus, the above argument implies that  $G(b)$  is a best reply. Q.E.D.

*Proof of Proposition 3.* First,  $r_0$  is an optimal reserve price at  $p = 0$ . Second, for  $r \in (r_0, \bar{r})$ , non-experts bid according to  $G_{r_0}(b)$  truncated below at  $r$  and the bidding of experts is independent of  $r$ . Thus, there exists  $\delta > 0$  such that if  $p \in (0, \delta)$  then the optimal reserve price is less than or equal to  $r_0$ . Third, for  $r \leq r_0$ ,  $b_2 \leq \bar{r}$  implies that  $B_r(z)$  is founded from above by

$$\bar{B}(z) = W(z, z) - \left[ \int_{x^*}^z \sigma_r(t) dW(t, t) - \bar{r} + W(x^*, x^*) \right] / \sigma_r(z)$$

and the bids of the non-experts are bounded from above by  $\bar{r}$ . Then, for  $p \in [0, \delta)$ ,  $\Gamma(p)$  can be bounded from above by

$$\begin{aligned} \bar{\Gamma}(p) &= (1-p)^n \bar{r} + np(1-p)^{n-1} [F^* \bar{r} + (1-F^*) \bar{B}(z)] \\ &\quad + [1 - (1-p)^n - np(1-p)^{n-1}] C, \end{aligned}$$

where  $F^*$  is the unconditional probability of bidder 1 observing a signal lower than  $x^*$  and  $C$  is a positive number and independent of  $p$ . Notice that  $d\bar{B}(z)/dp$  is bounded and  $\lim_{p \rightarrow 0} \bar{B}(z) = \lim_{p \rightarrow 0} \bar{r} = \bar{w}(x^*)$ . Simple calculations show that

$$\bar{\Gamma}'(0) = d\bar{r}/dp|_{p=0} = n \int_a^{x^*} [w_1(y) - \bar{w}(x^*)] dF_1(y) < 0,$$

where the inequality holds due to strict MLRP. The claim follows by noting that  $\Gamma(0) = \bar{w}(x^*) = \bar{\Gamma}(0)$ . Q.E.D.

## REFERENCES

1. M. Baye, A. Gillette, and C. de Vries, Limit orders, asymmetric information, and the formation of asset prices with a computerized specialist, *J. Econ.* **59** (1994), 71–96.
2. R. Engelbrecht-Wiggans, P. Milgrom, and R. Weber, Competitive bidding and proprietary information, *J. Math. Econ.* **11** (1983), 161–169.
3. K. Hendricks and R. Porter, Joint bidding and entry in federal offshore oil and gas lease auctions, University of British Columbia, 1994.
4. K. Hendricks, R. Porter, and C. Wilson, Auctions for oil and gas leases with an informed bidder and a random reservation price, *Econometrica* **62** (1994), 1415–1444.
5. P. McAfee and J. McMillan, Auctions with a stochastic number of bidders, *J. Econ. Theory* **82** (1987), 1–19.
6. P. McAfee and D. Vincent, Updating the reserve price in common-value auctions, *Amer. Econ. Rev.* **82** (1992), 512–518.
7. S. Matthews, Comparing auctions for risk averse buyers: A buyer's point of view, *Econometrica* **55** (1987), 633–646.
8. S. Matthews, Information acquisition in discriminatory auctions, in "Bayesian Models in Economic Theory" (M. Boyer and R. E. Kihlstrom, Eds.), pp. 181–206.
9. P. Milgrom, Good news and bad news: Representation theorems and applications, *Bell J. Econ.* **12** (1981), 380–391.
10. P. Milgrom and R. Weber, A theory of auctions and competitive bidding, *Econometrica* **50** (1982), 1089–1122.