

Expectations of Real Interest Rates and Aggregate Consumption: Empirical Tests

Wayne E. Ferson*

1. Introduction

Recently, the finance literature has included empirical analysis of consumption in asset pricing models based on the cross-equation restrictions implied by optimality of a representative agent's consumption and investment plan.¹ These studies have required some specification of an aggregate utility function, and power (constant relative risk aversion) utility has been predominant. The present paper extends this body of research by including models with constant absolute, as well as constant relative, risk aversion.

Evidence on the empirical magnitude of aggregate risk aversion is important for several reasons. In portfolio models, the relation of asset demand to risk depends on the degree of risk aversion. Aggregate risk aversion is a determinant of the predicted response of consumption to interest rate shifts and of optimal hedging behavior in dynamic asset pricing models. Risk aversion, therefore, influences the structure of prices in futures markets and the term structure of interest rates, to name but two examples. Recent studies of the volatility of stock prices (e.g., [16]) also depend on the magnitude of risk aversion.

This study employs quarterly data on U.S. aggregate consumption and Treasury bill returns for 1947-1980. The results do not appear consistent with the hypothesis of aggregate risk neutrality and they suggest that consumption-based asset pricing models should emphasize nonstationarities of "consumption betas" over intertemporal substitution in consumption.

The paper is organized as follows. Section II discusses the model relating equilibrium interest rates and anticipated real consumption changes. This is a discrete-time model, in the spirit of Breeden's [2] intertemporal asset pricing model. It assumes time-additive, state-independent utility with linear risk toler-

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¹ Recent examples include [21], [22], and [20]. Perhaps the earliest example using interest rates is [4].

ance and joint lognormality of price levels with the marginal utility of real consumption. As special cases, it includes the models of Rubinstein [37], Breeden and Litzenberger [3], Hall [19], and Fama [7], among others.

Section III discusses the econometric methodology used to test models with constant absolute and constant relative risk aversion utility functions. The methodology is a special case of one employed by Gibbons and Ferson [13], with the advantage that strong assumptions beyond those used in the theoretical development are not required.²

Section IV describes the data and Section V presents tests of the consumption models and estimates of aggregate risk aversion. Section VI summarizes the results.

II. The Models

Asset pricing models based on the optimality of consumption-investment plans imply that the stochastic process of consumption is related, through marginal utility, to the distribution of returns on assets. This paper examines a discrete-time model analogous to the continuous-time model of Breeden [2].³ A representative consumer-investor maximizes at each date the expectation of a concave, time-additive, state-independent utility function defined over a vector of dated consumption goods. Asset pricing derives from the first-order conditions for optimality. The time t price of a security paying random amounts \tilde{X}_{t+j} of the numeraire at future dates $t+j$ is given as

$$P_t = \sum_{j=0}^{\infty} \exp(-hj) E \left\{ \tilde{\Phi}_{t+j} \tilde{X}_{t+j} \mid \Omega_t \right\}, \quad (1)$$

$$\tilde{\Phi}_{t+j} = \left[\frac{u_c(\tilde{C}_{t+j})}{u_c(C_t)} \exp[-j\tilde{i}(t, t+j)] \right];$$

where $u_c(C)$ is a marginal utility of "real" consumption, h is the constant rate of pure time preference, and Ω_t is a sufficient statistic for the information available at time t . Expectations are assumed to be rational (i.e., mathematical conditional expectations) and markets efficient in the sense that Ω_t includes at least the true values of constant parameters and the information set Z_t subsequently employed in the tests. The term $\tilde{i}(t, t+j)$ is the j -period average rate of commodity price inflation.⁴ Define the time t price of a bond paying a single numeraire unit with

² Similar techniques have been employed recently by Hall [20] and Hansen and Singleton [22].
³ Similar models have appeared in [30], [14], [16], and others. The model described here is developed in detail by Ferson [8].

⁴ See [1] or [8] for technical discussions of the inflation index and real consumption.

certainly at time $t + j$ to be $\exp(-jr_{j,t})$, where $r_{j,t}$ is the nominal j -period spot rate. Equation (1) implies

$$(2) \quad 1 = E \left\{ \exp(-hj) \left[\frac{u_c(\tilde{C}_{t+j})}{u_c(C_t)} \right] \exp(j[r_{j,t} - \tilde{i}(t, t+j)]) \mid \Omega_t \right\} \quad \forall j, t.$$

In equation (2), $r_{j,t} - \tilde{i}(t, t+j)$ is the uncertain "real" interest rate on a j -period nominal bond.

Equation (2) has several advantages for empirical work. Since the equation must hold for each maturity j , it is legitimate to choose an observation interval and bond maturity without making the assumption that investors use that interval as a planning horizon. In particular, the use of quarterly data does not imply temporal aggregation bias if investors make decisions more frequently. This is important because the consistency and asymptotic efficiency of the estimators described in the next section would not be guaranteed if there were a problem with temporal aggregation bias.

A second advantage of equation (2) is that a partially-informed econometrician, with information Z_t contained in Ω_t , can take the conditional expectation of (2) given Z_t and obtain implications for moments that can be estimated.

Let $K(t) \equiv -\ln u_c(C_t)$ and assume that conditional on information Z_t , the joint distribution of $\{\tilde{K}(t+j) - K(t), \tilde{i}(t, t+j)\}$ is bivariate normal. Applying the normal moment generator to equation (2) then results in the following solutions, where the expectations are conditioned on Z_t .

$$(3) \quad \begin{aligned} r_{j,t} &= h + (1/j) E_t[\tilde{K}(t+j) - K(t)] + E_t[\tilde{i}(t, t+j)] - (1/2)\alpha_{jt}, \\ \alpha_{jt} &\equiv j \left[\text{var} \left\{ \left(\frac{\tilde{K}(t+j) - K(t)}{j} \right) + \tilde{i}(t, t+j) \mid Z_t \right\} \right]. \end{aligned}$$

Note that $K' = -u_{cc}/u_c$ is the coefficient of absolute risk aversion, so $K(t)$ is monotonically increasing in real consumption C_t if individuals are uniformly risk averse. Equation (3) implies that spot interest rates are positively related at each point in time to expected growth rates of a transformation of real consumption and to expected rates of inflation over the interval to bond maturity. Spot rates are negatively related to the conditional variances and the covariance of these terms.

Special cases of equation (3) are of particular interest. Hall [19] argued that the marginal utility of optimal consumption should approximate a random walk. If expected changes in $K(\cdot)$ per unit time are constant, then marginal utility is a trended random walk in the logarithms, and expected "real" interest rates depend only on pure time preference and the second moments in α_{jt} . In this case, changes in expected real returns would be associated only with changes in real consumption and inflation risks.

If investors are risk neutral, then changes in $K(\cdot)$ are zero regardless of the stochastic process followed by consumption. Aside from a Jensen inequality term, expected real returns on Treasury bills would be constant and identical for each maturity.

Any specific utility function $U(\cdot)$ implies a joint probability distribution assumption in equation (3). Equation (3) includes models with constant relative risk aversion and lognormal future real consumption; it also includes constant absolute risk aversion with normal consumption. In Figure (1), the cases investigated in this paper are tabulated. Case (iii) in the figure (log utility) obtains as constant relative risk aversion tends to unity.

Some Utility Functions and Distribution Assumptions Included in Equation (3)

Utility Function	Corresponding Form of $K(t)$	Probability Distribution Assumed for Per Capita Real Consumption
(i) $u(C) = (1/a) \exp(-aC)$ (constant absolute risk aversion equal to "a")	aC	$C \sim$ Normal
(ii) $u(C) = (1-a)^{-1} C^{1-a}$ (constant relative risk aversion equal to "a")	$a \ln C$	$C \sim$ Lognormal
(iii) $u(C) = \ln(C)$	nC	$C \sim$ Lognormal

FIGURE 1

The risk-neutral linear utility function has $a = 0$. The distribution of consumption is irrelevant to this utility, given the mean.

These particular assumptions about utility functions and probability distributions are motivated by several considerations. They provide a manageable set of solutions for empirical analysis and each has been a central feature of several previous studies.⁵ Also, specifying a constant utility function parameter may be more palatable than assuming stability of a reduced-form coefficient (an alternative criticized by Lucas [29]). For example, studies of the permanent income theory using aggregate time series data have typically assumed that the marginal propensity to consume is constant. However, when the real rate of interest is changing over time, there is no reason to expect that the portion of permanent income optimally consumed will be constant.⁶ Such tests might reject the null hypothesis when the permanent income theory is valid because of a changing marginal propensity to consume.

The test methodology employed here is particularly well suited to the many models that assume $\{K(t), t\}$ is normally distributed because, as will be shown in the next section, few additional assumptions are required.

III. Methodology

The following empirical analysis is limited to a three-month forecast period and Treasury bill maturity. It will be convenient to suppress the subscript $j = 1$ in equation (3) and to employ a shorthand notation to refer collectively to the

⁵ Studies that have made use of constant relative risk aversion and lognormality include [32], [3], [20], [16], [33], and [18]. Kraus and Litzenberger [27] and Rubinstein [36] derive results for the log utility case. Stapleton and Subrahmanyam [40] and Grossman [15], among others, have employed the assumptions of constant absolute risk aversion and normal distributions.

⁶ In models with perfect markets, permanent income (stock formulation) is equivalent to wealth. The fraction of wealth consumed is constant only when the consumption model reduces to a single-factor asset pricing model in terms of wealth.

models illustrated in Table 1, although separate tests are conducted. Equation (3) implies

$$(4) \quad \begin{aligned} E\{a\Delta\tilde{C}_{t+1} \mid Z_t\} &= \alpha_1 + E\{\tilde{R}_{t+1} \mid Z_t\}, \\ \alpha_1 &= (1 - 2) \text{var}\{a\Delta\tilde{C}_{t+1} - \tilde{R}_{t+1} \mid Z_t\} - h. \end{aligned}$$

In equation (4), $a\Delta\tilde{C}_{t+1}$ stands for the $K(t+1) - K(t)$ in equation (3). (For the constant absolute risk aversion model, $\Delta\tilde{C}_{t+1}$ equals the change in the level of real consumption; for the constant relative risk aversion case, it stands for changes in logarithms.) The "a" stands for the corresponding risk aversion parameter ($a = 1$ for log utility, $a = 0$ for risk neutrality). By definition, $\tilde{R}_{t+1} \equiv r_{t,t+1} - i(t, t+1)$ is the *ex post* real rate of return on a three-month Treasury bill.

The tests are conditional on the following stochastic specification

$$(5) \quad \begin{aligned} \Delta\tilde{C}_{t+1} &= \alpha_C + \beta'_C Z_t + \tilde{u}_{C,t+1} \\ \tilde{R}_{t+1} &= \alpha_R + \beta'_R Z_t + \tilde{u}_{R,t+1} \\ E(\tilde{u}_{C,t+1} \mid Z_t) &= E(\tilde{u}_{R,t+1} \mid Z_t) = 0; \quad t=0, \dots, T-1. \end{aligned}$$

The regression system (5) presupposes that conditional expectations may be modeled as linear functions on the instrumental variables Z_t . However, fairly general dependencies are allowed because it is only required that some smooth transformation influence the conditional expectation linearly (for example, by Taylor's expansion.) The value of Z_t should be available information at time t and should be correlated with changes in the expectations of economic agents. The linear projection of expectations on Z_t is assumed to have constant coefficients. The assumption that inflation rates and the log of marginal utility of consumption are jointly stationary and normally distributed with Z_t can justify both the form of the theoretical models (4) and the linearity of (5) with constant coefficients. This assumption also implies that the covariance matrix of the errors in (5) is constant (because of the homoscedasticity of the normal distribution), and, therefore, that the intercept α_1 of equation (4) is constant. (This assumption is convenient for exposition but stronger than necessary.)⁷

Rational expectation implies that the error vector should display no autocorrelation (assuming Z_t includes past error terms), because the observation interval corresponds to the bond maturity.⁸

⁷ For example, the error covariance matrix may be changing (e.g., multivariate student t distribution) provided α_1 is constant). Although the estimation procedures employed here do assume a constant covariance matrix, MacCurdy [31] describes procedures for inference when the covariance matrix is changing over time.

⁸ For example, assume $i < j$. Then

$$E(\tilde{u}_{C,t+i}, \tilde{u}_{C,t+j} \mid Z_t) = E(\tilde{u}_{C,t+i}, E(\tilde{u}_{C,t+j} \mid Z_{t+i}) \mid Z_t) = E(\tilde{u}_{C,t+i}, 0 \mid Z_t) = 0$$

The theoretical model (4) implies restrictions on the parameters of model (5). Equation (4) says that conditional expectations of marginal utility changes and of "real" interest rates are linearly related. If the coefficients are constant and if the conditional expectations change, then they must shift proportionately. First, differencing equation (4) implies

$$(6) \quad a\Delta E(\Delta\tilde{C}_{t+1} | Z_t) = \Delta E(\tilde{R}_{t+1} | Z_t).$$

Taking conditional expectations given Z_t of the statistical model (5), first differencing and substituting the resulting expressions into equation (6) implies

$$(7) \quad (a\beta'_C - \beta'_R)\Delta Z_t = 0.$$

Since this must be true for all realizations of ΔZ_t ,

$$(8) \quad a\beta_C = \beta_R.$$

Equation (8) will be called the "proportionality restriction."

Because expectations for R and ΔC must shift proportionately, information relevant for predicting one variable must be relevant for predicting the other, and the impact on each must be proportional. Thus, restriction (8) reduces the number of separate slope parameters in the forecasting equation (5) to one-half the original number. The model is tested by estimating system (5) unconstrained and separately with the proportionality restriction imposed. Imposing the constraint allows estimation of the risk aversion parameter, a . Comparing statistical fit of the two models allows a test of whether the data are inconsistent with the restriction. The risk aversion coefficient is identified if Z_t has at least one element in addition to the intercept. In that case, the utility parameter would be just identified: $a = \beta_R/\beta_C$. Increasing the length of the Z_t vector provides over-identifying restrictions. Smaller information sets utilize fewer over-identifying restrictions and will, therefore, result in less powerful tests.

The estimates of the coefficients of the forecasting equations, β_R and β_C , will be biased for the corresponding "true" coefficients (given Ω_t), unless the information omitted from Ω_t in choosing Z_t happens to be uncorrelated with Z_t . Because of "left-out-variables" bias, the β coefficients will not measure the marginal influence of Z_t on the "market's" expectations. However, the theory still may be tested using Z_t . If the "true" coefficient vectors satisfy a proportionality restriction, then bias in the β_R and β_C due to missing information will be offsetting, so that a proportionality restriction is preserved [8].

Substituting the conditional expectations from (5) and the proportionality restriction (8) into (4) implies

$$(9) \quad \begin{aligned} \alpha_t &= aE(\Delta\tilde{C}_{t+1} | Z_t) - E(\tilde{R}_{t+1} | Z_t) \\ &= (a\beta_C - \beta_R)'Z_t + (a\alpha_C - \alpha_R) \\ &= a\alpha_C - \alpha_R. \end{aligned}$$

Relation (9) identifies the pure time preference parameter, h . (Substitute the definition of α_1 from (4) into (9).)

When the parameter restrictions are substituted into the statistical model (5), the result is a restricted bivariate regression

$$(10) \quad \begin{aligned} \Delta \tilde{C}_{t+1} &= \alpha_C + \beta_C' Z_t + \tilde{u}_{C,t+1} \\ \tilde{R}_{t+1} &= -\alpha_1 + a(\alpha_C + \beta_C' Z_t) + \tilde{u}_{R,t+1} \end{aligned}$$

where a is the risk aversion coefficient and α_1 is defined in equation (4).

The unconstrained bivariate system (5) is a version of Zellner's [42] seemingly unrelated regression model. Since the predictor variables are the same in each equation, ordinary least squares (OLS) equation by equation is an efficient estimation procedure.

Under the null hypothesis, OLS is inefficient relative to a nonlinear multivariate estimation scheme because the number of slope parameters is reduced from $2K$ (in the unconstrained system (5)) to $K + 1$ in system (10). Nonlinear joint generalized least squares (JGLS) produces consistent and asymptotically efficient estimates of the regression coefficients and utility parameter.⁹ Gibbons and Ferson [13] have applied this approach to test different hypotheses about common stock returns.¹⁰

If the null hypothesis is true, the proportionality restriction is not binding and the fit of the multivariate regressions (5) and (10) should be asymptotically equivalent. The restrictions are tested using a likelihood ratio test statistic that is asymptotically distributed as chi-square [39].

$$(11) \quad T \text{Ln} \left\{ \frac{|\hat{\Sigma}_r|}{|\hat{\Sigma}_u|} \right\} \sim \chi^2(K-1)$$

where T is the sample size, K is the number of variables in the information vector, and the chi-square variable has $K - 1$ degrees of freedom. (Recall that imposing the proportionality restriction reduces by $K - 1$ the number of parameters to be estimated.) The matrices $\hat{\Sigma}_r$ and $\hat{\Sigma}_u$ are the estimated restricted and unrestricted residual covariance matrices, respectively, and $|\cdot|$ denotes the determinant function.

IV. The Data

The data include quarterly observations (1947 through 1980) of U.S. real aggregate consumption expenditures per capita, inflation rates, and three-month Treasury bill rates. Two measures of consumption are included: consumer nondurables and nondurables plus services. These consumption data from the De-

⁹ Gallant [12] shows that the JGLS method is almost surely consistent and is asymptotically efficient. (Although his results do not consider the case where the instruments Z_t are lagged dependent variables, the assumption of rational expectations implies that they extend to that case as well.)

¹⁰ All computations were performed using algorithms supplied by SAS Institute, Inc. Constrained estimation did not impose restrictions on the residual covariance matrix implied by the definition of α_1 , because this was not computationally feasible using these algorithms.

partment of Commerce have been employed by many previous writers.¹¹ The implicit price deflator for personal consumption expenditures (PCE) was chosen to measure inflation instead of a consumer price index (CPI). The PCE index employs aggregate expenditure weights, updated quarterly, whereas CPI weights are updated at infrequent intervals.¹² The consumption and PCE data are seasonally adjusted at annual rates.¹³

Two interrelated issues motivated a particular temporal alignment of the data. First, the instrumental variables Z_t must be known at the forecast date. Second, although the data refer to average consumption over an interval, the theory relates a bond price at a specific date to changes in consumption between two points in time. Differences of consumption averages over short intervals should approximate more closely point-to-point changes than do differences of averages taken over longer intervals. Therefore, quarterly changes in consumption were measured using differences over the shortest available observation interval (prior to 1959, the shortest available interval is one quarter; monthly data are available for later periods). Average measured consumption and inflation for a period were used as a proxy for "true" values at the midpoint of the interval, when the Treasury bill price is observed. For example, the difference between average measured consumption during February versus that during May is assumed to represent the change between mid-February and mid-May. This corresponds to the bill observation in mid-February. The first lag of measured consumption and inflation rates may not have been available on the date a Treasury bill price is observed and forecasts are assumed to be made. To minimize the chance of spurious predictive relations, only lags of two or more quarters were employed as predetermined variables.¹⁴

Two sets of predetermined instruments are used for the tests. The first consists solely of lagged values of the dependent variables while the second includes

¹¹ For example, Hall [19] and Grossman and Shiller [16] used nondurables plus services. Flavin [9] and Hall [20] used nondurables.

Aggregate personal consumption and the deflator are published in the National Income and Product Accounts in the *Survey of Current Business* and *Business Statistics* supplements. The data incorporated revisions through the Bureau of Labor Statistics seventh benchmark revision (see the December, 1980 *Survey of Current Business* for details). Treasury bill rates are from Salomon Brothers, *Analytical Record of Yields and Yield Spreads*, and were converted to continuously compounded annual rates. (Prior to the first quarter of 1950, Federal Agency securities are used in lieu of Treasury bills.)

¹² This measure of inflation is, of course, not without error. For example, although the analytical development does not require an explicit assumption of homothetic consumption preferences, this assumption is implicitly embraced by using the PCE deflator to measure inflation.

¹³ Seasonally adjusted data are not necessarily *a priori* preferred, but quarterly data on the PCE deflator are available only in seasonally adjusted form.

¹⁴ Consumption and PCE data are not normally published until the third week after the measurement interval. Also, the U.S. Commerce Department samples certain items at intervals greater than three months and interpolates; this procedure could induce spurious correlation between adjacent quarterly observations. Schwert [38] finds that common stocks may react to announcements of the CPI up to a month after the data are collected. Nonetheless, the approach of admitting only second- and higher-order lags probably results in prediction equations with lower R-squares than might be obtainable. Revised figures (data revised through 1980) are employed) would not have been available at the forecast origin.

per capita real expenditures on consumer durable goods, real returns on stocks and past inflation rates.¹⁵

Expenditures for consumer durable goods will, in part, determine the consumption flow from those goods in future periods. "Stock-adjustment" models of consumer expenditures have long emphasized this relation [23]. A change in the level of expenditures on durables could, therefore, signal future changes in consumption. Past values of inflation commonly have been employed in empirical studies as a proxy for inflationary expectations, and many studies have suggested that expected real returns may change with the level of expected inflation.

Common stock prices have long been regarded as efficient aggregators of economic information. If stock prices represent discounted expected future values, then changes in stock prices should reflect revisions of agents' expectations and thus provide a proxy for relevant new information.

V. Empirical Results

If expectations of changes in the consumption measure and of real returns are constant given Z_t , then the regression coefficients in equation (10) are zero except for an intercept and the risk aversion parameter is not identified.

Autocorrelations of real interest rates and consumption are examined first for evidence of changing expectations. Then the other predetermined variables are considered. Compared to traditional market efficiency studies, a maintained assumption and a tested hypothesis reverse roles. A traditional approach might maintain the assumption that expected returns are constant and view autocorrelated returns as evidence of inefficiency. These tests assume that weak-form efficiency and rational expectations hold; autocorrelation of returns then implies that expected returns are changing. The dynamic asset pricing model is a more interesting hypothesis if constant expectations are rejected.

The first panel of Table 1 shows sample autocorrelations of three-month Treasury bill returns and inflation for the 1953 to 1971 period. Fama [7] examined monthly real returns on Treasury bills over this period using CPI inflation rates; here the PCE deflator measures price changes. Consistent with Fama's findings, the individual sample autocorrelations of the *ex post* real rate are not large relative to their approximate standard errors (none exceeds three standard errors). However, the Box-Pierce adjusted Q statistic provides evidence against the joint hypothesis that all of the true autocorrelations are zero. Even when the first lag of the real rate is not considered (statistics in parentheses), the p -value of the Q statistic is .012.

Other studies have noted significant autocorrelation in real Treasury bill returns for periods other than 1953-1971.¹⁶ The second panel of Table 1 presents

¹⁵ Real stock returns are the Standard and Poor's (S & P) Composite total return index as reported in [26], and updated through 1980 from the S & P *Security Price Index Record*. Returns are measured at the end of the first month of each quarter by subtracting the inflation rate from nominal returns, both on a continuously compounded basis.

Table 6 contains sample means, standard deviations, and autocorrelations for each basic series and their first differences. Table 7 is a sample correlation matrix.

¹⁶ Studies that have rejected the hypothesis of constant expected real returns for Treasury bills include [34], [5], [24], and [25], among others.

evidence of changes in the mean real return for the 1947-1980 period. The second-order sample autocorrelation alone is greater than three approximate standard errors from zero. The Box-Pierce statistic rejects the null hypothesis that the true autocorrelations are zero at any conventional significance level.

Fama [7] chose to omit data from the post-1971 period, arguing that wage and price controls systematically distorted measured price levels. To the extent that spurious autocorrelation in the time series of measured real rates resulted, such patterns also should be reflected in the real consumption series because both employ the PCE price deflator. Note, however, that measured real consumption changes appear virtually uncorrelated over 1947-1980. This suggests that spurious autocorrelation in the PCE deflator does not explain the appearance of autocorrelation in real interest rates. The evidence in Table 1 does not reject the hypothesis that the expected change in consumption is constant for any of the measures of consumption examined.

Tables 2 and 3 extend the investigation to include the additional predetermined information described in Section IV. OLS estimation of system (5) is summarized for the constant relative risk aversion model in Table 2 and for the constant absolute risk aversion model in Table 3. Consumer nondurables plus services measure consumption. Either set of predetermined variables appears to be a significant explainer of variation in real interest rates (p -values of six percent or better in each case). However, only one of the six consumption regressions attains significance at ten percent. (This is the model of constant absolute risk aversion using combined instrumental variables.) The second column of p -values in Tables 2 and 3 presents a bivariate test for constant expectations. This hypothesis is rejected with p -values smaller than 0.08 in five of the six cases, and the p -values are less than .001 when both sets of instruments are combined. Consistent with the autocorrelation evidence, the tables suggest that the rejection of constant expectations is being driven mainly by the interest rate equation. The evidence against constant expected consumption growth is not very strong.

So far, the evidence suggests $\beta_C \approx 0$ and $\beta_R \neq 0$ in equation (5). The apparent lack of sensitivity of this result to the different consumption specifications reinforces the view that aggregate risk aversion will not be easy to estimate precisely.

Other studies have reported predictable patterns in consumption growth rates ($\beta_C \neq 0$). Hansen and Singleton [21], for example, find that monthly real consumption growth rates seem to be partly predictable. There are several possible explanations for the difference in results. First, there is the difference between quarterly data, employed here, and the monthly consumption data used by Hansen and Singleton. Monthly data are more susceptible to spurious correlation, not representative of true consumption. About 40 percent of the items in the monthly PCE deflator, for example, are not sampled monthly but are interpolated based on quarterly or annual figures. Second, this study employs a temporal alignment of the consumption and interest rate data designed to minimize spurious correlation because integrals of consumption, not true point-to-point consumption changes, are measured. Hansen and Singleton align the data in a way that assumes individuals know asset returns for the month when planning their consumption for that month. They also admit the first lagged value of the

TABLE 1
Means, Standard Deviations and Sample Autocorrelations of Interest Rates, Inflation,
and Changes in Real Consumption Expenditures Per Capita*
(Quarterly Data at Annual Rates)

Variable	Mean (%)	Std. Dev. (%)	Sample Autocorrelations:								Q	p-value	
			Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8			
First Quarter 1953 to Second Quarter 1971													
Nominal Interest Rate	3.47	1.62	.94	.86	.78	.70	.61	.53	.47	.43	.30*		*
Inflation Rate	2.31	1.61	.48	.48	.40	.37	.37	.45	.16	.21	.93		*
Ex Post Real Rate	1.16	1.32	.21	.17	.05	.17	.18	.34	-.02	.13	22.0 (18.1)		.005 (.012)
Second Quarter 1947 to Fourth Quarter 1980													
Nominal Interest Rate	4.08	2.62	.91	.85	.80	.74	.65	.58	.53	.49	.566		*
Inflation Rate	3.80	3.27	.65	.60	.53	.39	.32	.23	.19	.22	.203		*
Ex Post Real Rate	0.28	2.54	.40	.34	.24	.12	.06	.06	.08	.06	51.0 (28.4)		* (.0002)
Consumption Growth Rates:													
Nondurables	1.13	4.58	.13	.09	.12	.06	.01	.02	.06	.10	8.0 (5.6)		.434 (.589)
Nondurables & Services	1.80	3.03	.02	-.00	.06	.06	-.05	-.03	.04	-.07	2.5 (2.4)		.936 (.932)
Consumption Changes:													
Nondurables	.037	1.53	-.10	-.14	.12	-.02	.02	-.04	-.08	-.06	8.0 (6.5)		.434 (.479)
Nondurables & Services	1.19	2.08	.08	-.03	.12	.15	.02	.03	.02	.01	6.1 (5.2)		.636 (.634)

* Sample autocorrelations are Box-Jenkins estimates with approximate standard errors in the first and second panels equal to .12 and .09, respectively. Q is the adjusted Box-Pierce statistic (statistics in parentheses omit the first order autocorrelation). Under the null hypothesis that the true autocorrelations are zero, Q is approximately chi-square distributed with 0. (7) degrees of freedom. The p-value of the Q statistic equals the probability that a chi square variable exceeds the observed sample value. Asterisks indicate p-values less than .0001.

TABLE 3
 Summary of Unrestricted Least Squares Estimation of the Constant Absolute Risk Aversion Model Using Quarterly Data for 1947-1980.
 Number of Observations is 127. The Bivariate Regression Has the Form

$$\Delta C_{t+1} = \alpha_C + \beta_C z_t + \tilde{u}_{C,t+1}, \tilde{R}_{t+1} = \alpha_R + \beta_R z_t + \tilde{u}_{R,t+1} \quad \forall t = 0, \dots, T-1$$

where ΔC_{t+1} = change in per capita real expenditures for consumer non-durables and services.
 R_{t+1} = realized real return on 3-month Treasury bill and
 z_t = vector of predetermined variables known at date t .

Predetermined Variables	Dependent Variable	$R^2(a)$	F(b)	p value	Joint(c)	F Statistic	p-value	Durbin H Statistic	Residual Autocorrelations
Inflation Rates, Real Stock Market Returns and Per Capita Expenditures for Consumer Durable Goods at Lags 2, 3, and 4.	ΔC_{t+1}	.01	1.17	.320	1.22	.246	0.40	.03	.11
Values of the Two Dependent Variables at Lags 2, 3, 4, and 5.	R_{t+1}	.04	1.91	.056	1.55	.074	3.63	.30	.17
Lags 2 through 5 of the Two Dependent Variables and Three Lag Values of Inflation Rates, Real Stock Market Returns, and Durables Expenditures.	ΔC_{t+1}	.03	1.40	.203	2.60	<.0001	-0.29	.08	.01
	R_{t+1}	.09	2.20	.032	1.32		2.23	.19	.01
	ΔC_{t+1}	.12	1.81	.028			-0.92	-.08	-.03
	R_{t+1}	.15	1.74	.034			1.32	.11	.02

^a R^2 is the coefficient of determination adjusted for degrees of freedom.
^b F statistic for the significance of the regression; the null hypothesis is that the true β coefficients equal zero for a particular equation.
^c The joint F statistic is for the null hypothesis that $\beta_C = \beta_R = 0$ in the bivariate regression.

monthly consumption variable in their forecasting equation, although it would not have been available because of publication lag.¹⁷

If expected consumption growth is approximately constant ($\beta_C \approx 0$), then risk aversion will be difficult to identify. The condition $\beta_C = 0$ is consistent with the hypothesis that marginal utility of consumption follows a trended random walk with respect to Z_t . In this case, equation (4) implies that expected real returns on Treasury bills given Z_t will be constant except for fluctuations corresponding to changes in variance and the covariance of unexpected real returns with marginal utility. Thus, variation of expected real returns over time would be influenced by changing consumption betas, but would not be associated with intertemporal substitution through fluctuating planned growth rates of consumption. This result implies that explicit treatment of changing consumption betas is important for tests of intertemporal asset pricing, even in the context of Treasury bill returns. It suggests that surprises in inflation are not neutral but represent real risk premiums from the point of view of asset pricing.

The existence of changing expected real returns also rejects aggregate risk neutrality, ($a = 0$), which implies constant expected real returns for any pattern of consumption over time.¹⁸

Tests of the proportionality restriction (8) are conditional not only on the statistical specification of the forecasting equations (5), but also on a constant α_1 parameter in equation (4). Assuming constant pure time preference, α_1 is constant if and only if $\text{var}\{a\Delta\tilde{C}_{t+1} - R_{t+1}|Z_t\}$ is constant. Table 4 tests the hypothesis that this variance is equal in the 1947-1963 and 1964-1980 subperiods, conditional on different hypothetical values of relative risk aversion. The case $a = 0$ is equivalent to a standard F test for homogeneity of the variance of the Table 2 regression errors for the real return equation. It should not be surprising that the null hypothesis is rejected for $a = 0$ given previous evidence that the variance of inflation has not been constant over time [28], [25]. The hypothesis that α_1 is constant is rejected in Table 4 for extreme values of relative risk aversion (e.g., $a = 0$ or $a = 6$). For intermediate values (0.5 and 2.0), the evidence against the null hypothesis is mixed, and for the log utility case ($a = 1$) the hypothesis of constant α_1 is not rejected.

Table 5 presents the likelihood ratio tests of the proportionality restriction $\beta_R = a\beta_C$. Results are shown for the three combinations of predetermined variables in Tables 2 and 3. Both the constant absolute and constant relative risk aversion models are summarized.¹⁹

¹⁷ A comparison of the consumption autocorrelations in Table 1, with versus without the first lagged value, do not indicate that admitting the first lag would substantially alter the insignificant autocorrelations of quarterly consumption growth. Hansen and Singleton [21] do not provide evidence on the effect of ruling out the first lag for monthly data.

¹⁸ Grossman and Shiller [16] have employed similar logic to reject risk neutrality in the stock market. Of course, one could reject risk-neutral behavior in the stock market and not in the Treasury bond market if inflation risk is less important than the wider range of uncertainties to which operating firms are exposed.

¹⁹ An indirect estimate of the time preference parameter, h , is also produced. The six estimates of time preference implied by the two choices of risk aversion \times three choices of Z 's were all positive, but some of the point estimates were too large to be considered reasonable. The range of point estimates for time preference was 3.4 to 36.3 percent per annum.

TABLE 4
 Tests for Homogeneity of $\text{Var}\{a\Delta\tilde{C}_{t+1} - \tilde{R}_{t+1}|Z_t\}$ (Constant α_1 in Equation (4))
 in the 1947.2-1963.4 and 1964.1-1980.4 Subperiods

Pre-determined Conditioning Variables Z_t	Assumed Value of Coefficient of Relative Risk Aversion					F Statistic p-value
	$a=0$	$a=5$	$a=1$	$a=2$	$a=6$	
Real Stock Market Returns and per capita Expenditures for Consumer Durable Goods at Lags 2, 3, and 4.	2.48 .0001	1.65 .022	1.11 .342	0.87 .716	0.81 .799	
values of the Two Dependent Variables at Lags 2, 3, 4, and 5	2.70 *	2.07 .0002	0.93 .619	0.54 .993	0.42 .9997	F Statistic p-value

* Indicates the p-value is less than .0001. The measure of consumption is per capita real expenditures for consumer nondurables and services; quarterly differences in the natural logarithms are multiplied by 400. Real 3-month Treasury bill returns are continuously compounded at annual rates.

The constant relative risk aversion model is rejected with p -value .026 using lagged dependent variables as instruments (second panel), but using the set of instruments with no lagged dependent variables the p -value is .294.

When constant absolute risk aversion is assumed, the p -value of the chi-square statistic is less than .001 unless lagged dependent variables are excluded; then the significance level is 16 percent. The evidence against the null hypothesis is slightly stronger assuming constant absolute rather than constant relative risk aversion, and appears fairly strong for both models when the combined set of predictor variables is employed.

The estimates of absolute risk aversion in Table 5 exclude, with some confidence, values of an aggregate coefficient greater than 10 basis points for each (1972) dollar change in real consumption per capita. The relative risk aversion estimates appear significantly different from the logarithmic utility ($a = 1$) in two out of three cases, but these point estimates are in the region of convex (risk-loving) utility for consumption.

These estimates, together with the likelihood ratio tests and autocorrelation evidence, suggest a misspecification of the model. Conditional risks may be changing in important ways over time. It is also possible that changes in higher moments of consumption are driving the changes in expected real returns, but the model is not capturing moments beyond the second. If so, the assumption of lognormal marginal utility, employed in many papers for analytical convenience, may be inappropriate for empirical applications. Recent results reported by Brown and Gibbons [4] suggest that a utility-based model may be sensitive to deviations from the assumed probability distribution, but the general flavor of our results is quite similar for both the normal and the lognormal consumption models.

An alternative explanation, suggested by Brown and Gibbons, is that risk aversion is not precisely identified in these models apparently because quite different point estimates in statistical terms are not really that different in economic

TABLE 5
Likelihood Ratio Tests of the Proportionality Restriction (Equation (8)) and
Estimates of Risk Aversion Parameters.
(Quarterly data for 1947-1980. Number of observations is 127)

Predetermined Independent Variables ^a	Estimated Risk Aversion ^b Coefficients for Models with Constant		χ^2 df ^(c)	p-value ^(e)
	Relative Risk Aversion	Absolute Risk Aversion ^c		
Inflation Rates, Real Stock Market Returns and per capita Expenditures for Consumer Durable Goods at Lags 2, 3, and 4	5.38 (13.80)	2.68 (2.35)	9.6 8	.294
Values of the Two Dependent Variables at Lags 2, 3, 4, and 5	-0.38 (0.24)	1.84 (1.10)	15.9 7	.026
Lags 2 through 5 of the Two Dependent Variables and Three Lag Values of Inflation Rates, Real Stock Market Returns, and Durable Expenditures	-1.41 (0.51)	-2.79 (2.66)	29.5 16	.021
			40.6 16	.001

^a The measure of consumption is per capita real expenditures for consumer nondurables and services. Quarterly consumption changes are measured in the levels for the constant absolute risk aversion model and in the natural logarithms for constant relative risk aversion.

^b Asymptotic standard errors of the risk aversion estimates are in parentheses.

^c Absolute risk aversion estimates and their standard errors have been scaled by the ratio of average total per capita real expenditures for consumer nondurables plus services to the sum of these plus estimated consumption from the stock of durables plus real government expenditures for goods and services (from annual Commerce Department data). Units are basis points per (1972) dollar change in consumption per capita, at annual rates.

^d Degrees of freedom for the chi-square statistic.

^e * indicates the p-value is less than .0001.

terms. This is consistent with the hypothesis of Hall (19), [20] that expected real interest rates do not change very much in association with fluctuations in planned consumption growth rates. Since no previous study of which the author is aware has attempted to estimate a coefficient of constant absolute risk aversion using consumption, comparisons are not possible.

Several studies, however, have attempted to estimate risk aversion in constant relative risk aversion models. Friend and Blume (10) and Friend and Hasbrouck (11) estimate relative risk aversion for wealth, obtaining values near 2.0 and 6.0, respectively, depending on what is included in wealth. (Risk aversion for consumption and wealth are equivalent in general only for single period models). Grossman and Shiller (16), assuming a perfectly predictable stock mar-

ket in a consumption model, suggest that relative risk aversion is about 4.0. Hall [20] obtains values ranging from -13.3 to 25.6 using annual 1953-1979 consumption. Hansen and Singleton [21] obtain point estimates of relative risk aversion between .16 and 4.11. The latter two studies employed econometric methods similar to those in the present paper. Ferson [8] derives indirect estimates using the same data as in Table 5. These estimates are based on moments conditional on subperiods and do not rely on the validity of the forecasting equations as do the constrained estimates. These point estimates were 1.2 and 1.6. Brown and Gibbons [4] estimate relative risk aversion using nonlinear techniques with unconditional moments of stock returns. Their range of estimates is .09 to 7.29. It is interesting that in several of these studies the reported precision of the risk aversion estimates was quite high, yet the differences in the values using different methodologies are often large.

Apparently, a precise determination of aggregate risk aversion is difficult to obtain, based on the range of estimates in the literature and the estimates for both absolute and for relative risk aversion in Table 5. To obtain a feel for the economic difference in these estimates, one can approximately interpret the magnitude of relative risk aversion in terms of timeless gambles for real consumption to which a hypothetical individual would be indifferent. Pratt [35] showed that an individual would be indifferent between a coin toss over a fraction of current consumption versus paying $a\alpha^2/2$ of current consumption to avoid the gamble, where a is the coefficient of relative risk aversion. For example, with $a = 1.0$ (the log utility), a person would pay up to .005 percent of current consumption to avoid an even wager over 1 percent. A constant relative risk-averse individual with $a = 5$ would pay up to 2.5 basis points.

VI. Conclusions

The empirical results indicate the existence of changes in expected real returns to three-month Treasury bills, but do not strongly reject that expected real per capita consumption changes are constant. The evidence against the cross-equation restrictions implied by the asset pricing models was slightly stronger, as might be expected, when constant absolute risk aversion is assumed instead of constant relative risk aversion. The data do not appear consistent with the hypothesis of aggregate risk neutrality and there is no strong evidence that expected real returns on bonds fluctuate with changes in mean consumption growth rates. These results imply that changes in consumption betas must be important if consumption-based asset pricing models are to provide a useful characterization of the behavior over time of risk premiums associated with inflation.

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TABLE 6
Summary Sample Statistics*
(Quarterly Data (1947-1980) 135 Observations)

Series	Mean	STD DEV	Sample Autocorrelations								Q
			r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	
NON	1.266	0.18	.98	.96	.95	.92	.89	.87	.85	.83	932.0
Δ NON	0.37	1.53	-.10	-.14	.12	-.02	.02	-.04	-.08	-.06	8.0
$\Delta \ln$ NON	1.13	4.58	-.13	-.09	.12	-.06	-.01	-.02	-.06	-.10	8.0
SERV	1.26	0.37	.98	.96	.94	.92	.89	.87	.84	.82	924.0
Δ SERV	0.82	1.00	.02	.23	.02	.27	-.03	.16	.04	.16	25.5
$\Delta \ln$ SERV	2.48	2.86	.01	.11	-.10	.09	-.14	.05	.04	.13	9.9
NDS	2.53	0.62	.98	.96	.94	.92	.89	.87	.85	.82	930.0
Δ NDS	1.19	2.08	.08	-.03	.12	.15	.02	-.03	.02	-.01	6.1
$\Delta \ln$ NDS	1.80	3.03	.02	-.00	.08	.06	-.05	-.03	.04	-.07	2.5
DX	0.39	0.14	.98	.95	.94	.91	.88	.85	.82	.79	898.0
Δ DX	0.03	0.02	-.15	.07	.01	-.07	.05	-.02	-.12	-.18	11.0
RF	4.08	2.62	.91	.85	.80	.74	.65	.58	.53	.49	568.0
Δ RF	0.90	0.78	.01	-.23	-.02	.25	.03	-.18	-.16	.14	27.2
INF	3.80	3.27	.65	.60	.53	.39	.32	.23	.19	.22	203.0
Δ INF	0.04	2.73	-.45	.03	.09	-.09	.04	-.08	-.10	.01	32.2
RM	6.40	26.60	.13	.10	-.03	.10	-.07	-.06	-.14	.01	9.3
R_t	0.28	2.54	.40	.34	.24	.12	.06	-.06	-.08	.06	51.0
ΔR_t	0.05	2.75	.47	.04	.03	-.06	.05	-.06	-.12	.03	34.0

*The sample autocorrelations r_i are Box-Jenkins estimates with approximate standard errors equal to .09. Q is the Box-Pierce statistic for the first 8 autocorrelations. Under the null hypothesis that the true autocorrelations are zero, the expected value of Q is 8 and its standard error approximately equal to $4 \cdot 3 \cdot \ln$ denotes the natural logarithm.

Definition of Symbols

- NON Consumer nondurable goods component of real aggregate consumption expenditures per capita.
- SERV Real consumer services expenditures per capita.
- NDS $NON + SERV$
- DX Real consumer durables expenditures per capita.
- RF_t Nominal riskless interest rate, continuously compounded annual rates.
- INF_t Ex post inflation rate, continuously compounded annual percentage rate. $INF_t = 400 \ln(L_t/L_{t-1})$ where L_t is the level at time t of the implicit price deflator for total personal consumption expenditures.
- RM_t Continuously compounded total real return over $t-1$ to t on the S&P index, at annual percentage rate.
- R_t $RF_t - INF_t$, the ex post real return to the nominally riskless bond.

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TABLE 7
Sample Correlation Matrix*
(Quarterly Data 1947-1980)

	R-2	DX	R-1	RM-1	RM-2	DX-1	INF	RM
R-2	1.000							
DX	0.090	1.000						
R-1	0.409	0.072	1.000					
RM-1	0.017	-0.207	-0.013	1.000				
RM-2	-0.002	-0.209	0.025	0.127	1.000			
DX-1	0.084	0.990	0.054	-0.228	-0.217	1.000		
INF	-0.164	0.647	-0.192	-0.230	-0.252	0.655	1.000	
RM	0.042	-0.215	0.025	0.139	0.104	-0.241	-0.270	1.000
DX-2	0.065	0.983	0.065	-0.253	-0.236	0.990	0.673	-0.237
R	0.365	0.041	0.394	0.015	0.052	0.051	-0.617	0.010
INF-1	-0.206	0.584	-0.651	-0.265	-0.269	0.617	0.656	-0.237
Δ NON	-0.118	0.130	0.205	0.204	0.031	0.052	-0.142	0.152
Δ NDS	-0.077	0.265	0.204	0.144	0.051	0.182	-0.075	0.164
INF-2	-0.653	0.550	-0.212	-0.242	-0.288	0.573	0.615	-0.215
Δ NON-2	0.164	0.178	0.078	-0.052	0.157	0.168	0.069	-0.088
Δ lnNON	-0.131	0.107	0.248	0.222	0.042	0.033	-0.143	0.150
Δ lnNDS	-0.091	0.175	0.249	0.199	0.079	0.097	-0.120	0.175
Δ lnNON-2	0.147	0.099	0.068	-0.033	0.148	0.089	0.040	-0.057
	DX-2	R	INF-1	Δ NON	Δ NDS	INF-2	Δ lnNDS-2	Δ lnNON
DX-2	1.000							
R	0.038	1.000						
INF-1	0.626	-0.201	1.000					
Δ NON	0.052	0.168	-0.222	1.000				
Δ NDS	0.178	0.157	-0.165	0.890	1.000			
INF-2	0.607	-0.188	0.646	0.036	0.060	1.000		
Δ lnNDS-2	0.174	-0.048	-0.001	-0.085	-0.013	-0.138	1.000	
Δ lnNON	0.036	0.160	-0.257	0.989	0.876	0.042	-0.069	1.000
Δ lnNDS	0.097	0.153	-0.239	0.880	0.972	0.032	-0.003	0.896
Δ lnNON-2	0.103	-0.042	-0.029	-0.112	-0.074	-0.149	0.896	-0.088
	lnNDS	lnNON-2						
Δ lnNDS	1.000							
Δ lnNON-2	-0.042	1.000						

* Symbols are defined in Table 6. The notation "-n" means lagged n quarters. "Δ" means first difference and ln is natural logarithm.

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