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Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8

Anja Volk and Elaine Chew

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Brahms' Capriccio Op. 76, No. 8

- **David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8**
- Two independent mathematical models:
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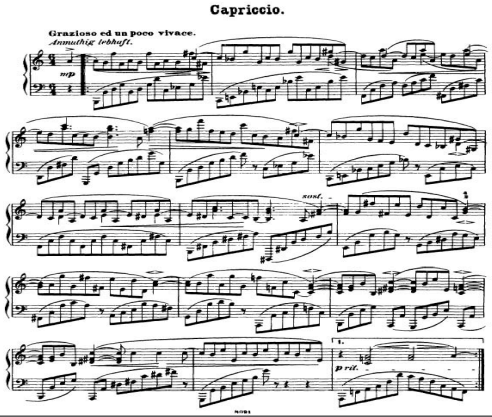
David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8



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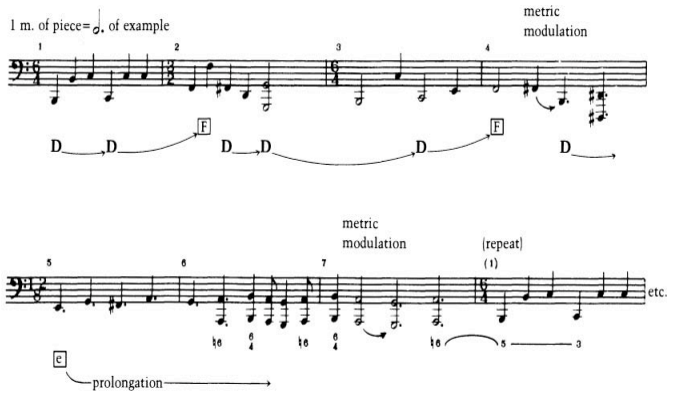

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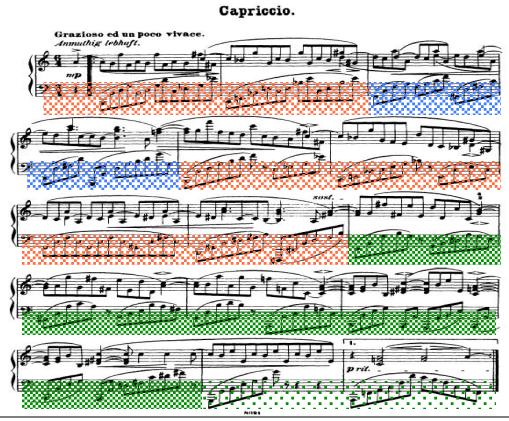

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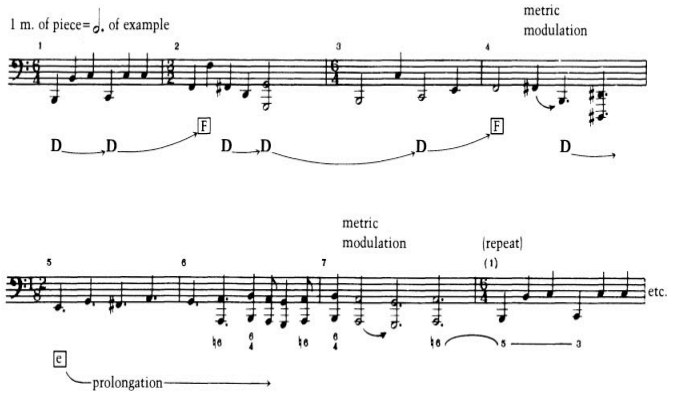

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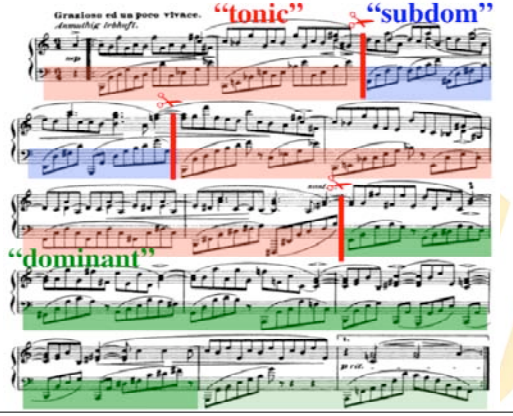
David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

… they contain the Cs in the bass-line, along with the bs that inflect them

… triggered by the strongest subdominant event of the passage, the bass-and-root F.

… triggered by and prolong the big dominant tonal event, bass-and root e.



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David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

-both the metric relation of 6/4 to 3/2 and that of 6/4 to 12/8 involve the play of the ratios 2:3 and 3:2. And these are the same ratios involved in pitch relationships of a fifth, the dominant and subdominant relations to a tonic pitch

-the relation of 6/4 to 12/8 inverts … the relation of 6/4 to 3/2, … is numerically analogous to the inversion of tonic-dominant and tonic-subdominant pitch relations

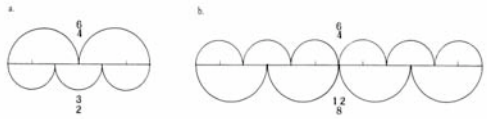

Figure 1

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David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

We must not try to push too far the tonal analogy for these figures.

The idea of a deep affinity between pitch and time is as Platonist as the Wendisch Mägdelein's vision of eternal love. … am I suggesting, that such relationships are more eternal than iron and steel? … I am not altogether sure.

Richard Cohn: **Complex Hemiolas, Ski-Hill-Graphs and Metric Spaces**

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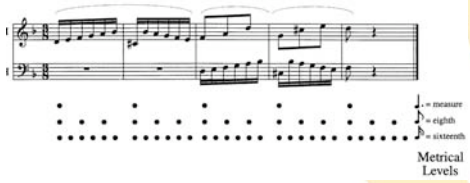
Mathematical Model: Inner Metric Analysis

Background:

–Mazzola, G. and Zahorka, O. (1993-95). *Geometry and Logic of Musical Performance*
–Fleischer, A., Mazzola, G. and Noll, T. (2000) *Computergestützte Musiktheorie*
–Nestke, A. and Noll, T. (2001). *Inner Metric Analysis*
–Fleischer, A. (2002). *A Model of Metric Coherence*
–Volk, A. (2004). *Metric Investigations in Brahms' Symphonies*



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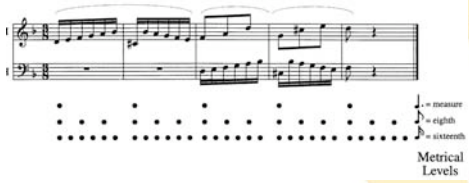
Mathematical Model: Inner Metric Analysis

Harald Krebs:

I define the meter of a work as the union of all layers of motion (i.e., series of regularly recurring pulses) active within it.



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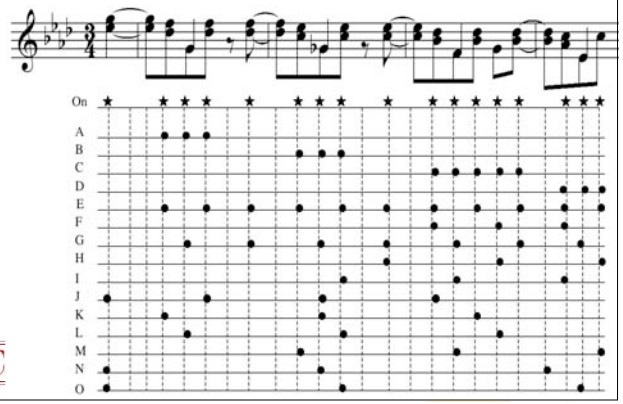
Mathematical Model: Inner Metric Analysis



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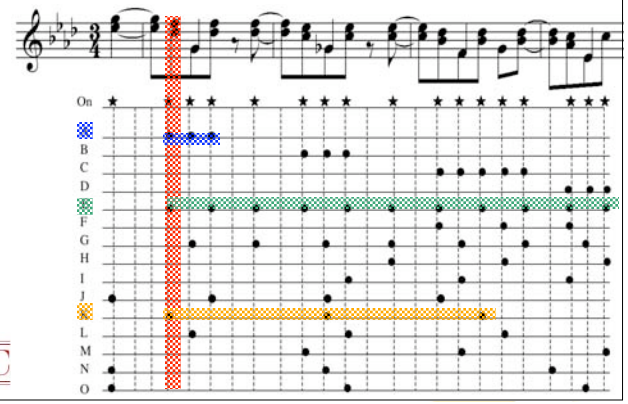
Mathematical Model: Inner Metric Analysis



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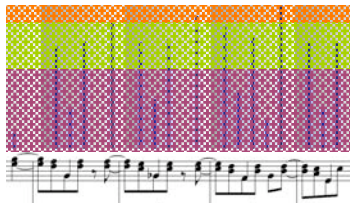
Mathematical Model: Inner Metric Analysis



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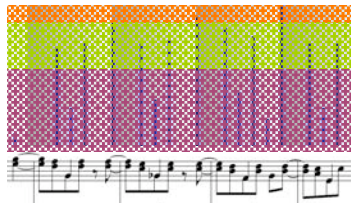
Mathematical Model: Inner Metric Analysis



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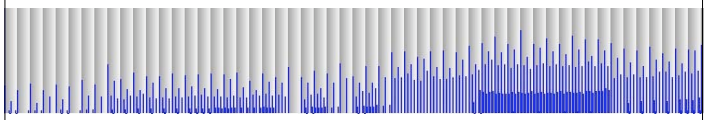
Mathematical Model: Inner Metric Analysis

Symphony C Major K. 551

1. Movement (4/4)

Wolfgang A. Mozart



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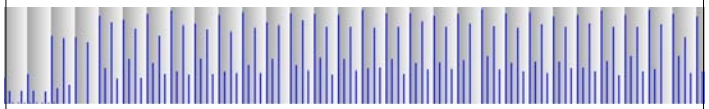
Mathematical Model: Inner Metric Analysis

Nonpareil Rag

Scott Joplin



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Mathematical Model: Inner Metric Analysis

Johannes Brahms !

Volk, A. (2004). *Metric Investigations in Brahms' Symphonies*

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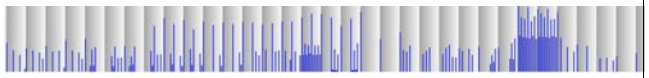
Inner Metric Analysis: Johannes Brahms, 3. Symphony

Johannes Brahms

Max Frisch:

The main theme … begins to project a metrical profile, but one that fits more clearly into 3/2 than 6/4. Only in bar 7 is the duple division of the bar firmly supported in all parts: the theme, the motto, and the harmonic voices move every half bar.



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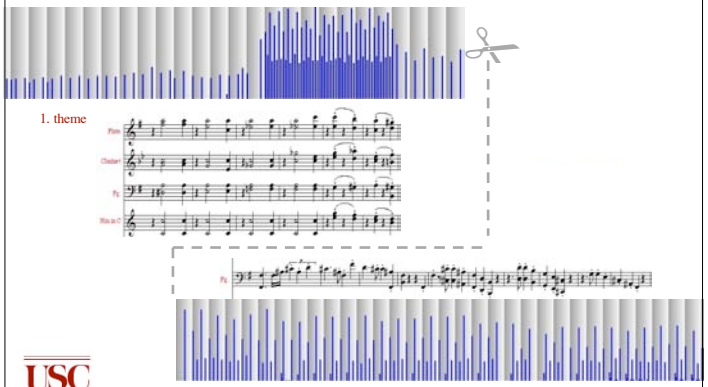
Inner Metric Analysis: Johannes Brahms, 4. Symphony



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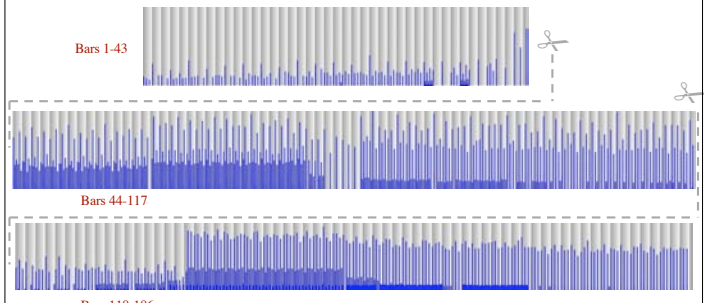
Inner Metric Analysis: Johannes Brahms, 2. Symphony

Analysis of the Exposition



Graham Phipps

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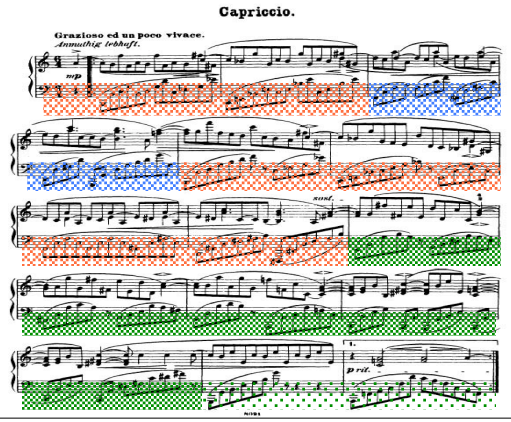
David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8



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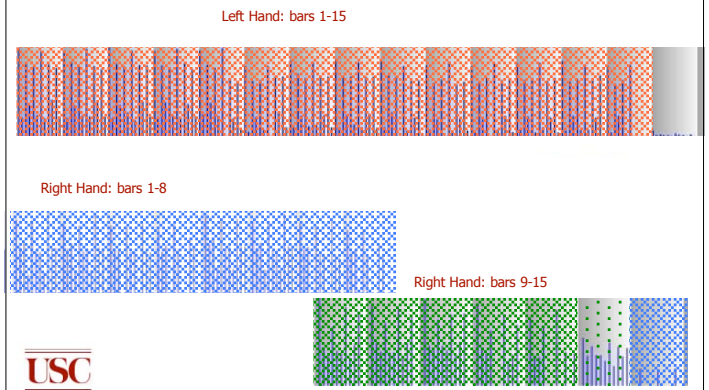
Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8



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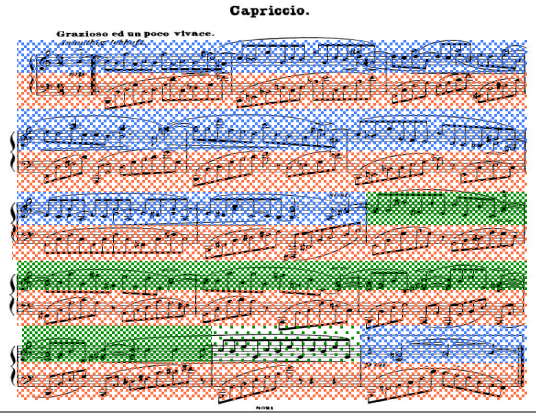
Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8



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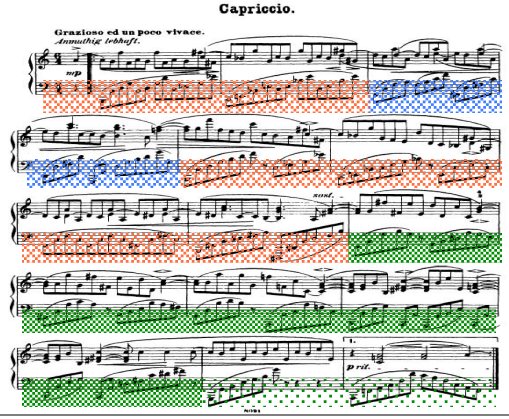

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The Spiral Array (Chew 2000)

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D				F#	C#	G#
Ab	Eb	Bb					A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Longuet-Higgins' (1962ab) harmonic network

The *tonnetz* in neo-Riemannian Theory

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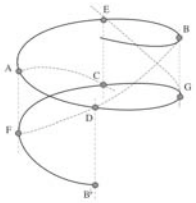
The Spiral Array (Chew 2000)

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Longuet-Higgins' (1962ab) harmonic network

The *tonnetz* in neo-Riemannian Theory



Chew's pc spiral in the spiral array (2000)

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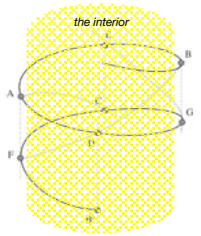
Modeling Tonality

Spiral Array (Chew 2000)



$$\mathbf{P}(k) \stackrel{def}{=} \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r \sin \frac{k\pi}{2} \\ r \cos \frac{k\pi}{2} \\ kh \end{bmatrix}$$

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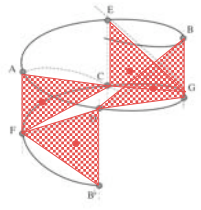
Modeling Tonality

Spiral Array (Chew 2000)



$$\mathbf{C_M}(k) \stackrel{def}{=} w_1 \cdot \mathbf{P}(k) + w_2 \cdot \mathbf{P}(k+1) + w_3 \cdot \mathbf{P}(k+4),$$
$$\text{where} \quad w_1 \geq w_2 \geq w_3 > 0 \text{ and } \sum_{i=1} w_i = 1.$$

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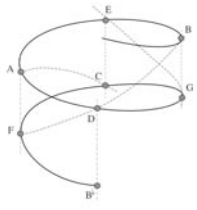
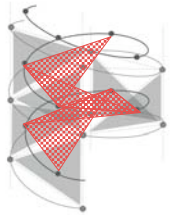
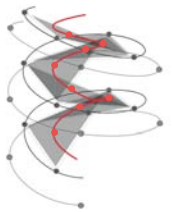
Modeling Tonality

Spiral Array (Chew 2000)







$$\mathbf{T_M}(k) \stackrel{def}{=} \omega_1 \cdot \mathbf{C_M}(k) + \omega_2 \cdot \mathbf{C_M}(k+1) + \omega_3 \cdot \mathbf{C_M}(k-1),$$
$$\text{where} \quad \omega_1 \geq \omega_2 \geq \omega_3 > 0 \text{ and } \sum_{i=1} \omega_i = 1.$$

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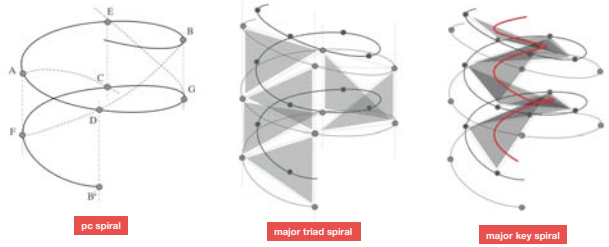
Modeling Tonality

Spiral Array (Chew 2000) discrete space ⇨ continuous space
exact membership ⇨ reduction to single-point representation



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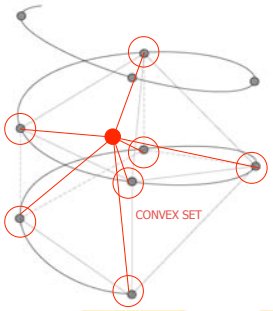
Key-Finding

Center of Effect Generator (Chew 2000)

- clustering of pitches in a key
- generate center of effect
- perform nearest neighbor search for closest key

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab



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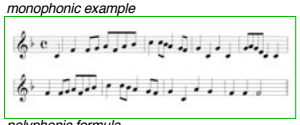
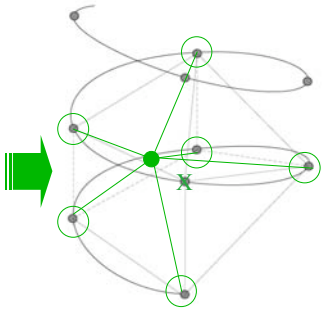
monophonic example



polyphonic formula

$$\mathbf{c}_{a,b} \stackrel{\text{def}}{=} \sum_{t=a}^{b} \sum_{j=1}^{n_t} \frac{d_{tj}}{D_{a,b}} \cdot \mathbf{p}_{tj}, \quad \text{where } D_{a,b} = \sum_{t=a}^{b} \sum_{j=1}^{n_t} d_{tj}.$$



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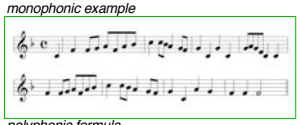

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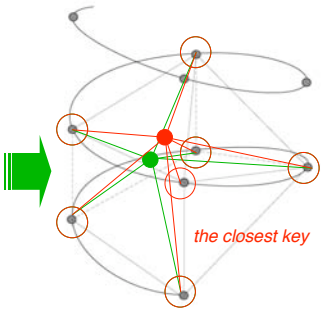
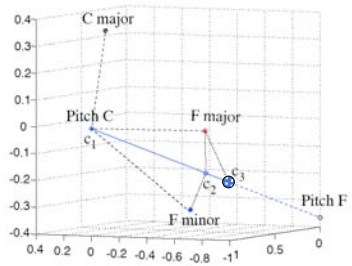


USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

Simple Gifts from Copland's *Appalachian Spring*



From Chew 2000 p.104.

USC Viterbi School of Engineering **Key-Finding**

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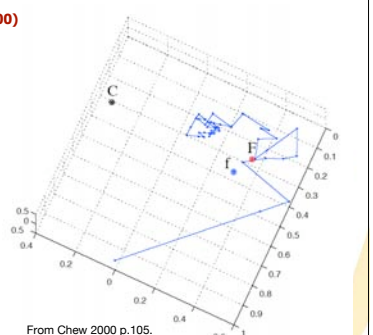

From Chew 2000 p.105.

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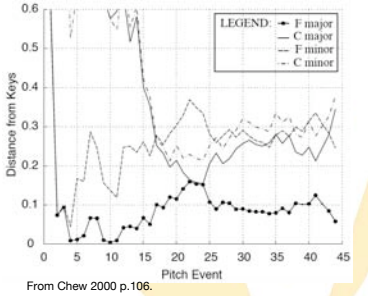


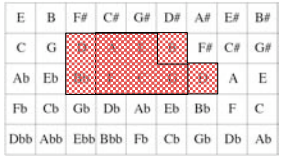
From Chew 2000 p.106.

USC Viterbi School of Engineering **Key-Finding: Comparisons**

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 1 in C major



Book 1 Fugue subjects	Steps to key CEG[a]	PTPM	SMA
1 C major	2	2	
2 C minor	5	5	
3 C♯ major	6	7	16
4 C♯ minor	3	3	4
5 D major	2	2	15†
6 D minor	3	3	8
7 E♭ major	2	6	11†
8 E♭ minor	2	6	12†
9 E major	14	12†	11
10 E minor	3	2	7†
11 F major	4	10	6
12 F minor	3	15	4†
13 F♯ major	3	2	8
14 F♯ minor	7	18	5†
15 G major	2	2	15
16 G minor	3	3	4
17 A♭ major	3	2	7†
18 G♯ minor	5	5	5
19 A major	2	4	7
20 A minor	5	5	5
21 B♭ major	4	4	14
22 B♭ minor	9	3	6†
23 B major	**3.75**	**5.25**	**8.71**
24 B minor			
Average		5.25 (4.79)	8.71 (8.21)

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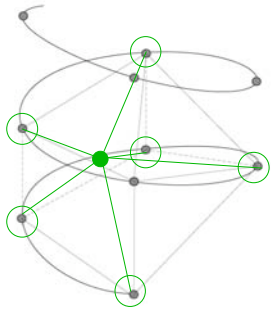
The *c.e.* as proxy for context

as key context



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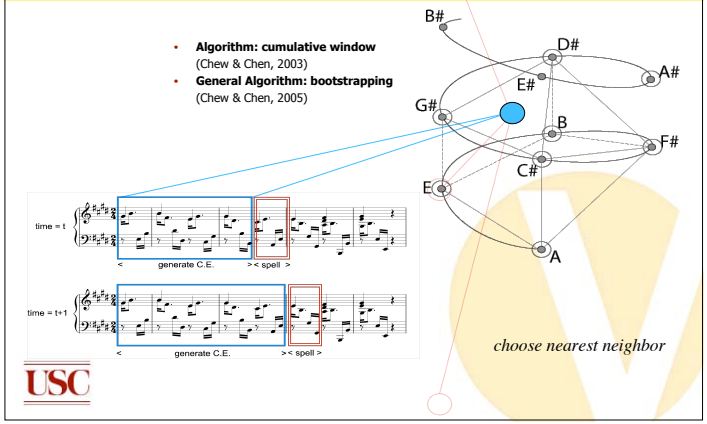
Pitch Spelling

- **Algorithm: cumulative window** (Chew & Chen, 2003)
- **General Algorithm: bootstrapping** (Chew & Chen, 2005)



choose nearest neighbor

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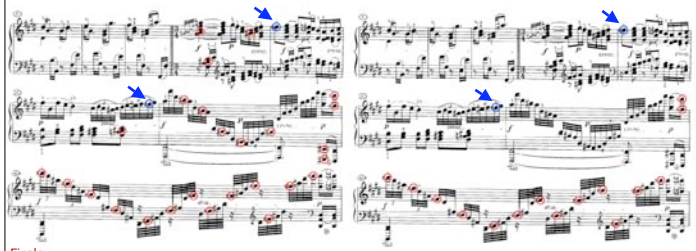
Pitch Spelling: Result (Chew & Chen, 2005)



Finale Sibelius

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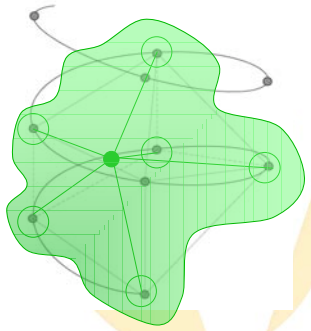
The *c.e.* as proxy for context

key context



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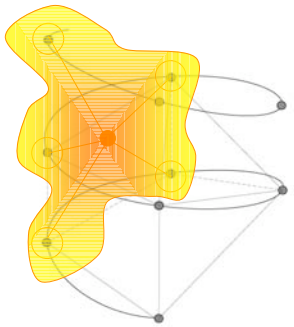
The *c.e.* as proxy for context

key context
any context



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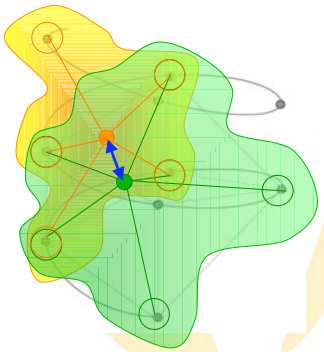
Measuring contextual distance



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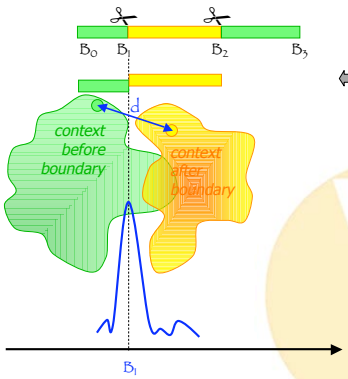
Distance peaks at boundary



Argus algorithm for automatic segmentation (Chew 2004)

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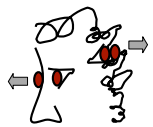
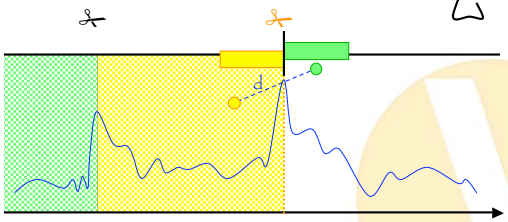
Argus: RT Segmentation (Chew 2004)





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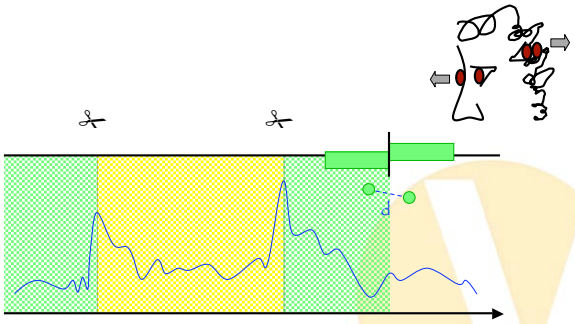
Argus: RT Segmentation (Chew 2004)



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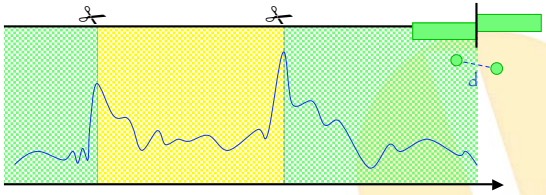
Argus: RT Segmentation (Chew 2004)

Advantages:
- Computes in real-time, *O(n)*
- Eliminates dependence on key representations
- Segments by pitch collection (more general)



Applied to two Regards in Messiaen's *Vingt Regards sur l'Enfant Jesus* (Chew 2004)

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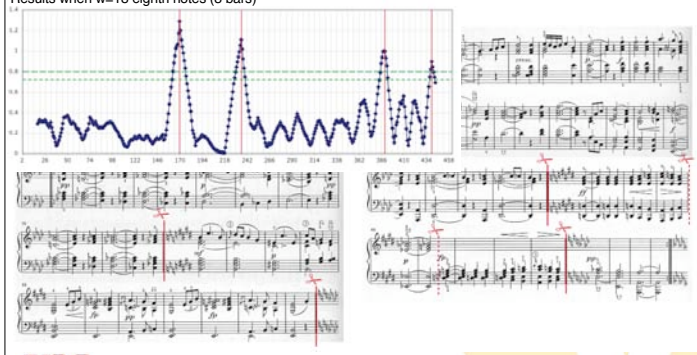
Argus: Schubert D780 No.6 (Chew 2005)

Results when w=18 eighth notes (3 bars)



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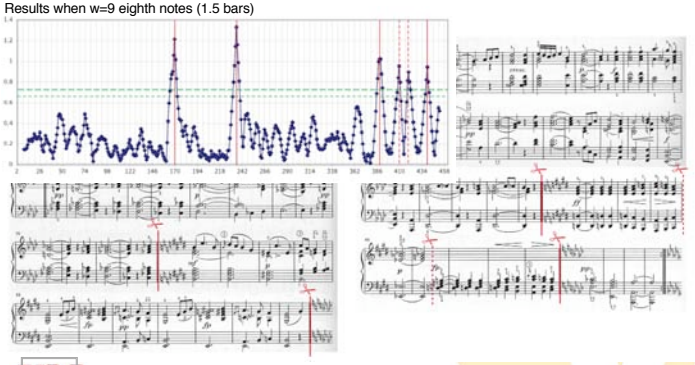
Argus: Schubert D780 No.6 (Chew 2005)

Results when w=9 eighth notes (1.5 bars)



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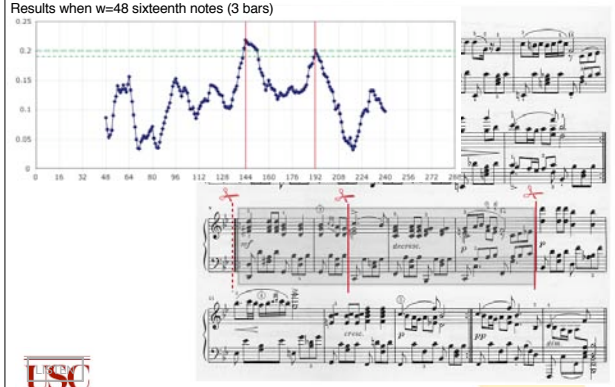
Argus: Schubert D935 No.3 (Chew 2005)

Results when w=48 sixteenth notes (3 bars)



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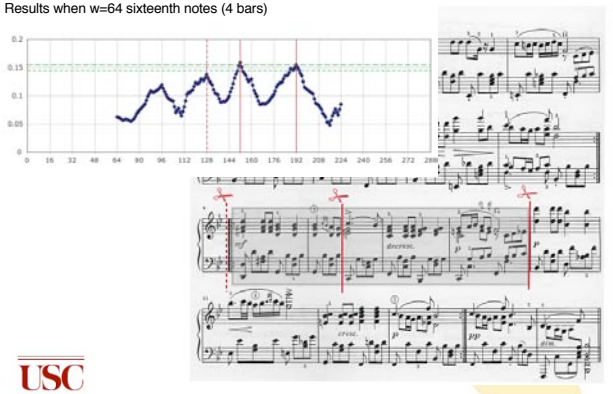
Argus: Schubert D935 No.3 (Chew 2005)

Results when w=64 sixteenth notes (4 bars)



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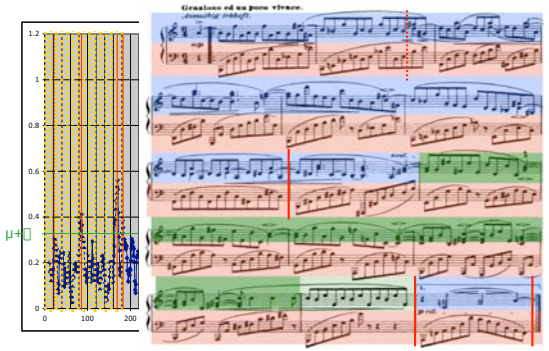
Argus: Brahms: $f = 12$, $b = 12$, $\mu = 0.1987$, $\sigma = 0.1304$



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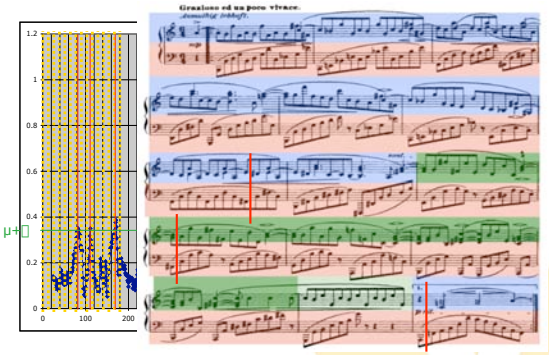
Argus: Brahms: $f = 24$, $b = 24$, $\mu = 0.2010$, $\sigma = 0.1434$



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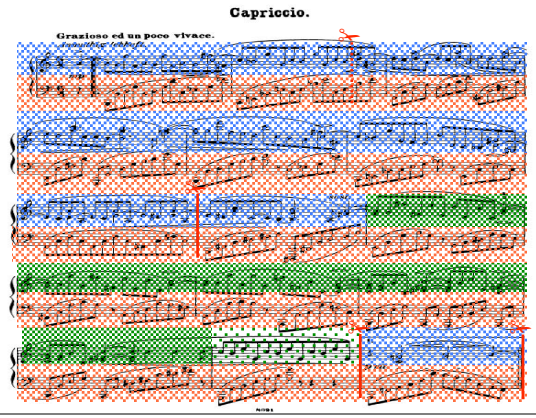
Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8



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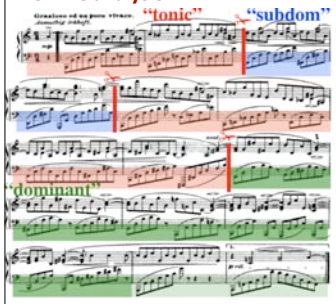
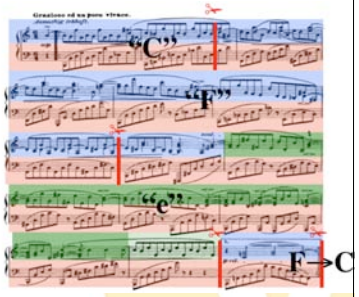
Comparison: Harmony and Meter in Brahms's Op. 76, No. 8

Lewin's analysis



Our analysis



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Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8

Brahms' Capriccio Op. 76, No. 8

- David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8
- Two independent mathematical models:
  - *Inner Metric Analysis (metric domain)*
  - *Spiral Array (tonal domain)*

Conclusions

- Computational models for analysis (metric & tonal)
- Real separation of metric & tonal information

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