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School of Engineering

Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8

Anja Volk and Elaine Chew




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Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8

Brahms' Capriccio Op. 76, No. 8

- David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8
- Two independent mathematical models:
 - Inner Metric Analysis (metric domain)
 - Spiral Array (tonal domain)





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David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

Capriccio.

*Gravioso ed un poco vivace.
Anmutig lebhaft.*

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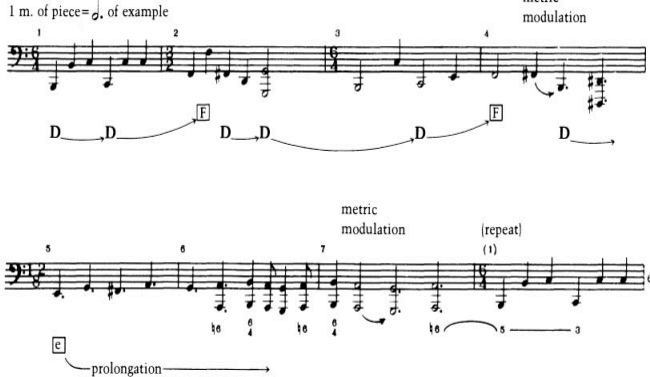
David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

1 m. of piece = $\frac{1}{2}$ of example

metric modulation

metric modulation (repeat)

prolongation



USC Viterbi School of Engineering David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

Capriccio.

Grazioso ed un poco vivace. Amabile e brioso.

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USC Viterbi School of Engineering David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

1 m. of piece = $\frac{1}{2}$ of example

metric modulation

metric modulation (repeat) (1) etc.

prolongation

USC Viterbi School of Engineering David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

Grazioso ed un poco vivace. Amabile e brioso.

... they contain the Cs in the bass-line, along with the bs that inflect them

... triggered by the strongest subdominant event of the passage, the bass-and-root F.

... triggered by and prolonging the big dominant tonal event, bass-and root e.

“tonic” **“subdom”**

“dominant”

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USC Viterbi School of Engineering David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

-both the metric relation of 6/4 to 3/2 and that of 6/4 to 12/8 involve the play of the ratios 2:3 and 3:2. And these are the same ratios involved in pitch relationships of a fifth, the dominant and subdominant relations to a tonic pitch

-the relation of 6/4 to 12/8 inverts ... the relation of 6/4 to 3/2, ... is numerically analogous to the inversion of tonic-dominant and tonic-subdominant pitch relations

Figure 1


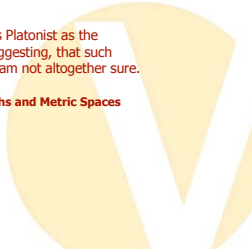
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USC Viterbi School of Engineering **David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8**

We must not try to push too far the tonal analogy for these figures.

The idea of a deep affinity between pitch and time is as Platonist as the Wendisch Mägdlein's vision of eternal love. ... am I suggesting, that such relationships are more eternal than iron and steel? ... I am not altogether sure.


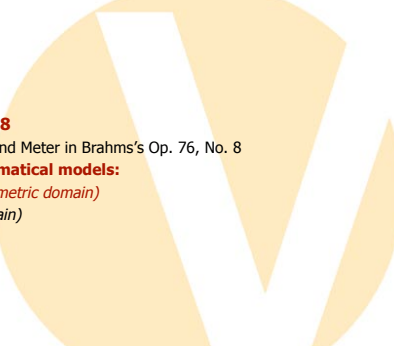
Richard Cohn: **Complex Hemiolas, Ski-Hill-Graphs and Metric Spaces**

USC Viterbi School of Engineering **Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8**

Brahms' Capriccio Op. 76, No. 8


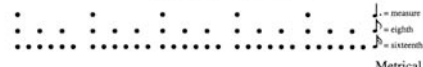
- David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8
- **Two independent mathematical models:**
 - *Inner Metric Analysis (metric domain)*
 - *Spiral Array (tonal domain)*

USC Viterbi School of Engineering **Mathematical Model: Inner Metric Analysis**



Background:

- Mazzola, G. and Zahorka, O. (1993-95). *Geometry and Logic of Musical Performance*
- Fleischer, A., Mazzola, G. and Noll, T. (2000) *Computergestützte Musiktheorie*
- Nestke, A. and Noll, T. (2001). *Inner Metric Analysis*
- Fleischer, A. (2002). *A Model of Metric Coherence*
- Volk, A. (2004). *Metric Investigations in Brahms' Symphonies*

Legend:
 J = 1/16 measure
 J = 1/8 eighth
 J = 1/4 sixteenth


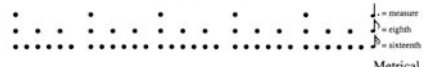
Metrical Levels

USC Viterbi School of Engineering **Mathematical Model: Inner Metric Analysis**



Harald Krebs:

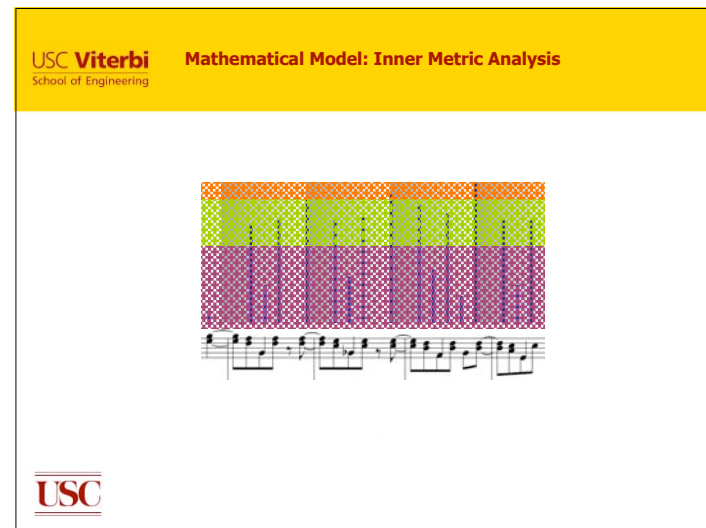
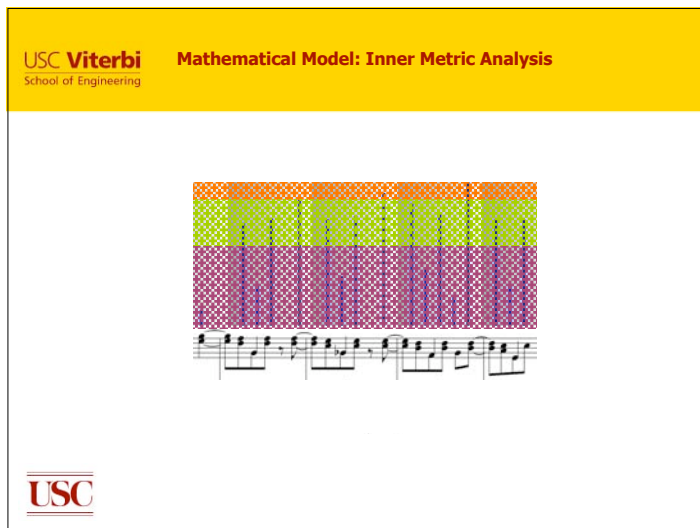
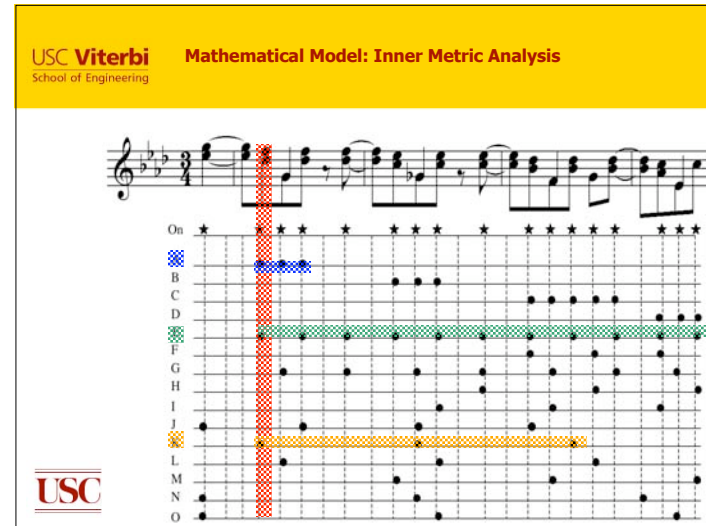
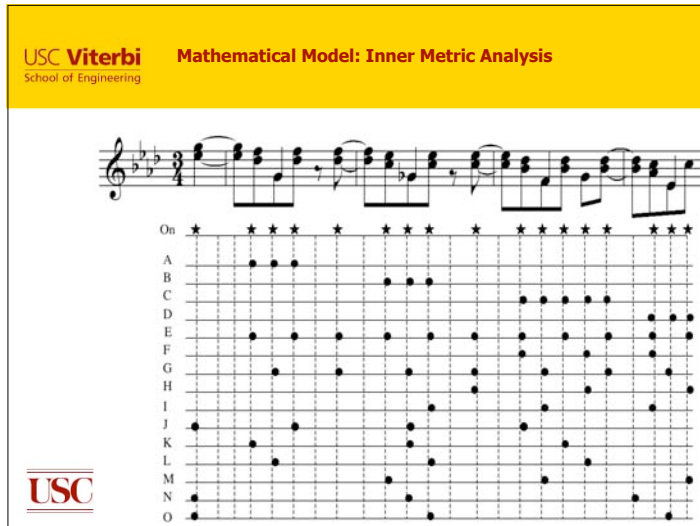
I define the meter of a work as the union of all layers of motion (i.e., series of regularly recurring pulses) active within it.

Legend:
 J = 1/16 measure
 J = 1/8 eighth
 J = 1/4 sixteenth

Metrical Levels

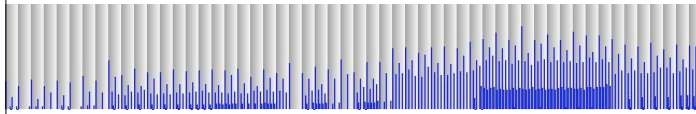





USC Viterbi School of Engineering **Mathematical Model: Inner Metric Analysis**

Symphony C Major K. 551

1. Movement (4/4)
Wolfgang A. Mozart

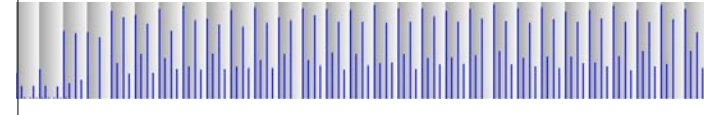


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USC Viterbi School of Engineering **Mathematical Model: Inner Metric Analysis**

Nonpareil Rag

Scott Joplin




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USC Viterbi School of Engineering **Mathematical Model: Inner Metric Analysis**

Johannes Brahms !


Volk, A. (2004). *Metric Investigations in Brahms' Symphonies*



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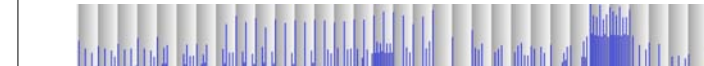
USC Viterbi School of Engineering **Inner Metric Analysis: Johannes Brahms, 3. Symphony**

Johannes Brahms



Max Frisch:

The main theme ... begins to project a metrical profile, but one that fits more clearly into 3/2 than 6/4. Only in bar 7 is the duple division of the bar firmly supported in all parts: the theme, the motto, and the harmonic voices move every half bar.



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USC Viterbi School of Engineering Inner Metric Analysis: Johannes Brahms, 4. Symphony

1. theme

2. theme

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Detailed description: This slide displays the inner metric analysis of the first two themes from the fourth movement of Johannes Brahms' Fourth Symphony. At the top, a spectrogram shows the overall energy profile of the music. Below it, the first theme is shown with musical notation for piano, clarinet, violin, and viola. A dashed box highlights a specific section of the first theme, which is then shown in a larger spectrogram below. The second theme is also shown with musical notation and a corresponding spectrogram. The USC logo is in the bottom left corner.

USC Viterbi School of Engineering Inner Metric Analysis: Johannes Brahms, 2. Symphony

Analysis of the Exposition

Bars 1-43

Bars 44-117

Bars 118-186

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Graham Phipps

Detailed description: This slide focuses on the exposition of the second movement of Johannes Brahms' Second Symphony. It features three stacked spectrograms corresponding to different sections: Bars 1-43, Bars 44-117, and Bars 118-186. Each spectrogram is accompanied by a pair of scissors icon, indicating a cut or transition point. The USC logo is in the bottom left corner, and the name Graham Phipps is in the bottom right corner.

USC Viterbi School of Engineering David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8

Capriccio.

Grazioso ed un poco vivace.
Anmutig lebhaft.

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Detailed description: This slide shows the musical score for Brahms' Capriccio, Op. 76, No. 8. The score is annotated with various colors (red, blue, green, orange) to highlight specific harmonic and metric features. The tempo and mood are indicated as 'Grazioso ed un poco vivace' and 'Anmutig lebhaft'. The USC logo is in the bottom left corner.

USC Viterbi School of Engineering Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8

Left Hand: bars 1-15

Right Hand: bars 1-8

Right Hand: bars 9-15

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Detailed description: This slide provides an inner metric analysis of the first 15 bars of Brahms' Capriccio, Op. 76, No. 8. It features three spectrograms: one for the left hand (bars 1-15) and two for the right hand (bars 1-8 and bars 9-15). The spectrograms use different colors (orange, blue, green) to distinguish between the different parts. The USC logo is in the bottom left corner.

USC Viterbi School of Engineering **Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8**

Capriccio.
Grave ed un poco vivace.
Amstüchtig lebhaft.

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USC Viterbi School of Engineering **David Lewin: On Harmony and Meter in Brahms's Op. 76, No. 8**

Capriccio.
Grave ed un poco vivace.
Amstüchtig lebhaft.

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USC Viterbi School of Engineering **Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8**

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USC Viterbi School of Engineering **The Spiral Array (Chew 2000)**

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D				F#	C#	G#
Ab	Eb	Bb				A	E	
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Longuet-Higgins' (1962ab) harmonic network

The tonnetz in neo-Riemannian Theory

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USC Viterbi School of Engineering **The Spiral Array** (Chew 2000)

E	B	F#	C#	G#	D#	A#	E#	B#
C	D	A	E	B	F#	C#	G#	
Ab	Eb	Bb	F	C	G	D	A	E
F#	C#	G#	D#	A#	E#	B#	F#	C#
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Dbb	Abb

Longuet-Higgins' (1962ab) harmonic network

The tonnetz in neo-Riemannian Theory

Chew's pc spiral in the spiral array (2000)

USC Viterbi School of Engineering **Modeling Tonality**

Spiral Array (Chew 2000)

the interior

$$P(k) \stackrel{\text{def}}{=} \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r \sin \frac{kh}{2} \\ r \cos \frac{kh}{2} \\ kh \end{bmatrix}$$

USC Viterbi School of Engineering **Modeling Tonality**

Spiral Array (Chew 2000)

$$C_{31}(k) \stackrel{\text{def}}{=} w_1 \cdot P(k) + w_2 \cdot P(k+1) + w_3 \cdot P(k+4),$$

where $w_1 \geq w_2 \geq w_3 > 0$ and $\sum_{i=1}^3 w_i = 1$.

USC Viterbi School of Engineering **Modeling Tonality**

Spiral Array (Chew 2000)

$$T_{31}(k) \stackrel{\text{def}}{=} w_1 \cdot C_{31}(k) + w_2 \cdot C_{31}(k+1) + w_3 \cdot C_{31}(k-1),$$

where $w_1 \geq w_2 \geq w_3 > 0$ and $\sum_{i=1}^3 w_i = 1$.

USC Viterbi School of Engineering **Modeling Tonality**

Spiral Array (Chew 2000) discrete space \square continuous space
exact membership \square reduction to single-point representation

pc spiral major triad spiral major key spiral

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USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

- clustering of pitches in a key
- generate center of effect
- perform nearest neighbor search for closest key

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

CONVEX SET

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USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

- clustering of pitches in a key
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monophonic example

polyphonic formula

$$c_{a,b} \stackrel{\text{def}}{=} \sum_{t=a}^{\text{def}} \sum_{j=1}^{n_a} \frac{d_{t,j}}{D_{a,b}} \cdot p_{t,j}, \quad \text{where } D_{a,b} = \sum_{t=a}^{\text{def}} \sum_{j=1}^{n_a} d_{t,j}.$$

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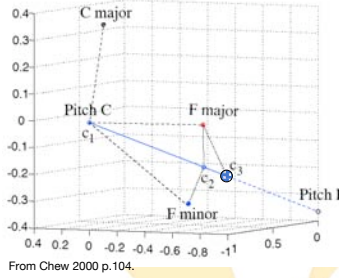
the closest key

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USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

Simple Gifts from Copland's *Appalachian Spring*

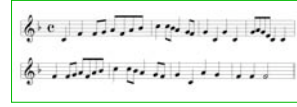
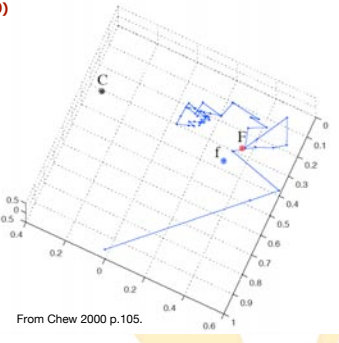



From Chew 2000 p.104.

USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

Simple Gifts from Copland's *Appalachian Spring*

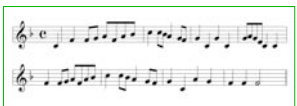
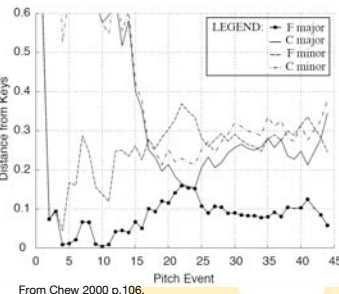



From Chew 2000 p.105.

USC Viterbi School of Engineering **Key-Finding**

Center of Effect Generator (Chew 2000)

Simple Gifts from Copland's *Appalachian Spring*

From Chew 2000 p.106.

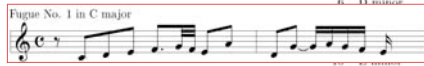
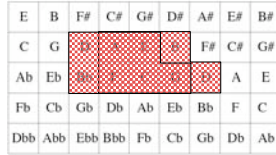
USC Viterbi School of Engineering **Key-Finding: Comparisons**

J.S. Bach's *Well-Tempered Clavier Bk 1*

Book 1	Steps to key		
Fugue subjects	CEG ^a	PTPM	SMA
1 C major	2	2	5
2 C minor	5	5	16
3 C ₂ major	6	7	4
4 C ₂ minor	3	3	15 ^b
5 D major	2	2	8
6 D minor	3	3	11 ^b
7 E major	2	6	12 ^b
8 E minor	14	12 ^b	11
9 F major	3	2	7 ^b
10 F minor	4	10	6
11 F major	3	15	4 ^b
12 F minor	3	2	8
13 F ₂ major	7	18	5 ^b
14 F ₂ minor	2	2	15
15 G major	3	3	4
16 G minor	3	2	7 ^b
17 A ₂ major	5	5	5
18 G ₂ minor	2	4	7
19 A major	5	5	5
20 A minor	4	4	14
21 B ₂ major	2	2	6 ^b
22 B ₂ minor	3	3	15
23 B major	3.75	5.25	8.71
24 B minor	3.75	5.25	8.71
Average	3.75	5.25 (4.79)	8.71 (8.21)

LISTEN

Fugue No. 1 in C major

USC Viterbi School of Engineering **The c.e. as proxy for context**

as key context

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USC Viterbi School of Engineering **Pitch Spelling**

- **Algorithm: cumulative window** (Chew & Chen, 2003)
- **General Algorithm: bootstrapping** (Chew & Chen, 2005)

choose nearest neighbor

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USC Viterbi School of Engineering **Pitch Spelling: Result** (Chew & Chen, 2005)

Finale Sibelius

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USC Viterbi School of Engineering **The c.e. as proxy for context**

key context

USC

USC Viterbi School of Engineering **The c.e. as proxy for context**

key context
any context

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The diagram illustrates a network of nodes connected by edges. A central region of nodes is highlighted in yellow, representing the context. The nodes are arranged in a roughly circular pattern with some internal connections.

USC Viterbi School of Engineering **Measuring contextual distance**

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The diagram shows a network of nodes similar to the previous slide. A central region is highlighted in green, and a blue arrow points from a node in this region towards another node, representing the measurement of contextual distance.

USC Viterbi School of Engineering **Distance peaks at boundary**

context before boundary
context after boundary

Argus algorithm for automatic segmentation (Chew 2004)

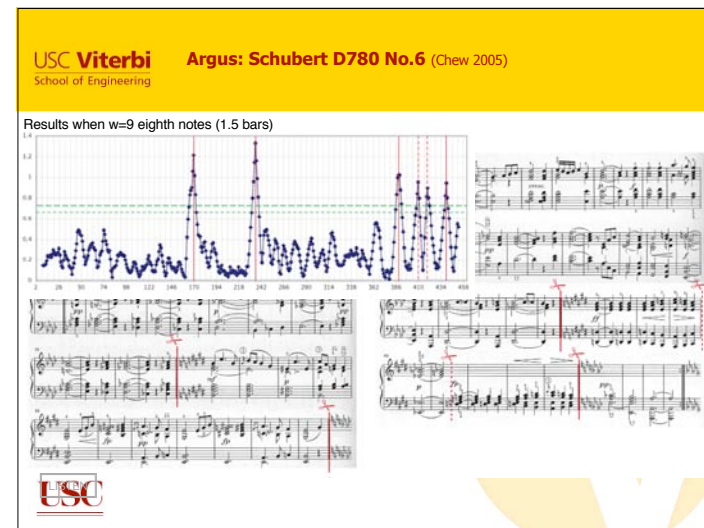
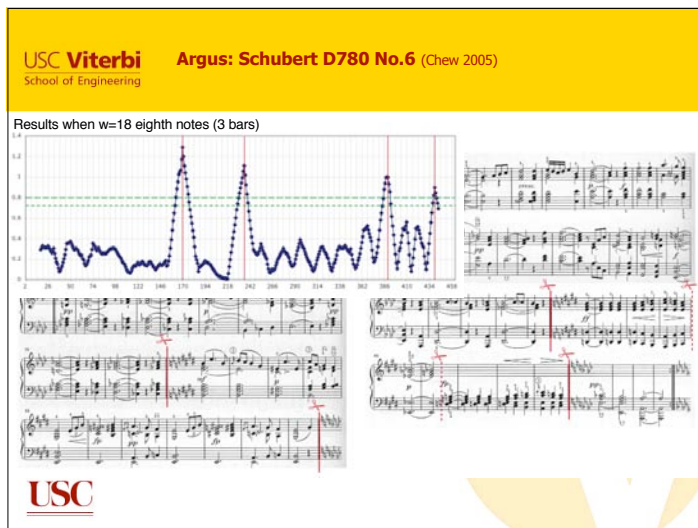
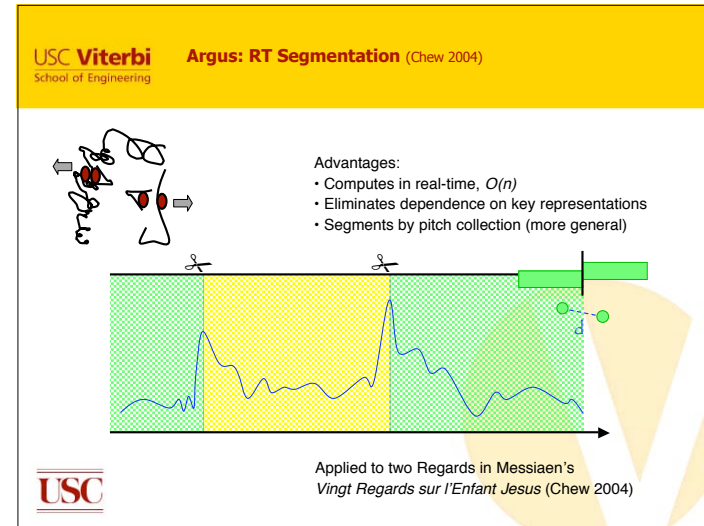
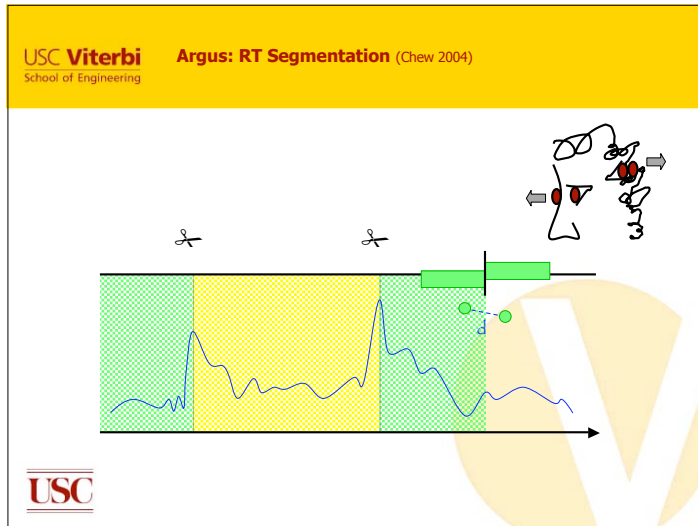
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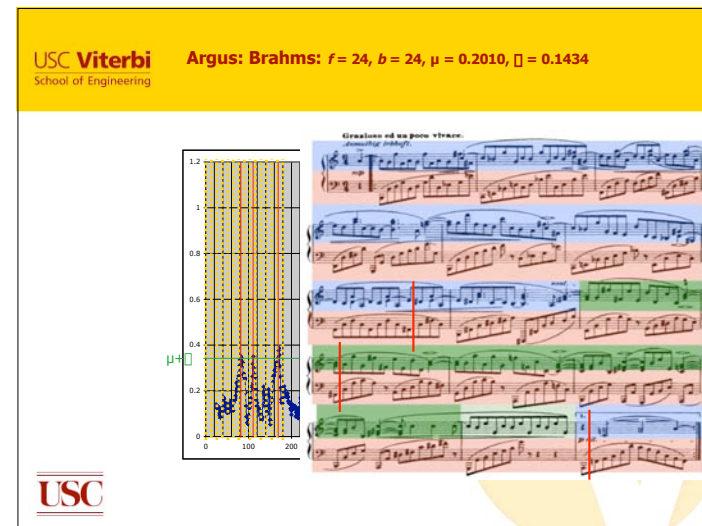
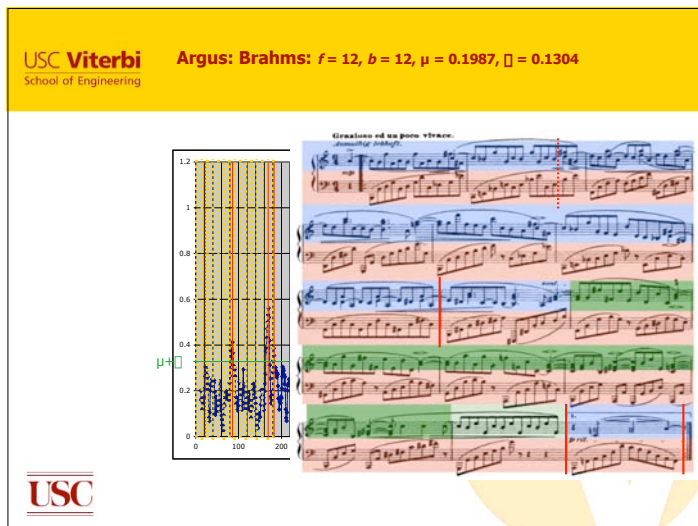
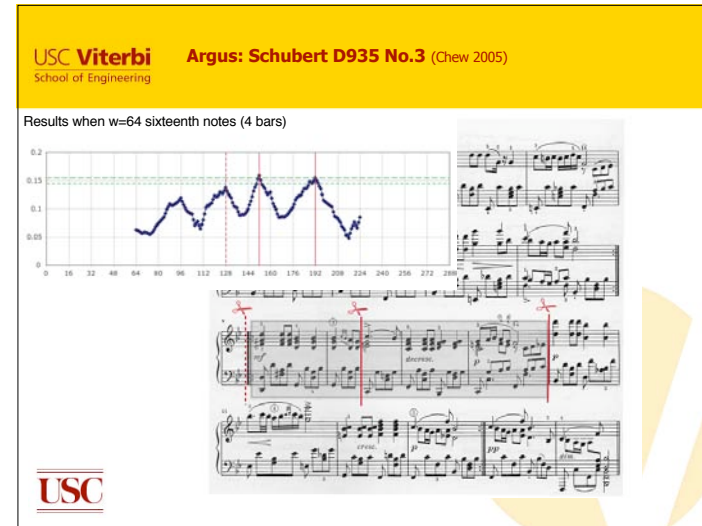
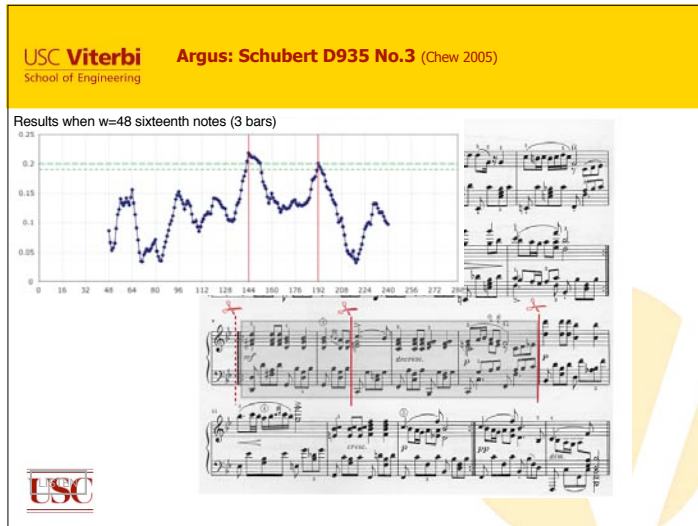
The diagram shows a network of nodes with a highlighted central region in green. A blue line graph below the network shows a peak at the boundary between the green and yellow regions. The graph is labeled with b_1 at the peak. Above the graph, a horizontal bar is divided into segments labeled b_0 , b_1 , b_2 , and b_3 . To the right, a small diagram shows a person's head with a red dot on the forehead and a red dot on the chin, with arrows pointing to the left and right, representing the Argus algorithm for automatic segmentation.

USC Viterbi School of Engineering **Argus: RT Segmentation (Chew 2004)**

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The diagram shows a network of nodes with a highlighted central region in green. A blue line graph below the network shows a peak at the boundary between the green and yellow regions. The graph is labeled with b_1 at the peak. Above the graph, a horizontal bar is divided into segments labeled b_0 , b_1 , b_2 , and b_3 . To the right, a small diagram shows a person's head with a red dot on the forehead and a red dot on the chin, with arrows pointing to the left and right, representing the Argus algorithm for automatic segmentation.





USC Viterbi School of Engineering **Inner Metric Analysis : Brahms' Capriccio op. 76, No. 8**

Capriccio.
Grave e un poco vivace.

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USC Viterbi School of Engineering **Comparison: Harmony and Meter in Brahms's Op. 76, No. 8**

Lewin's analysis **Our analysis**

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USC Viterbi School of Engineering **Re-Considering the Affinity between Metric and Tonal Structures in Brahms' Op. 76 No. 8**

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Conclusions

- Computational models for analysis (metric & tonal)
- Real separation of metric & tonal information

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