

# Welfare effects of spectrum management regimes

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## Abstract

Radio spectrum is a fundamental input in communications technology and the allocation of spectrum has gained ever increasing importance with technological advances. However, economic analysis mostly focused on perfecting the details of the competitive bidding process for spectrum licenses. This paper looks at spectrum management process prior to allocations by which the rules governing the access to and the use of spectrum are created. The choice of the spectrum management regime affects the market outcomes and consumer welfare as it determines the competitive and the technological environment in the market. We provide a first value estimate for unlicensed spectrum that had been overlooked and show that the value is commensurate with that of licensed spectrum found in previous studies. To assess the possible negative effects of excessive entry, we then model the unlicensed allocation regime where the use of spectrum is not restricted to license holders as a free entry equilibrium of a differentiated oligopoly model, as opposed the licensing regime where the use of spectrum is granted to pre-determined number of licensees which is modeled as a static oligopoly. We simulate this model in a wide range of model parameters and find the parameter thresholds where alternative regimes dominate in terms of consumer surplus. We conclude that the regime choice has to be thoroughly informed by the preference and technology structure and optimal regime choice must be an interior solution with a finely tuned mix of the two regimes.

**Keywords:** Radio Spectrum Policy, Telecommunications, Differentiated Oligopoly

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# 1 Introduction

Radio spectrum is one of the most important foundations of the contemporary communications technology. It is utilized to carry information in the absence of physical wired connections and is fundamental in the operation of countless devices and services in our everyday lives, from cell phones to TVs, from wireless networking to garage door openers, microwave ovens, baby monitors. Spectrum is a broadcast medium in the sense that the devices using it emit modulated electromagnetic waves for an intended recipient, but consequently, many devices coexisting in the same frequency band may interfere with each other. As the number of communications systems and devices has grown significantly over time, radio spectrum has become a scarce and precious commodity that needs to be allocated among competing users.

Spectrum regulatory authorities such as the Federal Communications Commission (FCC) in the US or the European Telecommunications Standards Institute (ETSI) in Europe handle the allocation and management of radio spectrum. These authorities divide spectrum into blocks with distinct geographic and frequency boundaries and dedicate these blocks to specific usages. The rights to operate devices in these blocks are then allocated to competing users. There are solid theoretical and empirical studies that have focused on perfecting the details of the competitive bidding process for the rights to use a spectrum block<sup>1</sup>. However, the spectrum management process prior to auctions, by which the rules governing the access to and the use of spectrum are created, has not achieved a comparable amount of interest from economists.

The process of spectrum management ascertains which users can obtain access to spectrum and what type of services can be provided using the spectrum subject to certain rules. There are three approaches to the management of spectrum. The first; command and control approach, whereby the regulatory authority assigns spectrum to specific users and uses subject to pre-engineered technological rules and on an as needed basis by comparative hearings, has been the dominant approach until recently. However, command and control approach is unanimously regarded as highly inefficient and is almost completely abandoned. Most of the recent debate on spectrum

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<sup>1</sup>See Cramton (2002), Klemperer (2002), Kwerel and Williams (2002) for excellent accounts of spectrum allocation mechanisms and spectrum auctions in particular.

management regimes focus on the remaining two approaches which are the licensing approach and the commons (unlicensed allocation) approach. The licensing alternative entails the creation and allocation of exclusive licenses to use the spectrum with protection from interference. The cell phone and the broadcast industry are the major examples of the products of a licensing regime and are proponents for further allocations in the same vein. Commons regime on the other hand entails specification of usage etiquettes but leaves access to spectrum open to any prospective user willing to accept and manage interference. Wireless networking through Wi-Fi and Bluetooth are the most obvious products of these unlicensed allocations, as well as cordless phones, wireless microphones, baby monitors and many more. Evidently, the choice of management regime greatly affects the final variety and technological characteristics of the communication devices in the market and shapes the nature of communications.

Another motivation for our work is the increased attention to whitespaces, i.e. the unused frequencies in the digital television broadcast bands. The gradual completion in early 2009 of the transition from analog to digital broadcast will leave even more unused spectrum beyond the currently unused or unassigned frequencies. This raises the question of how and to what type of uses should these whitespaces be allocated. One view is that the efficient way to allocate the spectrum is to assign exclusive licenses to use the spectrum with minimal restrictions on possible usages. It is argued that protection from interference granted by the exclusivity of the licenses, and the flexibility on the end use is the key to efficiency and incentives to invest. Previous allocations that most resemble the property rights approach like the Commercial Mobile Radio Services and Advanced Wireless Services frequencies (used mostly by cellular systems) are referenced as successes in terms of consumer surplus and government revenue created by the licensed allocation. The competing view credits commons approach for the success of unlicensed applications in certain spectrum bands like Wi-Fi and Bluetooth and argues that access to spectrum should be granted to any user provided that the user meets industry standards and usage etiquettes. One of the main concerns when thinking about management regimes is interference. It is argued that in a commons regime, interference would lead to a tragedy of commons scenario where a large number of interfering devices would degrade device or service quality. On the other hand, the variety and

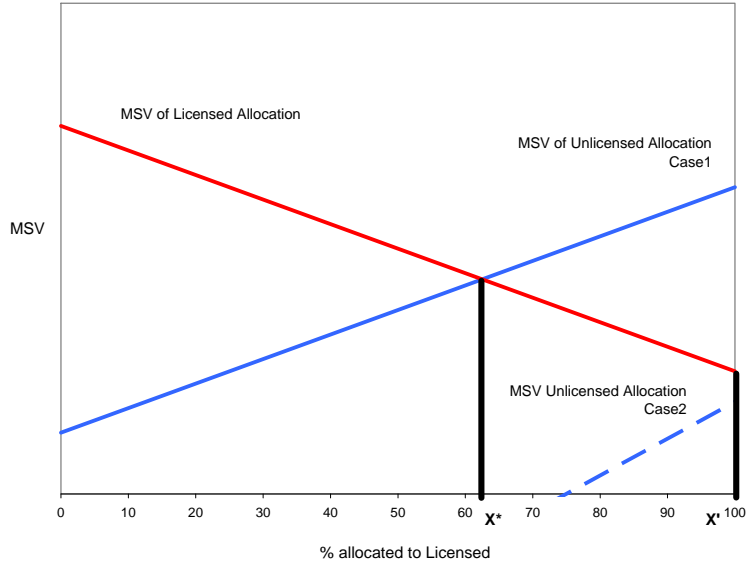


Figure 1: Marginal Social Value of Alternative Allocations

the distributed bottom up innovation resulting from open access in a commons scenario is argued to create considerable welfare. Therefore, the need for economic analysis that can enlighten the trade-offs faced in these dimensions is eminent. There are two main issues to be considered. The first is an empirical question that addresses whether the marginal social value of an unlicensed allocation is commensurate with that of licensed allocations so that an equally weighted future allocation is justified. The second question is more theoretical and looks at the welfare effects of quality degrading free entry under unlicensed allocation and compares that to a static oligopoly market with only licensed firms.

The right choice of regime requires an evaluation of the marginal welfare resulting from allocating an additional piece of spectrum under the two regimes. The simple economics behind efficient allocation dictates that the spectrum be allocated such that the marginal welfare of choosing licensing or commons to assign an additional piece of spectrum is equal. This argument is illustrated in figure one. An interior solution to such allocation problems require  $X^*$  percent to be allocated to licensed and  $1 - X^*$  percent to unlicensed. However there is the possibility that the marginal social value is lower under one of the regimes for all allocation possibilities, as illustrated by the dashed MSV curve in figure one. In this degenerate case, optimum is achieved by a cor-

ner solution where all marginal allocation favors the regime with the higher marginal social value, which is Licensing in the hypothetical example in figure one. So far under the digital transition, all of the spectrum space that has been re-assigned has been allocated to licensed usage. 74 MHz has been auctioned off and a further 24 MHz is licensed for public safety usage. If efficiency requires that all of the available spectrum, including the 98 MHz already allocated should be licensed, then we must be in a case where economic efficiency requires a corner solution. The empirical question then is whether the marginal value of a licensed allocation is always above the marginal value of an unlicensed allocation.

The valuation of consumer welfare derived from the use of licensed spectrum has been a fairly standard exercise with the availability of price and consumption histories and the quantity of licensed spectrum allocated to a particular service. For example Hazlett (2005) has estimated that the consumer surplus from the Commercial Mobile Radio Services bands which are exclusive assigned flexible licenses used to offer cellular service to be around \$80billion or about \$500 per subscriber in 2003. On the other hand, Hausman (1997) has estimated that the introduction of cellular communications created \$30 to \$50 billion consumer surplus per year. However, the evaluation of welfare derived from unlicensed spectrum has been overlooked. To our knowledge, there has not been a rigorous study that estimates the welfare gains from unlicensed spectrum usage. In particular this is a difficult exercise because unlike licensed spectrum that is used to provide few and relatively homogenous services like cellular communications, there are numerous and highly heterogenous devices and services that use unlicensed spectrum like Wi-Fi, Bluetooth, Wireless Internet Service Providers, and more importantly price and consumptions histories of such devices or services are rarely available. In fact the price is by definition zero for most services that use unlicensed spectrum. In section two we provide a simple estimation of part of the consumer surplus that can be attributed to unlicensed spectrum. We find that the consumer surplus from wireless home networking using Wi-Fi –a commons allocation- to be between \$824 and \$1362 per user per year which exceeds \$30billion in aggregate. This analysis establishes for the first time that the value of unlicensed spectrum should not be disregarded and is commensurate with the value of licensed spectrum on a per megahertz per population basis.

After establishing that there is value to be gained in unlicensed allocations, the second issue to be addressed is whether an unlicensed allocation would lead to a tragedy of commons result with excessive entry that degrades quality of devices or services because of interference. To root our paper in recent debate, we will use the whitespaces analogy but our model is applicable to any measure of spectrum on which prospective management approaches are to be evaluated. We address this concern by modeling a communications device market whereby the allocation of spectrum determines the technological environment, the competitive environment and consequently the quality and the variety of the devices in the market. We then go on to simulate the welfare consequences of instituting the alternative management regimes. The market is modeled as a differentiated goods oligopoly in three stages. In the first stage firms decide whether or not to enter and incur the entry cost. In the second stage firms choose the technological attributes resulting in a quality level. And in the third stage firms compete in quantities in the device market. We incorporate vertical differentiation through the indirect quality choice and horizontal differentiation by introducing imperfect substitutability among devices. In such a model the welfare effects of the alternative management regimes result from their effect on the final product offering of the industry. Under the licensing regime the access to and the use of any given whitespace is assigned to a single firm, thus there are as many firms as there are licenses. These firms also have complete control on the choice of device quality since they are not subject to any interference that can degrade quality. Consequently, the industry becomes a static differentiated goods oligopoly with as many firms as there are licenses. On the other hand, if the commons regime is chosen, any firm can access any given whitespace and deploy devices. This non exclusivity results in interference for firms using the same whitespace and consequently the device quality is degraded. Also, the number of firms becomes endogenous and is determined by zero profit condition subject to free entry. Assuming that consumers have preferences for both the quality and the variety of devices, the trade-off is then between a licensing regime with high quality devices but with fewer varieties, and a commons regime with a greater variety of lower quality devices.

Our work complements and extends the oligopolistic markets literature. Many of the previous works carry out welfare evaluations between price and quantity competition for a given number of

firms. The conventional wisdom in this literature upholds that price competition dominates quantity competition from a consumer welfare perspective. Hackner (2000) shows that the prices may be lower under quantity competition contrary to the prevailing wisdom in Singh and Vives (1984), and Vives (1985) when the market is bigger than a duopoly with vertical differentiation. Hsu and Wang (2005) on the other hand, show that consumer surplus under price competition in Hackner's model is still higher than quantity competition. Symeonidis (2003) shows that an increase in quality heterogeneity for a given number of firms increases consumer surplus regardless of the mode of competition. We complement these works by showing that with free entry, and endogenous quality choice subject to a negative externality, consumer surplus ceases to be monotonically increasing in the number of firms. In cases where free entry lowers quality rapidly because of a high interference elasticity, free entry may be detrimental to welfare. However, for high enough horizontal differentiation, the welfare enhancing effects of the increase in variety more than offsets the welfare degrading effects of the drop in quality as a result of interference. Ultimately, the choice on the management regime has to be informed by the preference and the technology structure. In cases where interference does not increase rapidly with entry, and consumers have a strong preference for variety, commons regime with no restriction on entry is welfare enhancing. Whereas, if interference is a concern and consumers have weak preferences for variety, restricting entry with licensing regime creates greater welfare.

This paper is organized as follows. In section two we provide a simple estimation of consumer surplus that can be attributed to unlicensed spectrum. In section three we provide a differentiated oligopoly model of a communications device market and analyze the welfare consequences of alternative management regimes. Fourth section describes the simulation results. Fifth section concludes.

## 2 An empirical valuation of Wi-Fi spectrum

### 2.1 Model

We consider a consumption problem posed in Goolsbee and Klenow (2006) where consumers maximize the following utility function which incorporates the time intensive nature of the Internet as a consumption good.

$$U = \theta(C_i^\alpha L_i^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\theta)(C_o^\beta L_o^{1-\beta})^{\frac{\sigma-1}{\sigma}}$$

where  $C_i$  is the consumption of Internet services and  $L_i$  is the time spent using the Internet. All other goods and services consumed form the composite good  $C_o$  with the time spent on the composite good being  $L_o$ .  $\theta$  represents the utility weight of the Internet bundle compared to that of the composite bundle. Finally  $(1-\alpha)$  and  $(1-\beta)$  represent the time intensities of the Internet and the composite bundles respectively.

The following is the budget that constrains consumers in their utility maximization.

$$P_i C_i + F + P_o C_o = W(1 - L_i - L_o)$$

where  $W$  is the wage,  $P_i$  and  $P_o$  are the prices of the internet service and the composite bundle respectively.  $F$  is the fixed cost of access to the Internet including the network setup.  $P_i$  can be interpreted as the price on marginal consumption which is zero in practice since internet access is usually priced as a flat monthly fee.

The combined Cobb-Douglas bundles are denoted as  $Y_i = C_i^\alpha L_i^{1-\alpha}$  and  $Y_o = C_o^\beta L_o^{1-\beta}$ . Letting the price on the bundles  $\rho_i$  and  $\rho_o$  to be the weighted average of the market price and the price of time (i.e. the wage), we have

$$\rho_i = \left(\frac{P_i}{\alpha}\right)^\alpha \left(\frac{W}{1-\alpha}\right)^{1-\alpha} \quad \text{and} \quad \rho_o = \left(\frac{P_o}{\beta}\right)^\beta \left(\frac{W}{1-\beta}\right)^{1-\beta}$$

Then, the optimal choices for the bundles become

$$Y_i = \frac{W - F}{\rho_i(1 + \Delta)} \quad \text{and} \quad Y_o = \frac{W - F}{\rho_o(1 + 1/\Delta)}$$

where

$$\Delta = \left( \frac{\rho_i}{\rho_o} \right)^{\sigma-1} \left( \frac{1 - \theta}{\theta} \right)^\sigma$$

Breaking down the bundles into their consumption and time counterparts gives the optimal choices as

$$\begin{aligned} C_i &= \frac{\alpha \rho_i Y_i}{P_i} & C_o &= \frac{\beta \rho_o Y_o}{P_o} \\ L_i &= \frac{(1 - \alpha) \rho_i Y_i}{W} & L_o &= \frac{(1 - \beta) \rho_o Y_o}{W} \end{aligned}$$

Using the optimal choices on the Internet bundle and the time spent on Internet we can get the following expression for  $\Delta$  :

$$\Delta = \frac{(1 - \alpha)(1 - \frac{F}{W}) - L_i}{L_i}$$

Observing that the cost of the access to the Internet is a small flat fee ( $F/W \approx 0$ ) and there is no marginal use pricing ( $\alpha = 0$ ), the above expression becomes approximately equal to the time spent on activities other than Internet relative to the time spent on Internet.

$$\Delta \approx \frac{1 - L_i}{L_i}$$

Using the prices of the bundles and rearranging, we get another expression for  $\Delta$  in terms of the wage

$$\Delta = AW^{(\beta-\alpha)(\sigma-1)} \left( \frac{1 - \theta}{\theta} \right)^\sigma$$

Where  $A = \left[ \frac{(P_i/\alpha)^\alpha (1-\beta)^\beta}{(P_o/\beta)^\beta (1-\alpha)^\alpha} \right]^{\sigma-1}$ . Equating the two expressions and taking the natural logarithm gives.

$$\ln \left( \frac{1 - L_i}{L_i} \right) \approx \ln(A) + (\beta - \alpha)(\sigma - 1) \ln(W) + \sigma \ln \left( \frac{1 - \theta}{\theta} \right) \quad (2.1)$$

The left hand side of the equation is the log of time spent on non-Internet activities relative to time spent on the internet which can be found in the data.  $\ln(A)$  is a constant across consumers.  $\ln(W)$  is the log of the wage and can be found in the data as well. The difference between the time intensities of the two bundles  $(\beta - \alpha)$  can also be measured from the data. So, from the estimation of this equation, assuming that the error term arises from the individual variation in the utility weight parameter  $\theta$ , the coefficient on the wage can be translated into an estimate of the elasticity of substitution between the two bundles which in turn can be used to calculate the consumer surplus.

## 2.2 Data and Estimation

We use the *North American Consumer Technographics* data from Forrester Research. The data comes from a survey conducted with a nationally representative sample of 60000 households. The survey includes various questions on ownership and use of various goods and services with a focus on telecommunications. Demographic, attitudinal and behavioral variables are present as well. We take the sample of 4865 respondents who report to have some type of home network and are online at least monthly. 2991 of the 4865 respondent have a wireless network and the remaining 1874 have other types of networks. We use data on the hours per week spent by the respondents on the Internet for personal reasons, income of the respondent and ownership and type of home networking devices. We also use data on the time spent on the Internet for work related reasons to contrast the implications of the model. We also include some demographic controls to refine the results in some of the regressions.

In the survey, answers to the questions regarding Internet time use are grouped as 1-4 hours, 5-9 hours and so on. For a conservative estimate we take the lower bounds of these intervals for our main results in the text but present the results taking the midpoints as well. The respondents with wireless networks spend an average of 10.66 hours per week on the Internet for personal reasons. This corresponds to 9.5 percent of the respondents' non-sleep time off 112 hours, assuming 8 hours of sleep per day. For respondents with wired networks, time spent on the Internet for personal reasons is 11.04 hours on average or 9.8 percent of non-sleep time. The numbers in the case where

	Average Internet Use	$(1 - \alpha)$	$(1 - \beta)$	Average full income*
Wireless Network Owners	10.66 hrs(9.5 %)	0.9877	0.6060	\$239295
Wired Network Owners	11.04 hrs(9.8 %)	0.9881	0.6045	\$190280
Wireless Network Owners (mp)	12.54 hrs(11.1 %)	0.9895	0.5986	\$234904
Wired Network Owners (mp)	12.92 hrs(11.5 %)	0.9898	0.5970	\$186762

(mp) : taking midpoints for time use calculations \*: work and leisure time valued at wage

Table 1: Summary Statistics

we use midpoints for time use calculations become 12.54 hours (11.1%) for wireless network owners and 12.92 hours (11.5%) for wired network owners.

The time intensities of the two bundles can be calculated as one minus the ratio of market expenditures on the bundle to market expenditures plus time expenditures

$$(1 - \alpha) = 1 - \frac{P_i C_i}{P_i C_i + W L_i} \quad \text{and} \quad (1 - \beta) = 1 - \frac{P_o C_o}{P_o C_o + W L_o}$$

Dividing the numerator and the denominator by  $W(1 - L_i - L_o)$  and letting the expenditure shares of the Internet and the composite bundle be  $E_i$  and  $E_o$  respectively gives

$$(1 - \alpha) = 1 - \frac{E_i}{E_i + \frac{L_i}{1 - L_i - L_o}} \quad \text{and} \quad (1 - \beta) = 1 - \frac{E_o}{E_o + \frac{L_o}{1 - L_i - L_o}}$$

We substitute 0.0033 for the expenditure share of the Internet ( $E_i$ ) since this is the expenditure for internet service providers within the national product accounts in 2005. We plug in the remaining 0.9967 for the expenditure share of the composite ( $E_o$ ) and 0.3570 for the share of work time in the non-sleep time of the consumer ( $1 - L_i - L_o$ ). The consumers with wireless networks spend 0.0951 of their time on the Internet and the remaining 0.5476 on leisure activities other than the Internet. These yield time intensities of  $(1 - \alpha) = 0.9877$  for the Internet and  $(1 - \beta) = 0.6060$  for the composite in the case of wireless network owners. Owners of wired networks spend 0.0986 of their time on the internet and 0.5442 of their time on other leisure activities. These yield time intensities of 0.9881 for the Internet and 0.6045 for the composite in the case of wired network owners. These statistics along with their counterparts in the case where we take midpoints for time use calculations are presented in table one.

We use the time intensities in the calculation of the elasticities from the coefficient on the log of

	Coefficient	Standard Error	R <sup>2</sup>	Implied Elasticity $\sigma$
Wireless Network Owners	0.2436	0.0327	0.0182	1.6381
Wired Network Owners	0.2003	0.0404	0.0129	1.5222
Internet for Work	-0.1507	0.0334	0.0055	N/A
Wireless Network Owners (c)	0.3131	0.0452	0.1219	1.8190
Wired Network Owners (c)	0.2558	0.0568	0.1439	1.6685
Wireless Network Owners (mp)	0.1893	0.0246	0.0194	1.4841
Wired Network Owners (mp)	0.1626	0.0305	0.0149	1.4139
Internet for Work (mp)	-0.1152	0.0253	0.0056	N/A
Wireless Network Owners (c) (mp)	0.2408	0.0340	0.1275	1.6150
Wired Network Owners (c) (mp)	0.1985	0.0428	0.1511	1.5066

(c) : controlling for value of assets, education and time spent on the internet for work related reasons (mp): midpoints

Table 2: Regression of  $\ln((1 - L_i)/L_i)$  on  $\ln(W)$

wage. The results of the regressions are reported in table two. The positive coefficients show that the respondents with higher incomes report spending less time on the internet. Wireless network owners are more responsive to changes in the opportunity cost of time with an elasticity of 1.6381. Wired network owners on the other hand have an elasticity of 1.5222. As a contrast, in the third and eighth row of table one, we replicate the regression taking the time spent on the internet for work related reasons as the independent variable. It can be assumed that respondents have little or no control on the time spent on the Internet for work related reasons, thus the coefficient need not be consistent with the models prediction for personal internet use. As indicated by the negative coefficient, people report to spend more time on the internet for work related reasons as the wage increases, but in this scenario it is not natural to think of the wage as the opportunity cost of the time spent on the Internet for work related reasons. These results are consistent with the findings of Goolsbee and Klenow (2006) that as the opportunity cost of time increases, people spend less time on the Internet for personal reasons but this is not true for the time spent on the Internet for work related reasons. However, conditional on having some type of network, we find the elasticities to be higher than those found in Goolsbee and Klenow (2006). Our benchmark regressions give estimates of the elasticity of 1.68 and 1.52 whereas Goolsbee and Klenow's larger sample of all respondents who are online at least monthly gives an elasticity of 1.32 which is not surprising since the larger sample includes respondents who rarely go online.

As a second attempt to refine the estimates we include some control variables. We include

education level of the respondent, number of hours spent on the internet for work related reasons and the combined value of owned assets of the respondent. The implied elasticities go up slightly. The regression including the wireless network owners posit an elasticity of 1.8190 compared to 1.6685 for wired network owners. Furthermore, in the last five rows we report the results of the regressions where we use midpoints for time use calculations. Elasticities go down slightly but the effect on welfare estimates is quite pronounced as we will illustrate in the next section.

### 2.3 Welfare

The consumer surplus can be approximated by equivalent variation. We use the expenditure function

$$E (P_o, P_i, F, W, u \mid Y_i > 0) = F + \frac{\rho_o}{(1 + 1/\Delta)^{1/(1-\sigma)}} \left( \frac{u}{1 - \theta} \right)^{\frac{\sigma}{\sigma-1}}$$

and its counterpart in the case when internet consumption is not available

$$E (P_o, W, u \mid Y_i = 0) = \rho_o \left( \frac{u}{1 - \theta} \right)^{\frac{\sigma}{\sigma-1}}$$

to calculate the equivalent variation as a percentage of full income

$$\frac{EV}{W} = \frac{E (P_o, W, u(P_o, P_i, F, W \mid Y_i > 0) \mid Y_i = 0)}{W} = \left[ \left(1 + \frac{1}{\Delta}\right)^{\frac{1}{\sigma-1}} \left(1 - \frac{F}{W}\right) - 1 \right]$$

The consumer surplus naturally depends on the elasticity of substitution  $\sigma$  between the Internet bundle and the composite bundle. Revoking the assumption that the Internet has a small flat subscription fee ( $F/W \approx 0$ ) and no price for marginal consumption ( $\alpha = 0$ ) the equivalent variation becomes

$$\frac{EV}{W} = (1 - L_i)^{\frac{-1}{\sigma-1}} - 1$$

Using the elasticity estimates we can calculate the equivalent variation relative to income. In table three we present the results of these calculations. For respondents with wireless networks, the consumer surplus turns out to be 16 percent of full income (wage income plus the value of leisure). For those respondents with other wired types of networks the consumer surplus is 22

	$\sigma$	EV/W	EV/W ( <i>l</i> )	EV/W at median income	EV/W at average income	Difference
Wireless Network Owners	1.6381	16%	3.2%	\$6755	\$7684	
Wired Network Owners	1.5222	22%	3.5%	\$6009	\$6840	\$844
Wireless Network Owners(c)	1.8190	13%	2.9%	\$6342	\$7285	
Wired Network Owners(c)	1.6685	16%	3.2%	\$5723	\$6461	\$824
Wireless Network Owners (mp)	1.4841	27%	4.2%	\$8762	\$9980	
Wired Network Owners (mp)	1.4139	34%	4.6%	\$7570	\$8618	\$1362
Wireless Network Owners(c) (mp)	1.6150	21%	3.9%	\$8404	\$9642	
Wired Network Owners(c) (mp)	1.5066	26%	4.2%	\$7415	\$8399	\$1242

(*l*): linearized (c):controlling for value of assets, education and time spent on the internet for work related reasons

Table 3: Consumer Surplus as a percentage of full income

percent of full income.

However, since the utility of consumption of the first unit is very high with a log demand the above calculations tend to overestimate the consumer surplus. To counter this effect and get a conservative estimate of the consumer surplus, we linearize the demand as in Hausman (1999) and use the fact that consumer surplus relative to full income is equal to the expenditure share divided by twice the elasticity which is equal to  $0.5 * L_i / \sigma(1 - L_i(1 - F/W))$  in our model. The calculations yield a consumer surplus of 3.2 percent of full income for wireless network owners. This corresponds to \$6755 per year for the wireless network owner with the median full income in the sample. On average wireless network owners realize a consumer surplus of \$7648. The median income consumer with a wired network realizes \$6009 of consumer surplus per year which is 3.5 percent of the full income and on average wired network owners realize \$6840 consumer surplus per year. The difference in the average consumer surplus is \$844 per year in the case without controls. Controlling for the value of assets, time spent on the Internet for work and education, slightly decrease the welfare estimates. The average consumer surplus of wireless network owners goes down to \$7285 per year. The average consumer surplus of consumers with a wired network goes down to \$6461 per year. The incremental consumer surplus realized by wireless network owners on average goes down to \$824 per year with the controls. The welfare estimates go up across the board in the case where midpoints are used in the calculation of time use. The Incremental consumer surplus that the wireless networks owners realize goes up to \$1362 in the benchmark case and is slightly lower at \$1242 when controlling for assets, education and Internet time use for

work related reasons.

It is important to point out that these estimates, although taking into account the time-intensities in a more accurate way, still have to be viewed with caution. First reason to be cautious is that all of the non-sleep time for the consumer is valued at the wage. If consumers value their leisure time less than the wage, then we would be over-estimating the welfare gains. We also do not take into account other time-intensive substitutes to the Internet except the composite. Taking into account other time intensive substitutes like watching TV or gym membership would increase elasticities and would reduce the welfare estimates.

### 3 An oligopoly model of a communications device market

Having established that unlicensed spectrum can create considerable welfare even with a very conservative valuation approach, we now go on to analyze whether interference resulting from free entry and the possibility of a tragedy of commons scenario in an unlicensed regime would be detrimental to welfare.

#### 3.1 Model Preliminaries.

We start by considering a market for communications devices. There are  $M$  consumers with preferences summarized by the following quasilinear utility function defined over a homogenous good and  $n$  varieties of differentiated communications devices:

$$U(q_0, q_1, \dots, q_n) = \sum_{i=1}^n \left( q_i - \frac{q_i^2}{T_i} \right) - \gamma \sum_i \sum_{j < i} \frac{q_i}{T_i} \frac{q_j}{T_j} + q_0 \quad (3.1)$$

where  $q_i$  and  $T_i$  are respectively the quantity and quality of variety  $i = 1, \dots, n$  and  $q_0$  is the quantity of a homogenous good that we also chose to be the numeraire. This specification incorporates both vertical and horizontal dimensions of product differentiation. The vertical attribute  $T_i$  measures the inherent objective quality of a device of variety  $i$ . On the other hand, the  $n$  varieties of devices differ according to a horizontal characteristic which is captured by the parameter  $\gamma \in [0, 2]$ . This parameter is exogenously given and measures the extent to which any two

of the  $n$  varieties are substitutable. As  $\gamma$  goes to zero, devices become independent and as  $\gamma$  goes to 2, devices become perfect substitutes. The utility function is strictly concave and decreasing in  $\gamma$  which reflects that the consumer has a taste for variety. The consumer also supplies one unit of labor inelastically for the production of the homogenous good in a perfectly competitive industry with constant returns to scale. One unit of labor is required to produce a homogenous good and marginal cost pricing under perfect competition equalizes labor income to unity. The quasilinearity isolates the utility in (1) from income effects and assuming that the consumer has a large endowment  $\tilde{q}_0$  of the homogenous good we focus on interior solutions. Following a standard utility maximization, the demand and the inverse demand are given respectively by

$$\frac{q_i}{T_i} = \frac{[2 + \gamma(n - 2)]T_i(1 - p_i) - \gamma \sum_{j \neq i} T_j(1 - p_j)}{(2 - \gamma)[2 + \gamma(n - 1)]} \quad (3.2)$$

$$p_i = 1 - \frac{2q_i}{T_i^2} - \frac{\gamma}{T_i} \sum_{j \neq i} \frac{q_j}{T_j} \quad (3.3)$$

The differentiated communications devices are produced by oligopolistic firms under increasing returns to scale. Marginal costs of devices are zero, however a fixed cost  $F$  is required to produce any level of output of any quality. This fixed cost represents the cost of obtaining access to a key resource or technology, in our case the access to spectrum. The presence of the fixed cost also implies a one-to-one mapping between varieties and firms. Thus, the subscript  $i$  interchangeably refers to a firm and the single variety it produces. This model subsumes the standard horizontal differentiation model that can be obtained by setting  $T_i = 1$  which reduces the model to the standard quadratic utility  $\sum_i (q_i - q_i^2) - \gamma \sum_i \sum_{j < i} q_i q_j$  with the inverse demand  $p_i = 1 - 2q_i - \gamma \sum_{j \neq i} q_j$ .

### 3.2 The device market

We model the device market as a three stage game. In the first stage, firms decide on whether to incur the fixed cost and enter the market. In the second stage, those firms that participate, noncooperatively choose the quality of their devices  $T_i(\cdot)$  and incur the associated costs  $K(T_i)$ . We will allow quality to be a direct choice variable as well as a function of an underlying choice

variable, namely device design. This will later allow us to incorporate quality externalities that are caused by free entry. In the third stage firms compete in quantities. The fixed cost and the rules of entry in the first stage represents the regulatory authority's choice on the management regime. Licensing regime entails the creation of a license to access and use the spectrum whitespace. Abstracting from any optimizing behavior on the part of the regulatory authority over the number of licenses and the fixed fee, we assume that a single license for each of the  $w$  whitespaces is created. These licenses are assumed to be allocated with a second price auction for the sake of simplicity. Assuming an outside option of zero, all firms would bid up to their prospective symmetric profit. This tie is then broken in favor of a random allocation of the licenses to  $w$  of the firms and those firms do participate in this case of indifference. The regulatory authority extracts all the profit with the access price  $F = \pi_i(w)$  where  $\pi_i(w)$  is the profit of each firm when there are  $w$  firms that are active. On the other hand, under commons regime, the access fee  $f$  is set close to zero and firms enter the market until net profit falls to zero. In both cases, firms end up with zero profit and we only focus on consumer surplus. We solve for the subgame perfect equilibria of the model for a given number of firms. After solving the model for a given number of firms, we provide equilibrium analysis and comparative statics as well as a simulation of the free entry equilibria under the two regimes.

### 3.2.1 Quantity competition

We solve the model by backward induction. In the last stage firms choose quantities simultaneously to maximize profits, taking the quantity choices of the rival firms as given.

$$\max_{q_i} \pi_i = M \left( 1 - \frac{2q_i}{T_i^2} - \frac{\gamma}{T_i} \sum_{j \neq i} \frac{q_j}{T_j} \right) q_i - K(T_i) \quad (3.4)$$

Taking the first order conditions and rearranging gives the reaction functions as

$$\frac{q_i}{T_i} = \frac{1}{4} \left( T_i - \gamma \sum_{j \neq i} \frac{q_j}{T_j} \right)$$

We impose symmetry and solve for the Cournot quantities and the resulting prices

$$q_i^c = \frac{T_i \left( [4 + \gamma(n-2)]T_i - \gamma \sum_{j \neq i} T_j \right)}{[4 + \gamma(n-1)](4 - \gamma)} \quad p_i^c = \frac{2 \left( [4 + \gamma(n-2)]T_i - \gamma \sum_{j \neq i} T_j \right)}{T_i [4 + \gamma(n-1)](4 - \gamma)}$$

Plugging the Cournot quantities and prices back into the profit function gives the second stage profit as a function of qualities. In the second stage firms maximize profit by choosing the quality. The maximization problem at the second stage is:

$$\max_{T_i} \pi_i = \frac{2M}{a^2 b^2} \left( (a - \gamma)T_i - \gamma \sum_{j \neq i} T_j \right)^2 - K(T_i)$$

where  $a = [4 + \gamma(n-1)]$  and  $b = (4 - \gamma)$ . Differentiating with respect to quality gives the first order condition:

$$\frac{4M}{a^2 b^2} \left( (a - \gamma)T_i - \gamma \sum_{j \neq i} T_j \right) (a - \gamma)T_i' - K'(T_i) = 0$$

Summing up over  $i$ , imposing symmetry, rearranging and substituting for  $a$  and  $b$  yields

$$T_c'(\cdot)T_c(\cdot) \left( \frac{4 M [4 + \gamma(n-2)]}{[4 + \gamma(n-1)]^2(4 - \gamma)} \right) = K'(T_c) \quad (3.5)$$

For well behaved functional forms of quality and cost of quality, this equation can be solved to obtain the Cournot-Nash equilibrium quality  $T_c$ . We can then characterize the subgame perfect equilibrium by plugging quality back into the Cournot quantities and prices. For the symmetric case at hand, these become:

$$q_c = \frac{(T_c)^2}{[4 + \gamma(n-1)]} \quad p_c = \frac{2}{[4 + \gamma(n-1)]}$$

The equilibrium profit of each firm is given by

$$\pi_c = \frac{2M(T_c)^2}{[4 + \gamma(n-1)]^2} - K(T_c)$$

The equilibrium number of firms will be determined in the first stage by the rules and the fixed cost of entry. When licensing regime is implemented number of firms will be exogenous and equal to the number of licenses  $n^* = w$ . Under commons regime, equilibrium number of firms will be determined by zero profit condition  $\pi_c(n^*) - f = 0$ . The consumer surplus is then obtained by calculating how much consumers would have to be paid to forgo the entire consumption bundle, which is expressed as

$$CS_c(n^*) = n^* M \left( q_c - \left( \frac{q_c}{T_c} \right)^2 - \frac{\gamma(n^* - 1)}{2} \left( \frac{q_c}{T_c} \right)^2 - p_c q_c \right)$$

### 3.2.2 Specification of quality

Quality specification is motivated by the engineering aspects of communications devices. We treat the quality choice as a convex problem in its own stage. Firms indirectly choose quality by choosing the design of their devices with a decreasing marginal quality. When carried over to the quantity competition stage, this configuration results in a concave net profit as a function of design. We assume design is increasingly costly with a convex marginal cost. This amounts to assuming that cost increases faster than gross profit and admits an interior optimum design. For the specification of quality we turn to Shannon-Hartley theorem of Information Theory. In this configuration the quality of a device is measured by the information throughput of the device in kilobits per second. The amount of information that can be reliably transmitted over a communications channel by any device is bounded above by a channel capacity. This upper bound is specified by the Shannon-Hartley theorem as  $C = W \log_2(1 + \frac{S}{N})$  where  $C$  is the channel capacity in kilobits per second (kbps),  $W$  is the bandwidth of the channel in kilohertz (kHz),  $S$  is the total signal power in watts over the bandwidth and  $N$  is the total noise power over the bandwidth. Shannon (1949) calls the system that transmits without errors at rate  $C$  an ideal system and goes on to add that such a system can not be achieved with any finite encoding process but can be approximated as closely as desired by efficiently designing the device and the encoding properties. Firms in our model aim to approximate the Shannon upper bound by investing in the design  $d_i \geq 0$  of the devices they produce. Thus the quality  $T_i$  of a device of variety  $i$ , measured by the attained infor-

mation throughput is given by  $T_i(d_i) = (1 - e^{-d_i}) C$  with  $T_i' > 0, T_i'' < 0$  and  $\lim_{r_i \rightarrow \infty} T_i(d_i) = C$ . The cost of design is  $K_i(d_i) = e^{d_i} - d_i - 1$ , with  $K_i(0) = K_i'(0) = 0$  and  $K_i' > 0, K_i'' > 0$ . The convexity of the design cost reflects the idea that as one aims to get closer to the Shannon upper bound, the necessary encoding becomes increasingly complex resulting in higher marginal cost<sup>2</sup>. This assumption guarantees the existence of a symmetric interior equilibrium.

The quality degrading effects of interference are captured through the noise power over the bandwidth. Interference is an increase in noise that a system becomes subject to with the operation of another system in the same bandwidth. Assuming that the  $n$  firms will be distributed uniformly over the whitespaces, the number of firms that operate on the same whitespace at any given time is defined as  $m = \frac{n - (n \bmod w)}{w}$  where  $w$  is the number of whitespaces. Because of the discreteness of the number of firms we use a modular approach and we assume that total noise power over the bandwidth is increasing in  $m : \partial N(m) / \partial m > 0$ , and that the interference elasticity is  $d \log N(m) / d \log m = \varepsilon$ . With these specifications, the quality of a device is expressed as

$$T_i(d_i | W, w, S, N, n) = (1 - e^{-d_i}) W \log_2 \left( 1 + \frac{S}{Nm^\varepsilon} \right) \quad (3.6)$$

### 3.3 Equilibrium analysis

Setting quality equal to one reduces the model to the standard horizontally differentiated Cournot model with the inverse demand  $p_i = 1 - 2q_i - \gamma \sum_{j \neq i} q_j$ . The equilibrium quantities and prices are given by:

$$q_c = \frac{1}{[4 + \gamma(n - 1)]} \quad p_c = \frac{2}{[4 + \gamma(n - 1)]}$$

For any positive substitutability, the profit  $\pi_c = 2M/[4 + \gamma(n - 1)]^2$  is monotonically decreasing in the number of firms. Furthermore the consumer surplus  $nM(2 + \gamma(n - 1))/2[4 + \gamma(n - 1)]^2$  is monotonically increasing in the number of firms. Therefore, a reduction in the fixed cost of entry that allows more firms to enter would increase the consumer surplus in a scenario without vertical differentiation. This conventional wisdom that greater competition leads to greater consumer

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<sup>2</sup>See James Martin's book *Telecommunications and the Computer*, ch 16 for an account of encoding and channel capacity.

surplus fails to hold in scenarios with vertical differentiation and the threshold where free entry starts to decrease consumer surplus depends on the substitutability and the speed of decrease in quality due to interference that comes about with free entry.

Substituting the quality  $T_i(d_i) = C(1 - e^{-d_i})$  where  $C = W \log_2(1 + \frac{S}{Nm^\epsilon})$  and the cost of design  $K(d_i) = e^{d_i} - d_i - 1$  into the first order condition yields

$$\frac{4MC^2(a - \gamma)^2}{a^2b^2} (e^{-d_i}) (1 - e^{-d_i}) - \frac{4MC^2\gamma(a - \gamma)}{a^2b^2} (e^{-d_i}) \sum_{j \neq i} (1 - e^{-d_j}) = e^{d_i} - 1$$

Summing over  $i$  and solving for the symmetric design gives the equilibrium design as

$$d_c = \frac{1}{2} \ln \left( \frac{4 M C^2 [4 + \gamma(n - 2)]}{[4 + \gamma(n - 1)]^2 (4 - \gamma)} \right) \quad (3.7)$$

With this closed form expression for device design we can characterize the subgame perfect equilibrium by setting the design choice to its equilibrium level which then implies the equilibrium quality

$$T_c(d_c|W, w, S, N, n) = (1 - e^{-d_c}) W \log_2(1 + \frac{S}{Nm^\epsilon}) \quad (3.8)$$

Cournot output and price then becomes

$$q_c = \frac{(T_c)^2}{[4 + \gamma(n - 1)]} \quad p_c = \frac{2}{[4 + \gamma(n - 1)]}$$

The symmetric equilibrium profit is given by

$$\pi_c = \frac{2M(T_c)^2}{[4 + \gamma(n - 1)]^2} - K(d_c)$$

Finally the consumer surplus is obtained by finding how much the consumers would have to be paid to forgo the entire consumption bundle. This is given by  $CS^c = M(U(Q) - np^c q^c)$  where  $Q = \{q_i | i \in 1, \dots, n; q_i = q^c\}$ . Then the consumer surplus becomes

$$CS^c = nM \left( q^c - \left( \frac{q^c}{T^c} \right)^2 - \frac{\gamma(n-1)}{2} \left( \frac{q^c}{T^c} \right)^2 - p^c q^c \right)$$

## 4 Simulations

The rich preference and technology specification comes at the cost of complex closed form comparative statics. However, we can simulate the equilibria and analyze the effects of key parameters and regime choices. Having solved for the subgame perfect equilibria, we go on to simulate these equilibria for a range of parameter choices. There are seven parameters of the model. These are the number of whitespaces  $w$ , the bandwidth of whitespaces  $W$ , signal power over the bandwidth  $S$ , noise power over the bandwidth  $N$ , substitutability  $\gamma$ , interference elasticity  $\varepsilon$ , and the fixed cost  $F$ . For a given number of firms  $n$  and a set of parameter values  $\{w, W, S, N, \gamma, \varepsilon, F\}$  we can calculate the equilibria of the second and the last stages. The free entry condition of the first stage can be simulated by iterating the number of firms until profit is exhausted.

Under the licensing regime, the equilibrium number of firms is always equal to the number of whitespaces since there is a single license to obtain access to each whitespace. The profit of the active firms will be extracted in the entry stage by the regulatory authority with  $F = \pi_i$  obtained by the auction mechanism. So given the set of parameter values except  $F$ , the equilibrium under licensing regime is obtained simply by pinning down  $n = w$ . The fixed cost parameter is obtained in equilibrium by setting  $\pi_i = F$ . Under the commons regime, access cost is close to zero and number of firms will increase until profit equals this minimal fixed entry cost. So, the equilibrium under commons regime for a given set of parameter values is obtained by setting the fixed cost sufficiently low at  $F = 10^{-6}$  and iterating over the number of firms until equilibrium profit is zero.

We pin down the bandwidth  $W$  of a given whitespace to  $6000kHz$  since this is the bandwidth of a TV whitespace that is under question. And we pin down the number of whitespaces  $w$  to 10, since this is roughly the average number of whitespaces in a given geographic area. Another convenient way to reduce the parameter space without loss of generality is to make use of the

fact that the noisiness of a communication channel is expressed in decibels(dB) by Signal to Noise Ratio,  $SNR = 10 \log_{10}(\frac{S}{N})$ . A typical signal to noise ratio for a wireless networking device is about 30dB which corresponds to a signal power that is 1000 times the noise power over the channel. To be able to analyze equilibria in different native noise environments in the simulations we let this parameter vary within the set  $S = \{0, 10, 20, 30, 40, 50, 60, 70, 80, 90\}$ . In the same vein, we want to analyze different values for how fast interference in a given whitespace increases with the entry of a firm. So we let the interference elasticity  $\varepsilon$  vary within the set  $E = \{0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$ . For an interference elasticity of  $\varepsilon = 0.2$ , the increase in noise due to the entry of new firms would be relatively slow, whereas for  $\varepsilon = 1$  it would be linear, and for  $\varepsilon = 2$  it would be quadratic. We also do not want to pin down substitutability to be able to allow different preferences on the demand side. We let the substitutability parameter  $\gamma$  vary within the set  $\Gamma = \{0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9\}$ . To avoid degenerate equilibria under price competition, we choose not to include the extreme cases of perfect substitutability at  $\gamma = 2$  and no substitutability at  $\gamma = 0$ . Table four in the appendix summarizes these parameter choices.

Consistent with the parameter choices there are three levels of iterations in the simulation algorithm. Given the number and the bandwidth of the whitespaces and the fixed cost of access, first the algorithm picks the native signal to noise ratio. Then, the algorithm picks the interference elasticity at the next level. Finally, the algorithm picks the substitutability in the innermost level. It then continues by calculating the equilibrium design, qualities, quantities, prices, profits and consumer surplus as it iterates over the number of firms. To find the equilibria under licensing regime one iteration suffices since number of firms equals that of the whitespaces with a single firm operating in a given whitespace. Effectively, licensing regime creates an industry with Cournot competitors that operate without interference and that are equal in number to that of whitespaces. To simulate the commons regime, the algorithm iterates over the number of firms until the profit per firm is equal to the access fee. The algorithm is summarized in pseudocode in table five.

With these specifications we simulate the model in 10 cases of native signal to noise ratios, and in each case, one hundred times for the combinations of substitutability and interference elasticity

parameters. We obtain one thousand equilibria for each parameter combination in  $S \times E \times \Gamma$ . All of the equilibria are included in the appendix in figures thirteen through twenty two. Since the fixed cost is lower under commons than it is under licensing, and since profit is decreasing in the number of firms, there are always at least as many firms under commons regime as there are under licensing. So, moving from an oligopoly with a fixed number of firms to a free entry scenario would be welfare enhancing according to the conventional wisdom. This would ideally lead to a welfare improvement through tougher competition, lower prices and a greater variety. However, the presence of interference complicates the conventional comparative statics. In environments like ours that are prone to congestion or interference, care must be taken when evaluating welfare consequences of policies that shape the market.

Figures two through eleven show the consumer surplus under commons and licensing regimes for each of the ten native signal to noise ratios that we simulated the model for. In each figure the x and y axis are the combinations of substitutability and interference elasticity, and the two dimensional manifolds plot the consumer surplus under licensing regime and the consumer surplus under commons regime measured at the vertical axis. The gray surface is the consumer surplus under licensing regime, and the colored surface is the consumer surplus under commons. Starting at the lowest native SNR, we observe in only one of the one hundred combinations of substitutability and interference elasticity that the commons regime with free entry dominates the licensing regimes with number of firms restricted to the number of licenses. Except the combination of the lowest substitutability at 0.1 and lowest interference elasticity at 0.2, all other combinations result in higher consumer surplus under licensing. So in environments that are inherently noisy, and with low return to design investment, instituting a commons regime with free entry would result in higher welfare only in cases where interference does not increase quickly with entry and consumers have strong preferences for variety. As we go to higher native signal to noise ratios, the combinations of substitutability and interference elasticity for which commons regime dominates expands. At higher native SNRs firms have more room to respond to increases in noise due to entry through their choice on device design. As native SNR increases, the marginal quality improvement for a unit of design investment becomes higher and more firms prevail in a free entry equilibrium.

At the highest native signal to noise ratio, about half of the combinations result in commons dominating licensing in terms of consumer surplus. The regions where commons dominates can more clearly be seen in the contour plots of the consumer surplus ratio in table six. The tension between the preference for quality and the preference for variety determines the thresholds in the parameter space where the dominance goes from one regime to the other. The regions where commons regime creates more surplus fall towards the regions with lower substitutability and low interference elasticity and the region grows as one move towards higher native SNR. The boundary in the space of substitutability, interference elasticity and native SNR, where welfare dominance changes from one regime to the other is depicted in figure twelve. At any combination that lies above this manifold, commons creates higher consumer surplus whereas any combination below favors licensing.

Comparison of the free entry and static equilibria for a given native SNR, and combinations of substitutability and interference elasticity can be carried out by analyzing figures thirteen through twenty two. These figures presents the key variables for each of the 100 equilibria that is simulated for each of combination of substitutability and interference elasticity parameters. Licensing equilibria on the right all have 10 firms since that is the number of licenses. Commons equilibria on the left block have higher number of firms prevailing in equilibrium, thus higher varieties. Not surprisingly, prices are uniformly lower under the free entry equilibria, so are the qualities. Quantities on the other hand show a non monotonic pattern as one moves from licensing to commons. For high interference elasticities, as one moves to a free entry equilibria, the drop in quality because of interference is pronounced. The number of firms that enter is not very high and each active firm produces less compared to the licensed firms. Consequently, the welfare degrading effects of the drop in quality and quantity more than offset the welfare enhancing increase in varieties and the decrease in prices. However, as one moves to lower interference elasticities, a switch from licensing to commons allows more and more firms to prevail in equilibrium and causes the usual drop in price and qualities. The drop in prices becomes sharper and the drop in qualities become less pronounced. At the same time, along the substitutability dimension, lower substitutability leaves more market power to each prevailing firm and allows higher number

of firms to prevail in equilibrium. Depending on the intensity of entry, prices may decrease or increase. For high interference elasticities, the intensity of entry is low, thus prices increase as substitutability declines, however, for lower interference elasticities, intensity of entry is high, and prices go down. The final effect on welfare of going from a static equilibrium to a free entry equilibrium is a priori ambiguous. We observe that for low enough interference elasticity, there are low levels of substitutability for which free entry equilibria and the welfare enhancing effects of greater varieties therein, more that offsets the welfare degrading effects of lower qualities. As we move to higher native SNRs, the decrease in the noisiness of the channel allows more firms to operate for any given parameter combination without degrading quality. This pushes prices down and increases quantities leaving the consumer with a higher gain in consumer surplus for any combination of substitutability and interference elasticity. Therefore, the regions of substitutability and interference elasticity plane where commons dominates in terms of consumer surplus expands as one moves to less noisy environment indicated by higher native SNRs. This can be seen in the contour plots in table six. We can summarize all these observations in figures twelve. This figure shows the manifold in the three dimensional parameter space that is obtained by combining the contour plots for each native SNR. The separating manifold divides the space into two regions and any parameter combination above this manifold the free entry equilibria of the commons regime creates more consumer surplus than the static equilibria of licensing regime.

## 5 Conclusion

We establish a lower bound on the value of unlicensed spectrum and show that marginal welfare of unlicensed allocation should not be disregarded. To address the concerns about a tragedy of commons scenario we provide a model in which interference is incorporated as a quality degrading externality and in which the choice of spectrum management regime affects consumer welfare through basic market mechanisms. We modelled consumers as having preferences, over a range of varieties, for communications devices and device qualities. Devices are produced and sold by competitive firms. Choice of management regime affects the welfare outcome in two ways. First it directly affects the level of competition in the market and secondly by indirectly affecting the

quality of devices produced. We find that, if interference increases rapidly with number of firms in the market and consequently quality is degraded quickly with free entry, than restricting entry with a licensing regime might be welfare enhancing. Otherwise, despite the lower qualities, the increase in variety and the consequent increase in utility more than offsets the drop in quality. In addition to the empirical finding that establish commensurate values for similar licensed and unlicensed spectrum bands, the oligopoly model also prescribes a mix of regime choices informed by the technological (interference) and economic (substitutability) characteristics of the market and application at hand. Favoring one regime unanimously over the other without regard to the technological and economic environment would likely add to the inefficiencies that current allocations suffer from.

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# Appendix

## Tables and Figures

	Parameter	Value
$W$	Bandwidth of a whitespace	$6000kHz$
$w$	Number of whitespaces	10
$F$	Fixed cost of access	$10^{-6}$ under commons, $\pi_i$ under licensing
$SNR$	Signal to Noise Ratio(dB)	{0, 10, 20, 30, 40, 50, 60, 70, 80, 90}
$\varepsilon$	Interference elasticity	{0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2}
$\gamma$	Substitutability	{0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9}

Table 4: Model Parameters

---

### Algorithm : Equilibria

**Input:**  $\{w, W, SNR, \gamma, \varepsilon, F\}$

**Output:** design  $d$ , quality  $T(d)$ , quantity  $q$ , price  $p$ , profit  $\pi$ , consumer surplus  $CS$

**for each**  $SNR$  **in the set** {0, 10, 20, 30, 40, 50, 60, 70, 80, 90}

**for each**  $\varepsilon$  **in the set** {0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2}

**for each**  $\gamma$  **in the set** {0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9}

**while**  $\pi_i > F$  for  $i:1,2,\dots,n_{max}$

**calculate** design  $d_i$ , quality  $T_i(d_i)$ , quantity  $q_i$ , price  $p_i$ , profit  $\pi_i$ , Consumer Surplus  $CS$

**if**  $\pi_i = F$  **stop**

**record output**

**terminate**

---

Table 5: Summary of Simulation Algorithm

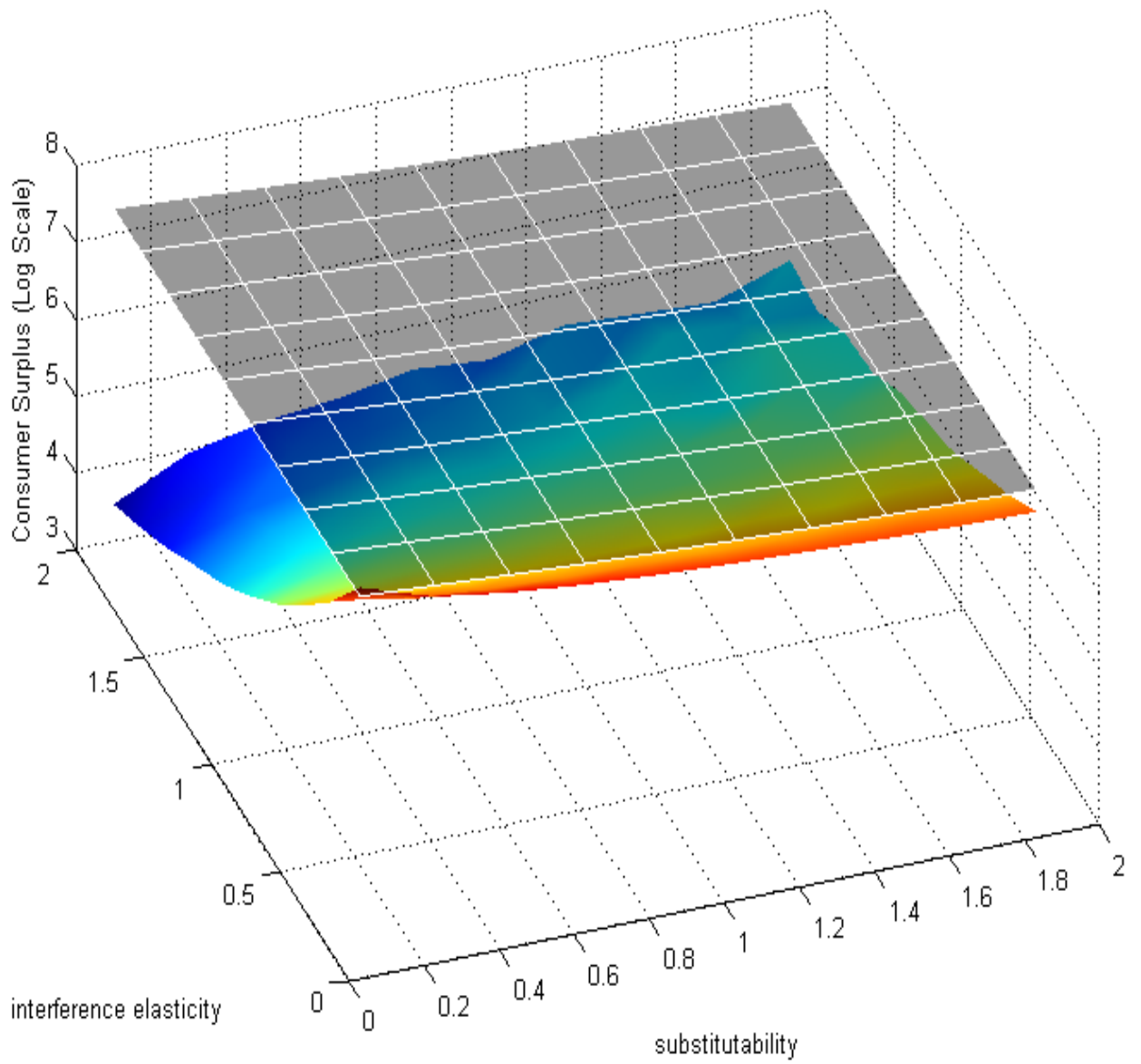


Figure 2: Cournot Consumer Surplus at SNR=0dB (Licensing in gray, Commons in color)

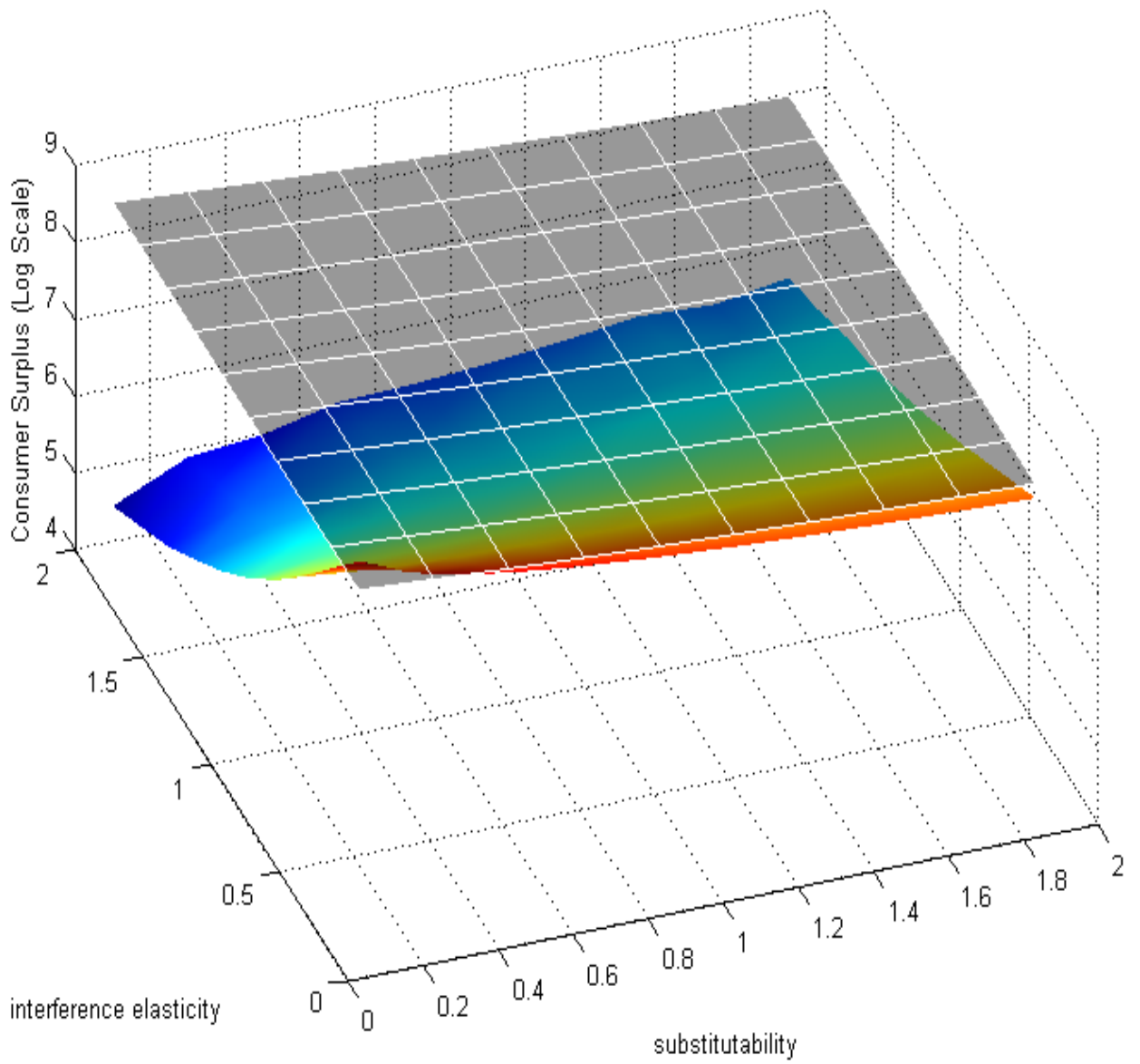


Figure 3: Cournot Consumer Surplus at SNR=10dB (Licensing in gray, Commons in color)

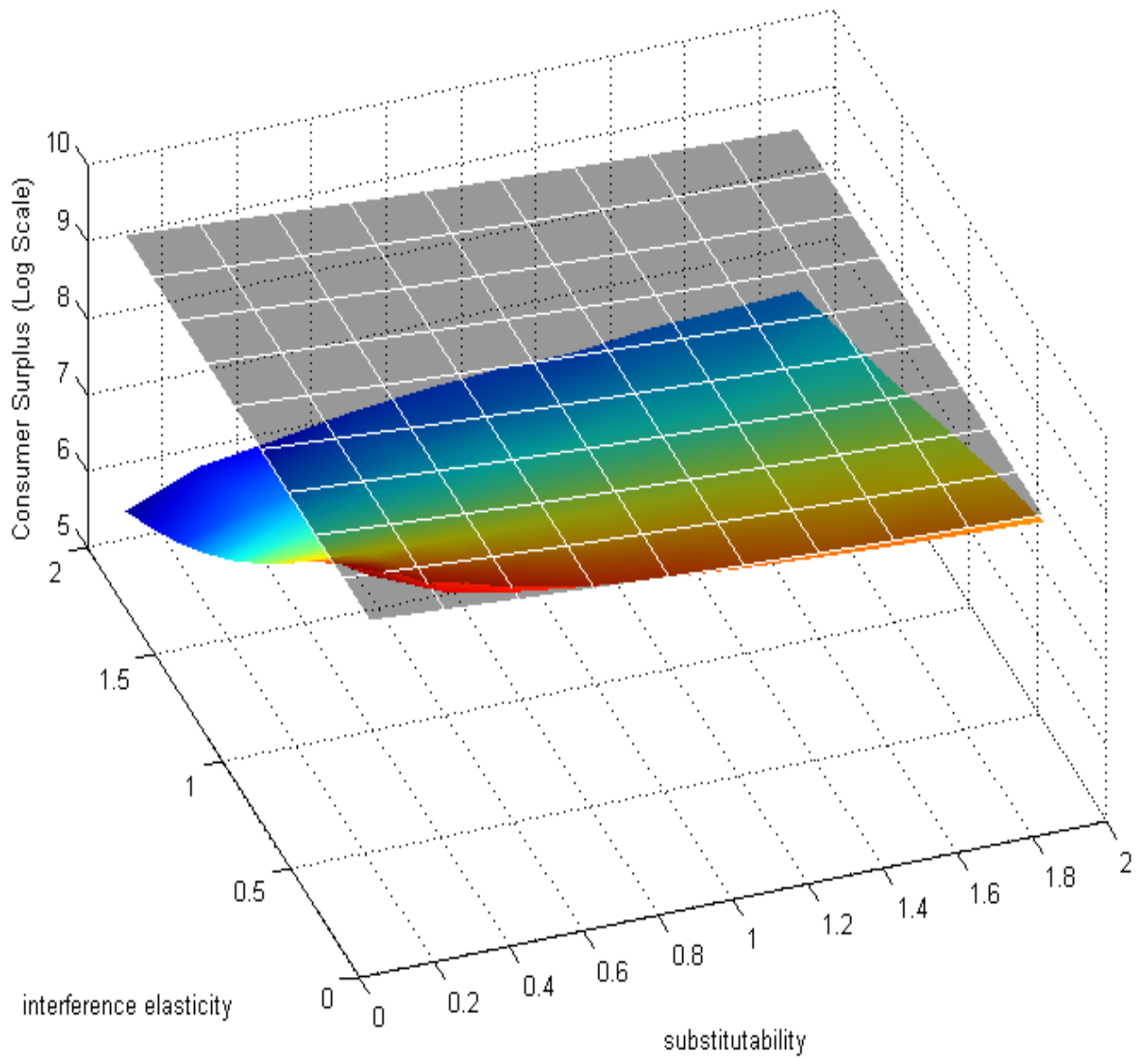


Figure 4: Cournot Consumer Surplus at SNR=20dB (Licensing in gray, Commons in color)

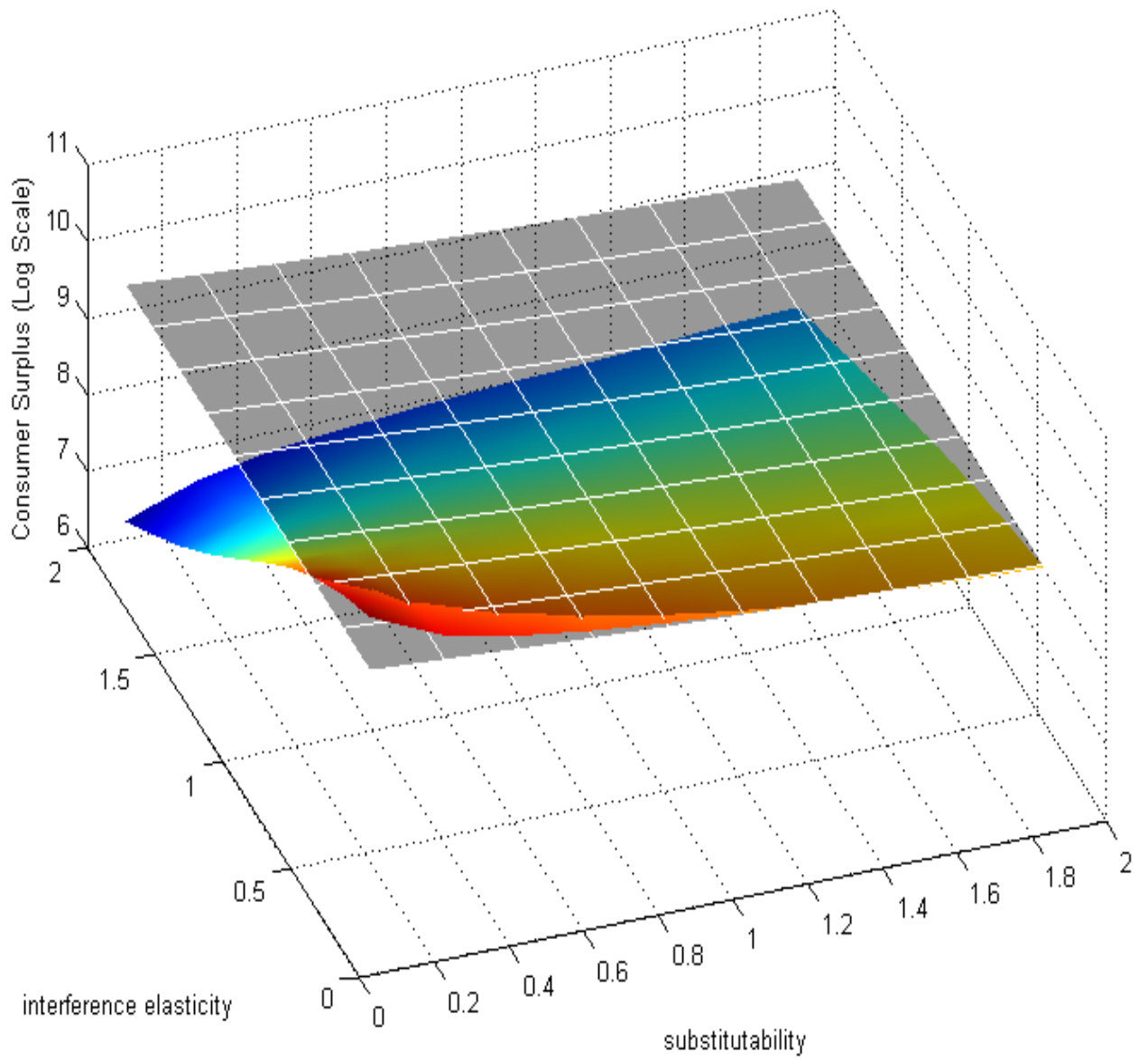


Figure 5: Cournot Consumer Surplus at SNR=30dB (Licensing in gray, Commons in color)

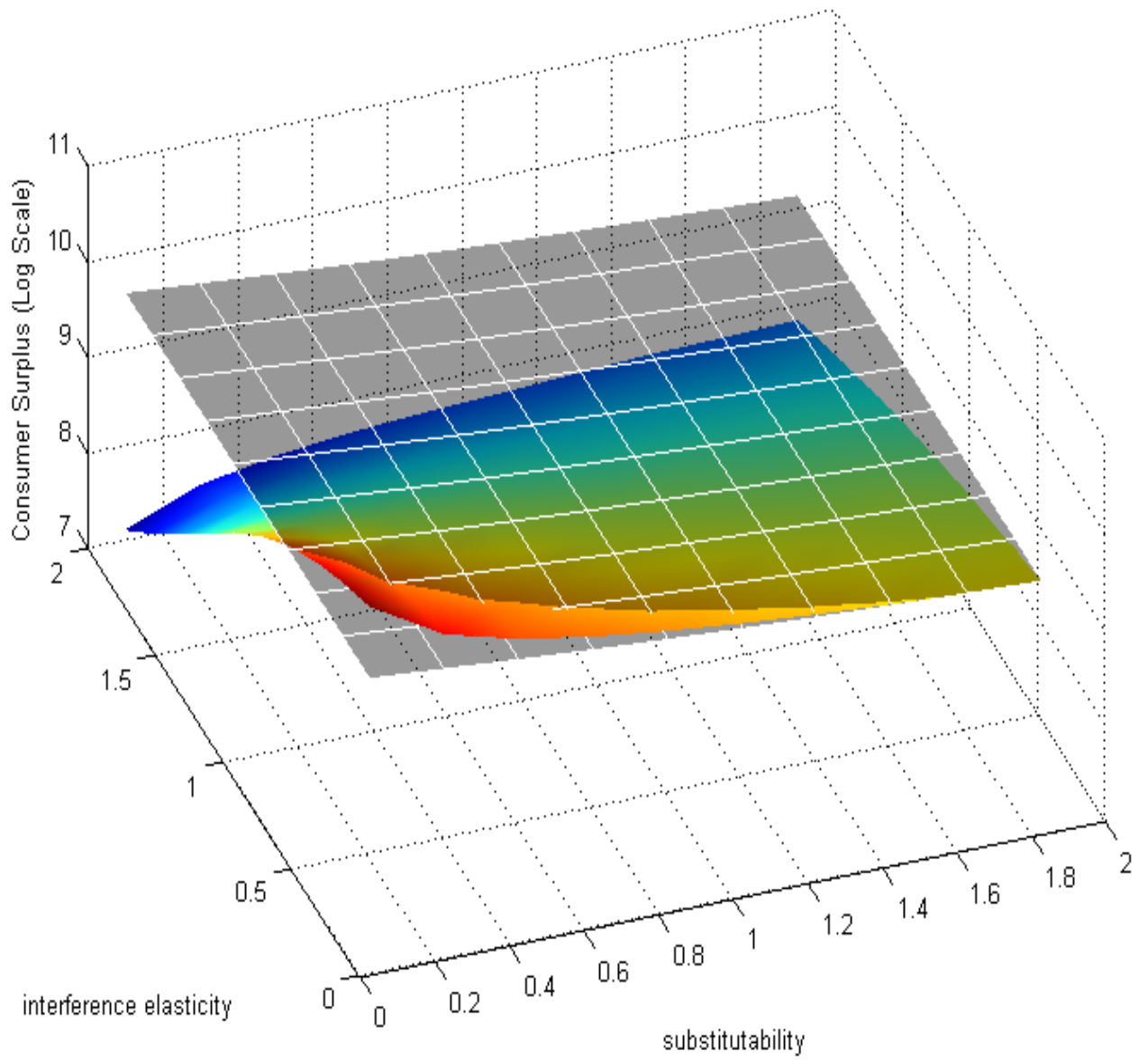


Figure 6: Cournot Consumer Surplus at SNR=40dB (Licensing in gray, Commons in color)

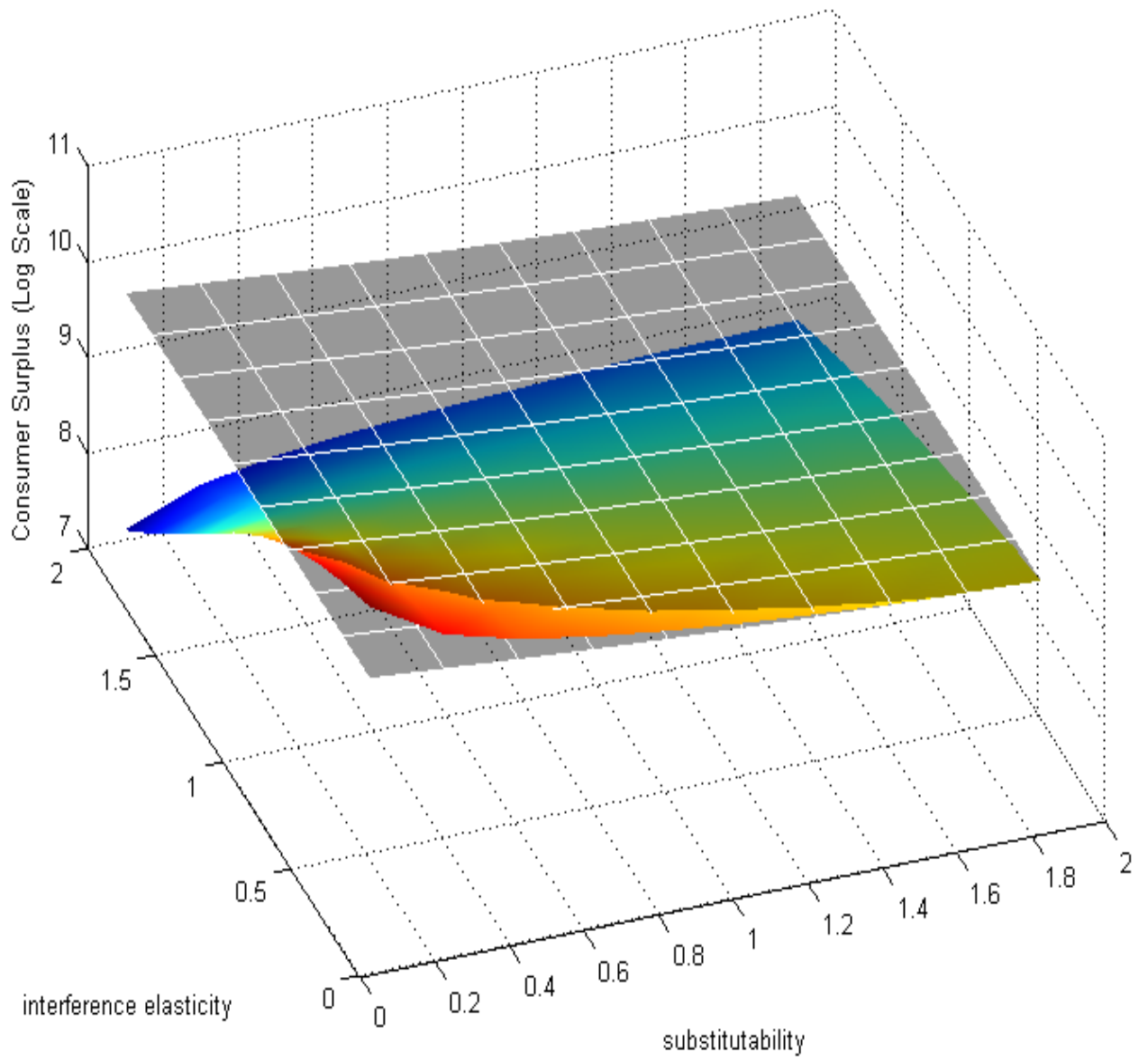


Figure 7: Cournot Consumer Surplus at SNR=50dB (Licensing in gray, Commons in color)

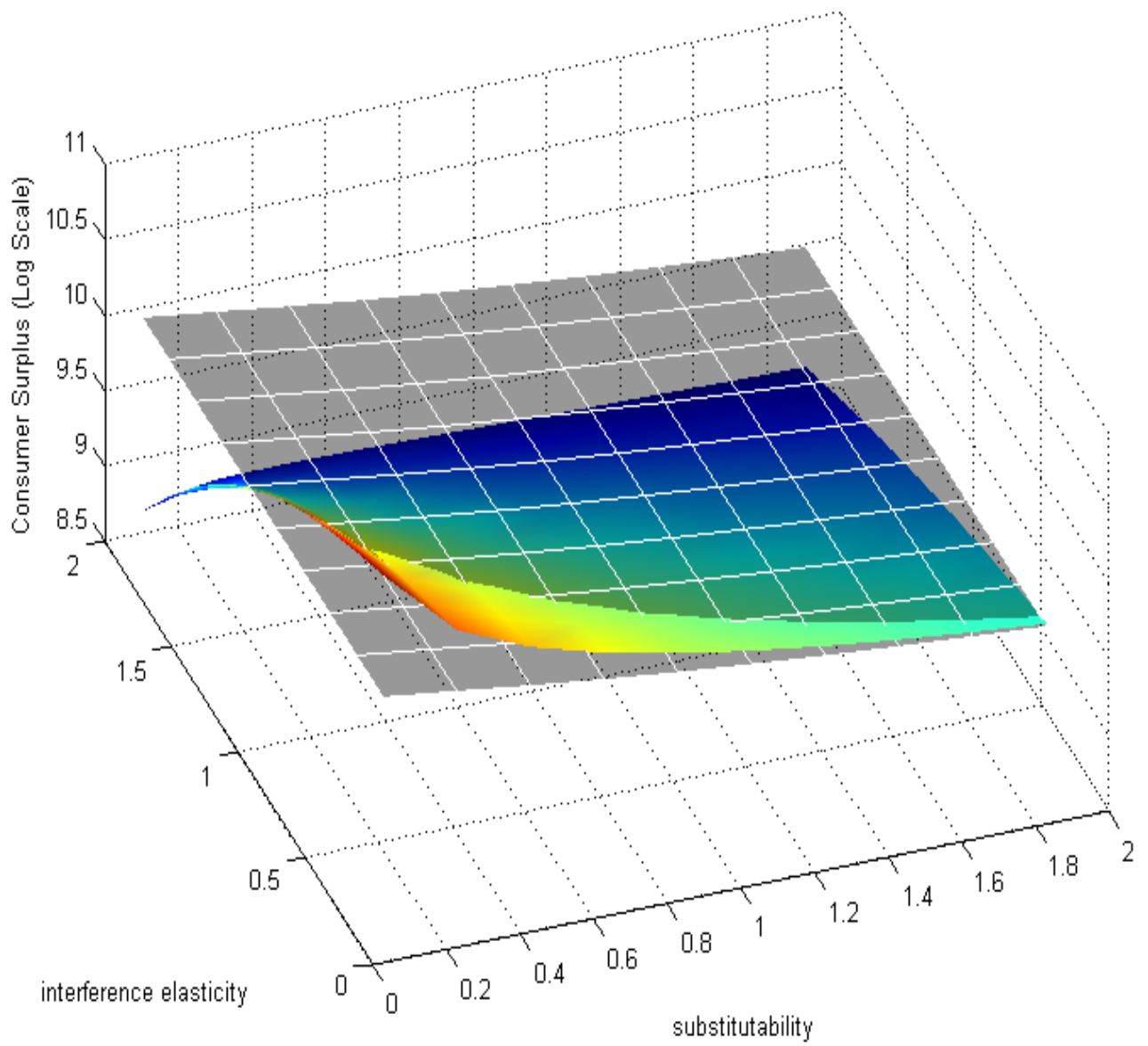


Figure 8: Cournot Consumer Surplus at SNR=60dB (Licensing in gray, Commons in color)

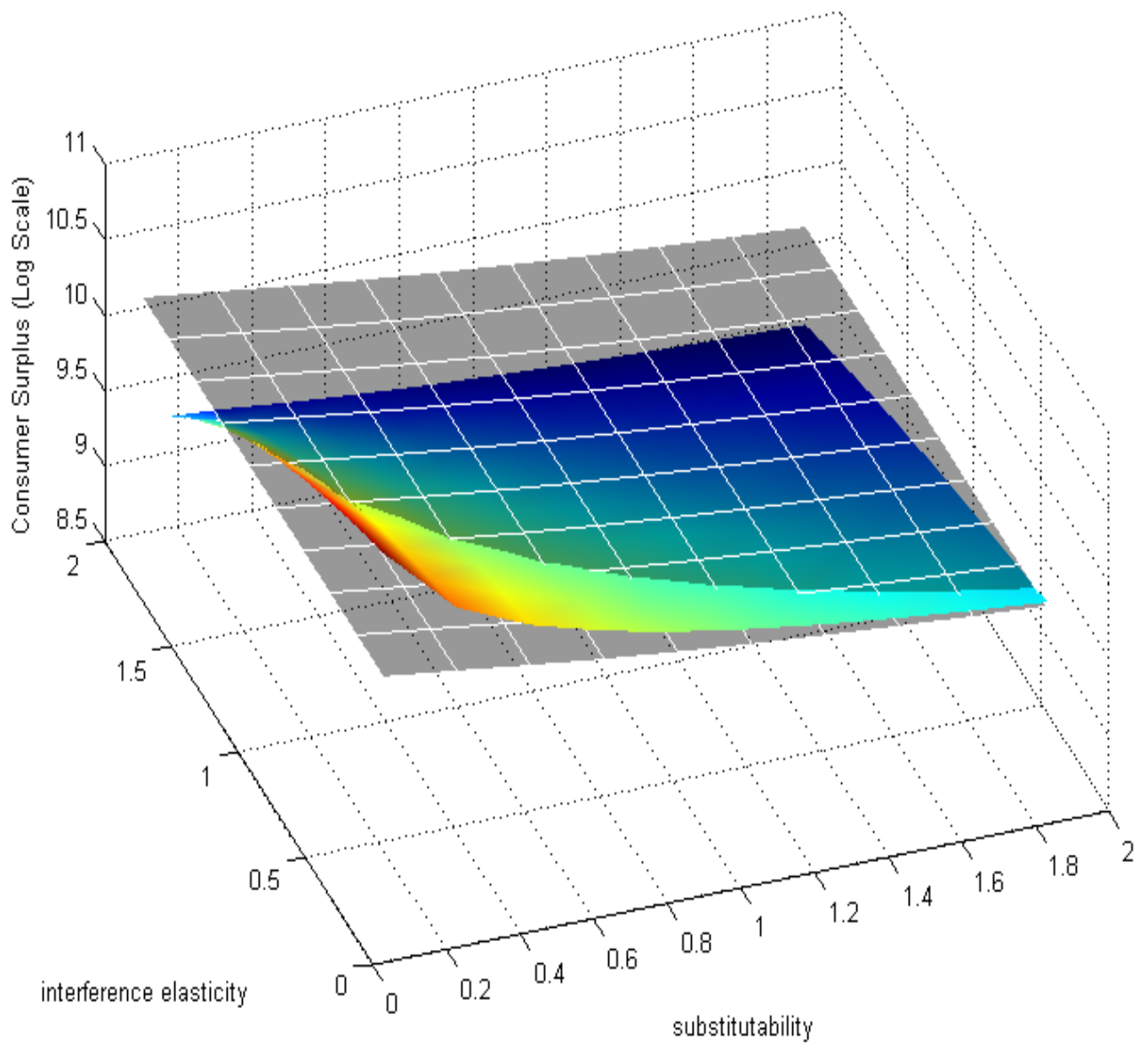


Figure 9: Cournot Consumer Surplus at SNR=70dB (Licensing in gray, Commons in color)

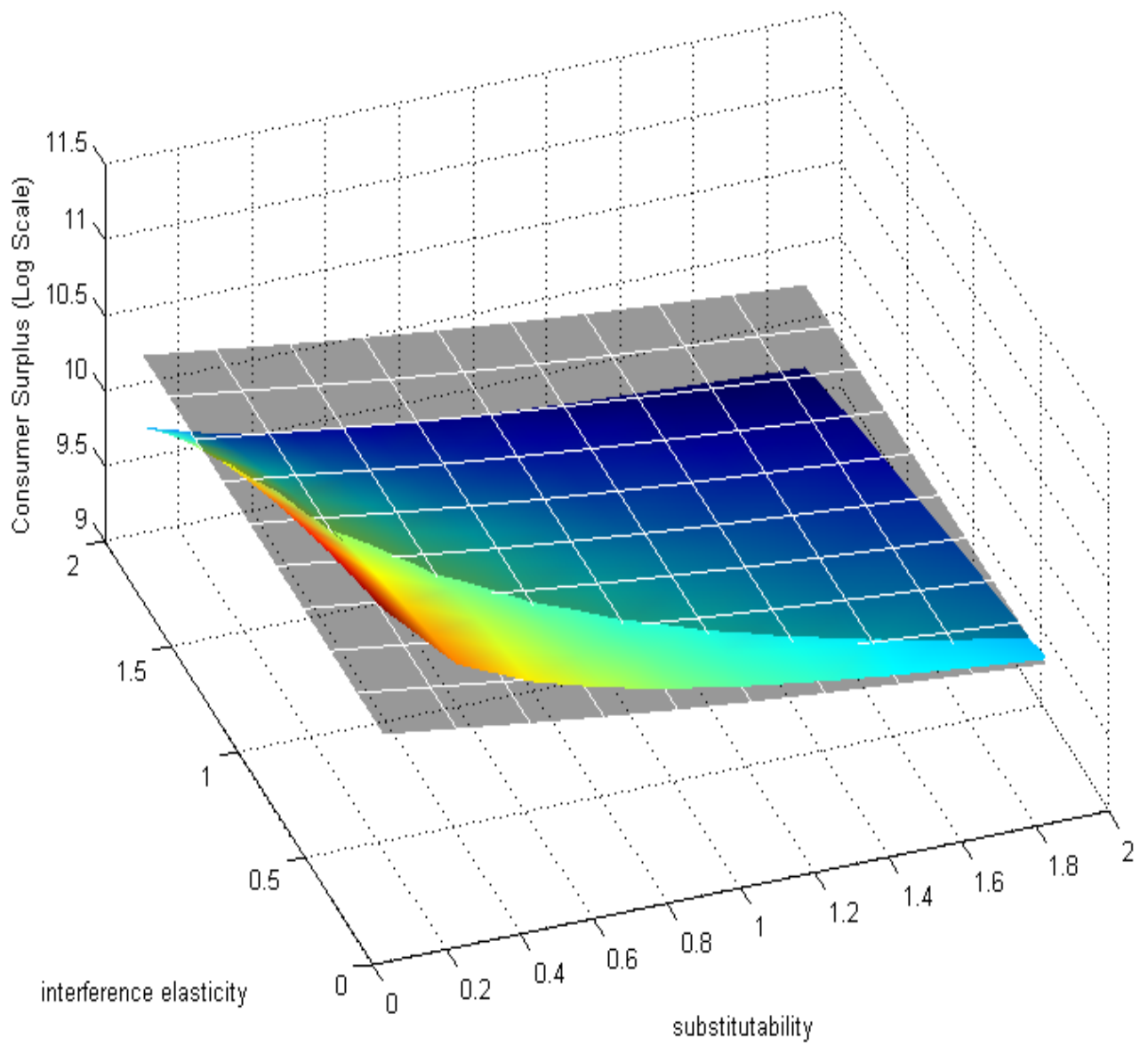


Figure 10: Cournot Consumer Surplus at SNR=80dB (Licensing in gray, Commons in color)

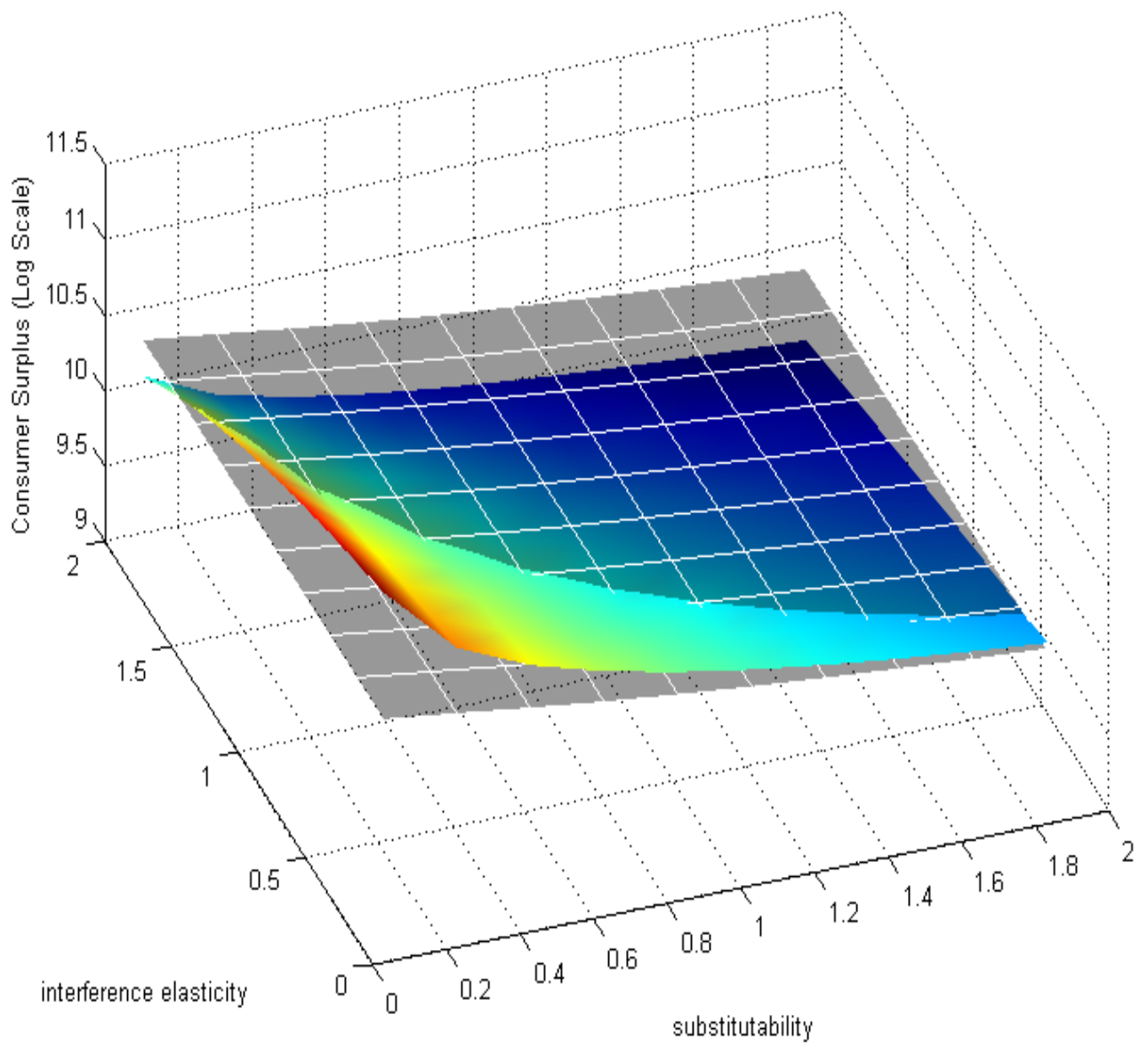


Figure 11: Cournot Consumer Surplus at SNR=90dB (Licensing in gray, Commons in color)

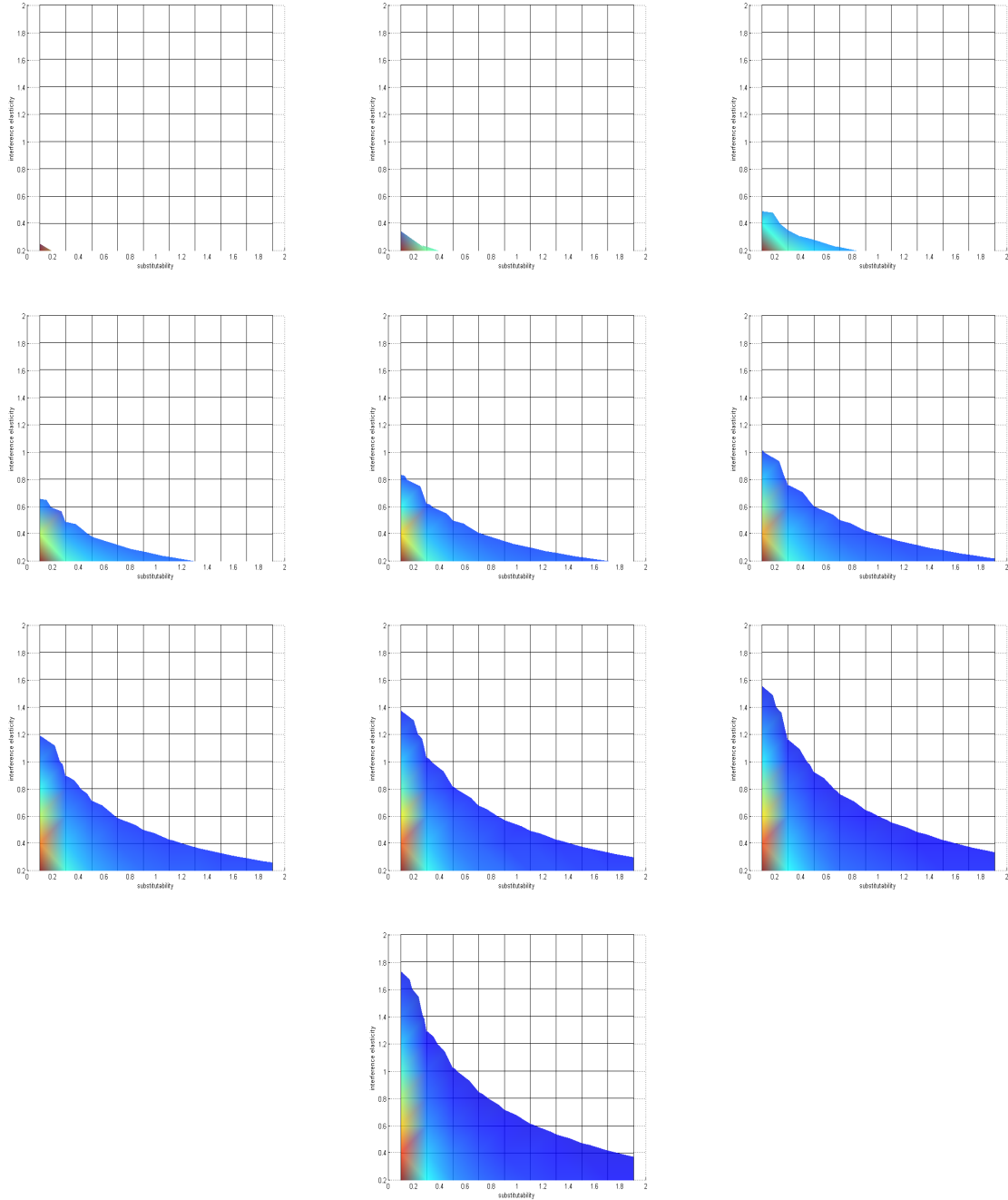


Table 6: Regions where Cournot Consumer Suplus is greater under Commons for all SNRs

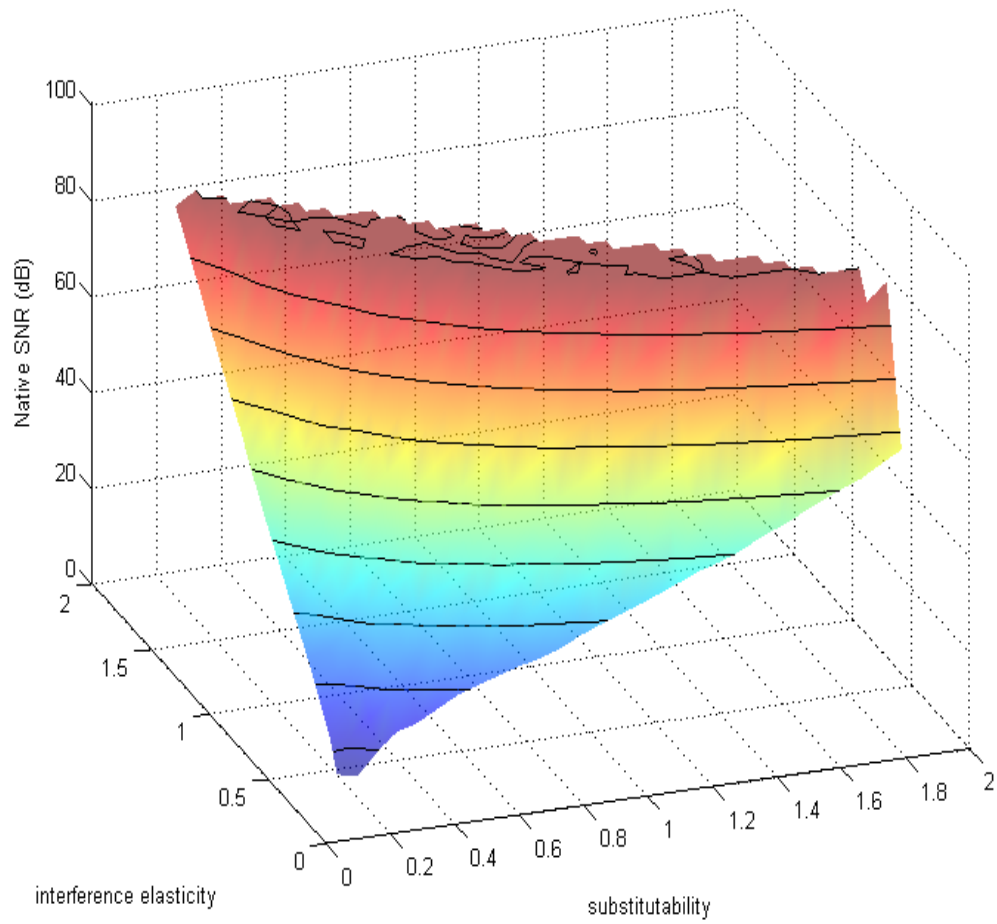


Figure 12: The boundary of welfare dominance under Cournot competition

**Cournot Equilibria when native SNR is 0dB**

COMMONS

LICENSING

COMMONS										LICENSING														
		Interference Elasticity										Interference Elasticity												
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2	Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2	
Substitutability	1.9	67325	52848	99520	113818	227799	336524	480466	960001	1.8E+6	4.0E+6	1.9	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6	7.7E+6
	1.7	30906	58696	111007	126530	177739	291106	535676	966121	1.9E+6	4.2E+6	1.7	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6	8.4E+6
	1.5	34763	66169	70453	93758	200908	262803	517507	910476	2.0E+6	4.5E+6	1.5	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6	9.1E+6
	1.3	39772	39473	80924	107520	169118	246810	452395	966699	2.0E+6	5.0E+6	1.3	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6	10.0E+6
	1.1	22374	46263	58080	87320	150845	241534	472006	919286	2.1E+6	5.6E+6	1.1	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6	11.1E+6
	0.9	27006	32003	45918	76353	143055	248286	462105	934677	2.3E+6	6.4E+6	0.9	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6	12.4E+6
	0.7	18223	24931	39891	72442	118290	205001	446888	975617	2.5E+6	7.5E+6	0.7	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6	14.1E+6
	0.5	14373	22147	28612	59858	95991	198620	425037	1.0E+6	2.8E+6	9.5E+6	0.5	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6	16.2E+6
	0.3	9049.3	12192	20907	42884	78238	169610	398068	1.1E+6	3.5E+6	13.4E+6	0.3	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6	18.8E+6
	0.1	3111	5572.9	10714	23561	48855	125310	359070	1.2E+6	5.3E+6	28.3E+6	0.1	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6	21.7E+6
Prices		Interference Elasticity								Prices		Interference Elasticity												
Substitutability	1.9	0.0228	0.0206	0.0202	0.0172	0.0155	0.0138	0.0116	98.3E-4	79.7E-4	61.9E-4	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	
	1.7	0.0229	0.0225	0.0195	0.0189	0.0165	0.0145	0.012	98.6E-4	78.8E-4	61.2E-4	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	
	1.5	0.0253	0.0235	0.0216	0.0186	0.0165	0.0145	0.0122	0.0101	79.1E-4	60.2E-4	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	
	1.3	0.0279	0.0248	0.0223	0.0205	0.0177	0.0151	0.0126	0.0102	80.1E-4	59.6E-4	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	
	1.1	0.029	0.0273	0.025	0.022	0.019	0.0161	0.0132	0.0105	80.9E-4	59.3E-4	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	
	0.9	0.0334	0.0303	0.0266	0.0238	0.0201	0.0169	0.0139	0.0109	82.5E-4	59.3E-4	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	
	0.7	0.0382	0.0337	0.0302	0.0263	0.0225	0.0185	0.0148	0.0116	85.4E-4	59.7E-4	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942		
	0.5	0.046	0.0408	0.0342	0.0303	0.0255	0.0208	0.0165	0.0125	89.5E-4	60.7E-4	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353		
	0.3	0.0593	0.0504	0.0438	0.0382	0.0314	0.0252	0.0194	0.0141	97.3E-4	62.8E-4	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985			
	0.1	0.1005	0.0873	0.0743	0.0612	0.0489	0.0378	0.0274	0.0188	0.0119	69.1E-4	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082				
Quantities		Interference Elasticity								Quantities		Interference Elasticity												
Substitutability	1.9	137795	107920	203145	231627	462750	682488	972164	1.9E+6	3.6E+6	8.0E+6	1.9	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6	1.7E+6		
	1.7	63261	120090	226428	257926	361438	590754	1.1E+6	2.0E+6	3.9E+6	8.5E+6	1.7	1.9E+6	1.9E+6	1.9E+6	1.9E+6	1.9E+6	1.9E+6	1.9E+6	1.9E+6	1.9E+6			
	1.5	71332	135526	144020	191070	408569	533365	1.0E+6	1.8E+6	4.0E+6	9.1E+6	1.5	2.1E+6	2.1E+6	2.1E+6	2.1E+6	2.1E+6	2.1E+6	2.1E+6	2.1E+6				
	1.3	81829	80953	165534	219538	344319	501173	916337	2.0E+6	4.1E+6	10.1E+6	1.3	2.3E+6	2.3E+6	2.3E+6	2.3E+6	2.3E+6	2.3E+6	2.3E+6					
	1.1	46085	95122	119143	178569	307537	490994	956650	1.9E+6	4.3E+6	11.2E+6	1.1	2.6E+6	2.6E+6	2.6E+6	2.6E+6	2.6E+6	2.6E+6						
	0.9	55881	66002	94349	156426	291984	505111	937190	1.9E+6	4.7E+6	12.8E+6	0.9	3.0E+6	3.0E+6	3.0E+6	3.0E+6	3.0E+6							
	0.7	37895	51603	82263	148794	242038	417717	907226	2.0E+6	5.1E+6	15.2E+6	0.7	3.5E+6	3.5E+6	3.5E+6	3.5E+6								
	0.5	30132	46180	59249	123456	197002	405692	864301	2.1E+6	5.7E+6	19.1E+6	0.5	4.2E+6	4.2E+6	4.2E+6	4.2E+6								
	0.3	19240	25677	43727	89178	161548	347996	811854	2.2E+6	7.0E+6	26.9E+6	0.3	5.4E+6	5.4E+6	5.4E+6	5.4E+6								
	0.1	6917.2	12212	23150	50192	102734	260467	738398	2.5E+6	10.7E+6	57.1E+6	0.1	7.3E+6	7.3E+6	7.3E+6	7.3E+6								
Qualities		Interference Elasticity								Qualities		Interference Elasticity												
Substitutability	1.9	517.92	457.8	627.96	669.48	945.37	1147.1	1367.4	1929.4	2640.2	3915.5	1.9	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5				
	1.7	332.35	457.79	627.5	669.48	791.4	1010.6	1367.1	1831.8	2572.6	3807.4	1.7	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5					
	1.5	332.41	457.66	471.2	541.69	791.06	902.7	1263.3	1671.8	2455.1	3717	1.5	5996.5	5996.5	5996.5	5996.5	5996.5	5996.5						
	1.3	332.49	329.97	471.03	541.77	677.16	815.51	1100.8	1604.6	2310.9	3639.5	1.3	5996.6	5996.6	5996.6	5996.6	5996.6	5996.6						
	1.1	230.05	330.07	368.77	450.44	589.81	743.67	1035.8	1440.7	2193.6	3531.5	1.1	5996.7	5996.7	5996.7	5996.7	5996.7							
	0.9	230.31	249.65	297.61	382.32	520.81	683.25	928.43	1315.4	2066.5	3410.6	0.9	5996.8	5996.8	5996.8	5996.8	5996.8							
	0.7	168.26	195.58	246.17	329.97	419.49	549.17	806.83	1186.9	1896.4	3276.9	0.7	5997	5997	5997	5997								
	0.5	128	157.69	177.51	255.31	321.09	458.83	667.06	1026.2	1708	3102.8	0.5	5997.2	5997.2	5997.2	5997.2								
	0.3	80.524	92.167	119.47	169.68	226.83	330.92	502.6	817.82	1463	2859.7	0.3	5997.5	5997.5	5997.5	5997.5								
	0.1	29.331	38.365	52.034	75.491	106.57	167.69	279.31	513.36	1045.4	2405.2	0.1	5997.8	5997.8	5997.8	5997.8								
Varieties		Interference Elasticity								Varieties		Interference Elasticity												
Substitutability	1.9	45	50	51	60	67	75	90	106	131	169	1.9	10	10	10	10	10	10	10	10				
	1.7	50	51	59	61	70	80	97	118	148	191	1.7	10	10	10	10	10	10	10					
	1.5	51	55	60	70	79	90	108	130	167	220	1.5	10	10	10	10	10	10						
	1.3	53	60	67	73	85	100	120	149	190	256	1.3	10	10	10	10	10	10						
	1.1	60	64	70	80	93	110	135	170	222	304	1.1	10	10	10	10	10	10						
	0.9	63	70	80	90	107	128	157	200	266	371	0.9	10	10	10	10	10	10						
	0.7	70	80	90	104	122	150	188	242	330	474	0.7	10	10	10	10	10	10						
	0.5	80	91	110	125	150	185	236	314	440	652	0.5	10	10	10	10	10	10						
	0.3	100	120	140	162	200	252	332	460	673	1050	0.3	10	10	10	10	10	10						
	0.1	160	190	230	288	370	490	690	1027	1644	2855	0.1	10	10	10	10	10	10						

Colors indicate whether the value is LOWER or HIGHER than its

**Cournot Equilibria when native SNR is 10dB**

COMMONS

LICENSING

COMMONS										LICENSING											
Consumer Surplus	Interference Elasticity									Consumer Surplus	Interference Elasticity										
	2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4		0.2	2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	398826	584124	960001	1.4E+6	2.4E+6	4.1E+6	7.7E+6	15.2E+6	30.2E+6	60.0E+6	1.9	92.4E+6	92.4E+6	92.4E+6	92.4E+6	92.4E+6	92.4E+6	92.4E+6	92.4E+6	92.4E+6
	1.7	284982	457869	811574	1.3E+6	2.3E+6	3.9E+6	7.7E+6	15.5E+6	32.0E+6	65.3E+6	1.7	1.0E+8	1.0E+8	1.0E+8	1.0E+8	1.0E+8	1.0E+8	1.0E+8	1.0E+8	1.0E+8
	1.5	321611	517507	710136	1.2E+6	2.0E+6	3.7E+6	7.5E+6	15.8E+6	34.2E+6	72.0E+6	1.5	1.1E+8	1.1E+8	1.1E+8	1.1E+8	1.1E+8	1.1E+8	1.1E+8	1.1E+8	1.1E+8
	1.3	246810	430493	643910	1.2E+6	1.8E+6	3.7E+6	7.5E+6	16.5E+6	36.6E+6	80.4E+6	1.3	1.2E+8	1.2E+8	1.2E+8	1.2E+8	1.2E+8	1.2E+8	1.2E+8	1.2E+8	1.2E+8
	1.1	200667	375962	607319	982949	1.7E+6	3.5E+6	7.6E+6	16.9E+6	39.9E+6	91.8E+6	1.1	1.3E+8	1.3E+8	1.3E+8	1.3E+8	1.3E+8	1.3E+8	1.3E+8	1.3E+8	1.3E+8
	0.9	173427	267596	492365	896199	1.6E+6	3.3E+6	7.4E+6	17.8E+6	44.4E+6	1.1E+8	0.9	1.5E+8	1.5E+8	1.5E+8	1.5E+8	1.5E+8	1.5E+8	1.5E+8	1.5E+8	1.5E+8
	0.7	161400	268346	438510	778934	1.5E+6	3.1E+6	7.5E+6	19.0E+6	50.8E+6	1.3E+8	0.7	1.7E+8	1.7E+8	1.7E+8	1.7E+8	1.7E+8	1.7E+8	1.7E+8	1.7E+8	1.7E+8
	0.5	97635	194745	325725	625529	1.3E+6	3.0E+6	7.5E+6	20.7E+6	60.5E+6	1.7E+8	0.5	1.9E+8	1.9E+8	1.9E+8	1.9E+8	1.9E+8	1.9E+8	1.9E+8	1.9E+8	1.9E+8
	0.3	79574	132001	249614	470219	1.1E+6	2.7E+6	7.5E+6	23.3E+6	79.1E+6	2.6E+8	0.3	2.3E+8	2.3E+8	2.3E+8	2.3E+8	2.3E+8	2.3E+8	2.3E+8	2.3E+8	2.3E+8
	0.1	29378	58981	117168	285688	726291	2.1E+6	7.4E+6	29.9E+6	1.4E+8	6.2E+8	0.1	2.6E+8	2.6E+8	2.6E+8	2.6E+8	2.6E+8	2.6E+8	2.6E+8	2.6E+8	2.6E+8
Prices	Interference Elasticity									Interference Elasticity											
	1.9	0.013	0.0116	98.3E-4	86.9E-4	73.0E-4	61.2E-4	49.9E-4	39.9E-4	31.8E-4	25.3E-4	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.0129	0.0116	0.0104	89.6E-4	74.3E-4	61.5E-4	50.2E-4	39.8E-4	31.3E-4	24.8E-4	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.0142	0.0122	0.011	93.5E-4	77.7E-4	63.0E-4	51.0E-4	40.1E-4	31.1E-4	24.3E-4	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.0151	0.0131	0.0116	96.1E-4	80.1E-4	66.0E-4	52.1E-4	40.4E-4	31.1E-4	24.0E-4	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.0161	0.0141	0.0122	0.0104	85.5E-4	69.0E-4	53.7E-4	41.1E-4	31.2E-4	23.7E-4	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.018	0.0155	0.0136	0.0112	91.3E-4	73.2E-4	56.5E-4	42.4E-4	31.4E-4	23.5E-4	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.0204	0.0175	0.0149	0.0124	0.01	78.3E-4	59.9E-4	44.2E-4	32.0E-4	23.4E-4	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.024	0.0211	0.0176	0.0144	0.0115	87.9E-4	65.5E-4	47.0E-4	33.0E-4	23.4E-4	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.0317	0.0272	0.0223	0.0179	0.0141	0.0105	75.8E-4	52.4E-4	35.1E-4	23.8E-4	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
0.1	0.0573	0.0472	0.0378	0.0295	0.0222	0.0158	0.0107	67.9E-4	41.2E-4	25.2E-4	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	
Quantities	Interference Elasticity									Interference Elasticity											
	1.9	808140	1.2E+6	1.9E+6	2.8E+6	4.8E+6	8.3E+6	15.4E+6	30.4E+6	60.6E+6	1.2E+8	1.9	20.4E+6	20.4E+6	20.4E+6	20.4E+6	20.4E+6	20.4E+6	20.4E+6	20.4E+6	20.4E+6
	1.7	577400	926492	1.6E+6	2.6E+6	4.7E+6	7.8E+6	15.5E+6	31.1E+6	64.2E+6	1.3E+8	1.7	22.3E+6	22.3E+6	22.3E+6	22.3E+6	22.3E+6	22.3E+6	22.3E+6	22.3E+6	22.3E+6
	1.5	652510	1.0E+6	1.4E+6	2.4E+6	4.1E+6	7.5E+6	15.1E+6	31.7E+6	68.6E+6	1.4E+8	1.5	24.6E+6	24.6E+6	24.6E+6	24.6E+6	24.6E+6	24.6E+6	24.6E+6	24.6E+6	24.6E+6
	1.3	501173	872450	1.3E+6	2.3E+6	3.7E+6	7.4E+6	15.2E+6	33.2E+6	73.5E+6	1.6E+8	1.3	27.4E+6	27.4E+6	27.4E+6	27.4E+6	27.4E+6	27.4E+6	27.4E+6	27.4E+6	27.4E+6
	1.1	407919	762704	1.2E+6	2.0E+6	3.5E+6	7.1E+6	15.2E+6	33.9E+6	80.0E+6	1.8E+8	1.1	31.0E+6	31.0E+6	31.0E+6	31.0E+6	31.0E+6	31.0E+6	31.0E+6	31.0E+6	31.0E+6
	0.9	353212	543613	998303	1.8E+6	3.2E+6	6.7E+6	15.0E+6	35.7E+6	89.1E+6	2.2E+8	0.9	35.6E+6	35.6E+6	35.6E+6	35.6E+6	35.6E+6	35.6E+6	35.6E+6	35.6E+6	35.6E+6
	0.7	329540	546226	890288	1.6E+6	2.9E+6	6.3E+6	15.0E+6	38.1E+6	1.0E+8	2.6E+8	0.7	41.8E+6	41.8E+6	41.8E+6	41.8E+6	41.8E+6	41.8E+6	41.8E+6	41.8E+6	41.8E+6
	0.5	200062	397866	663134	1.3E+6	2.6E+6	6.1E+6	15.0E+6	41.5E+6	1.2E+8	3.4E+8	0.5	50.7E+6	50.7E+6	50.7E+6	50.7E+6	50.7E+6	50.7E+6	50.7E+6	50.7E+6	50.7E+6
	0.3	164358	271377	510639	957584	2.2E+6	5.4E+6	15.1E+6	46.9E+6	1.6E+8	5.2E+8	0.3	64.3E+6	64.3E+6	64.3E+6	64.3E+6	64.3E+6	64.3E+6	64.3E+6	64.3E+6	64.3E+6
0.1	62329	123801	243544	588770	1.5E+6	4.4E+6	14.9E+6	60.3E+6	2.8E+8	12.4E+8	0.1	87.9E+6	87.9E+6	87.9E+6	87.9E+6	87.9E+6	87.9E+6	87.9E+6	87.9E+6	87.9E+6	
Qualities	Interference Elasticity									Interference Elasticity											
	1.9	1247.7	1507.7	1929.4	2315.9	3031.6	3991.4	5428.8	7618.9	10750	15133	1.9	20753	20753	20753	20753	20753	20753	20753	20753	20753
	1.7	998.17	1263.5	1679.9	2096.5	2824.8	3646.6	5139.8	7290.3	10469	14937	1.7	20753	20753	20753	20753	20753	20753	20753	20753	20753
	1.5	998.25	1263.3	1477.8	1909.9	2480.4	3357.2	4768.6	6913.4	10162	14739	1.5	20753	20753	20753	20753	20753	20753	20753	20753	20753
	1.3	815.51	1074.6	1311.7	1749.5	2205.1	3110.7	4455.1	6590.5	9795.7	14498	1.3	20753	20753	20753	20753	20753	20753	20753	20753	20753
	1.1	677.84	925.49	1173.4	1489.5	1980.8	2802.4	4104.8	6121.1	9403.3	14256	1.1	20753	20753	20753	20753	20753	20753	20753	20753	20753
	0.9	571.85	708.02	958.03	1288.5	1713	2475.7	3684.5	5690.9	8976.3	13959	0.9	20753	20753	20753	20753	20753	20753	20753	20753	20753
	0.7	488.6	627.45	799.32	1061.8	1444.8	2107.3	3256.5	5186.6	8472	13609	0.7	20754	20754	20754	20754	20754	20754	20754	20754	20754
	0.5	323.12	454.47	584.91	806.94	1161.9	1758.6	2759	4576.6	7816	13145	0.5	20754	20754	20754	20754	20754	20754	20754	20754	20754
	0.3	228.86	292.78	399.75	545.08	822.1	1281.1	2144.2	3770.7	6924.1	12486	0.3	20754	20754	20754	20754	20754	20754	20754	20754	20754
0.1	83.768	116.77	162.15	249.95	394.05	670.83	1233.1	2471.7	5282.6	11141	0.1	20754	20754	20754	20754	20754	20754	20754	20754	20754	
Varieties	Interference Elasticity									Interference Elasticity											
	1.9	80	90	106	120	143	171	210	263	330	415	1.9	10	10	10	10	10	10	10	10	10
	1.7	90	100	112	130	157	190	233	294	374	473	1.7	10	10	10	10	10	10	10	10	10
	1.5	92	108	120	141	170	210	260	331	427	547	1.5	10	10	10	10	10	10	10	10	10
	1.3	100	115	131	158	190	231	293	379	493	640	1.3	10	10	10	10	10	10	10	10	10
	1.1	110	126	147	172	210	261	336	440	581	766	1.1	10	10	10	10	10	10	10	10	10
	0.9	120	140	160	195	240	300	390	521	704	944	0.9	10	10	10	10	10	10	10	10	10
	0.7	135	159	187	225	280	360	472	642	889	1218	0.7	10	10	10	10	10	10	10	10	10
	0.5	160	183	220	270	341	448	604	844	1204	1700	0.5	10	10	10	10	10	10	10	10	10
	0.3	198	233	286	360	460	620	867	1260	1885	2789	0.3	10	10	10	10	10	10	10	10	10
0.1	310	385	490	638	862	1226	1834	2907	4810	7893	0.1	10	10	10	10	10	10	10	10	10	

Colors indicate whether the value is LOWER or HIGHER than its counterpart under the alternative management regime

Figure 14: Cournot Equilibria when Native SNR is 10dB



**Cournot Equilibria when native SNR is 30dB**

COMMONS

LICENSING

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	16.1E+6	24.5E+6	40.7E+6	64.7E+6	1.1E+8	1.7E+8	2.5E+8	3.7E+8	5.2E+8	7.1E+8
	1.7	14.7E+6	23.9E+6	37.9E+6	64.3E+6	1.1E+8	1.7E+8	2.7E+8	4.0E+8	5.7E+8	7.8E+8
	1.5	12.5E+6	20.8E+6	36.1E+6	63.5E+6	1.1E+8	1.8E+8	2.8E+8	4.3E+8	6.3E+8	8.8E+8
	1.3	11.0E+6	19.9E+6	35.2E+6	62.6E+6	1.1E+8	1.9E+8	3.1E+8	4.8E+8	7.1E+8	10.0E+8
	1.1	10.0E+6	17.4E+6	32.9E+6	60.9E+6	1.1E+8	2.0E+8	3.3E+8	5.4E+8	8.1E+8	11.7E+8
	0.9	8.9E+6	16.1E+6	30.1E+6	58.2E+6	1.1E+8	2.1E+8	3.7E+8	6.2E+8	9.6E+8	14.1E+8
	0.7	7.3E+6	13.8E+6	27.8E+6	55.2E+6	1.1E+8	2.3E+8	4.2E+8	7.3E+8	11.8E+8	17.8E+8
	0.5	5.9E+6	11.6E+6	24.1E+6	52.2E+6	1.1E+8	2.5E+8	5.0E+8	9.2E+8	15.6E+8	24.3E+8
	0.3	3.9E+6	8.5E+6	19.0E+6	45.7E+6	1.1E+8	2.8E+8	6.4E+8	13.1E+8	23.8E+8	39.1E+8
	0.1	1.8E+6	4.3E+6	11.1E+6	31.9E+6	99.9E+6	3.3E+8	10.1E+8	26.8E+8	58.4E+8	1.1E+10

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8	7.7E+8
	1.7	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8	8.3E+8
	1.5	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8	9.0E+8
	1.3	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8	9.9E+8
	1.1	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8	11.0E+8
	0.9	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8	12.3E+8
	0.7	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8
	0.5	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8
	0.3	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8	18.7E+8
	0.1	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8	21.6E+8

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	39.1E-4	33.8E-4	28.8E-4	24.4E-4	21.0E-4	18.1E-4	15.8E-4	13.9E-4	12.4E-4	11.2E-4
	1.7	40.5E-4	34.6E-4	29.3E-4	24.9E-4	21.0E-4	18.0E-4	15.6E-4	13.6E-4	12.1E-4	10.9E-4
	1.5	42.8E-4	35.9E-4	30.2E-4	25.4E-4	21.3E-4	18.0E-4	15.5E-4	13.4E-4	11.9E-4	10.6E-4
	1.3	45.0E-4	38.0E-4	31.5E-4	26.0E-4	21.6E-4	18.1E-4	15.4E-4	13.3E-4	11.7E-4	10.4E-4
	1.1	48.8E-4	40.2E-4	33.1E-4	27.1E-4	22.2E-4	18.4E-4	15.4E-4	13.2E-4	11.5E-4	10.2E-4
	0.9	53.2E-4	43.8E-4	35.6E-4	28.7E-4	23.1E-4	18.8E-4	15.6E-4	13.2E-4	11.4E-4	10.0E-4
	0.7	60.2E-4	48.9E-4	39.0E-4	30.9E-4	24.5E-4	19.5E-4	15.9E-4	13.2E-4	11.3E-4	9.9E-4
	0.5	70.7E-4	56.7E-4	44.7E-4	34.7E-4	26.7E-4	20.8E-4	16.5E-4	13.4E-4	11.3E-4	9.7E-4
	0.3	92.3E-4	72.8E-4	55.9E-4	42.1E-4	31.2E-4	23.2E-4	17.6E-4	13.9E-4	11.4E-4	9.7E-4
	0.1	0.0167	0.0127	93.5E-4	66.5E-4	45.8E-4	31.1E-4	21.4E-4	15.5E-4	12.0E-4	9.8E-4

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	32.2E+6	49.2E+6	81.7E+6	1.3E+8	2.1E+8	3.3E+8	5.0E+8	7.4E+8	10.4E+8	14.2E+8
	1.7	29.5E+6	47.9E+6	76.0E+6	1.3E+8	2.2E+8	3.4E+8	5.3E+8	7.9E+8	11.4E+8	15.7E+8
	1.5	25.1E+6	41.7E+6	67.0E+6	1.3E+8	2.2E+8	3.6E+8	5.7E+8	8.6E+8	12.6E+8	17.6E+8
	1.3	22.1E+6	39.9E+6	60.7E+6	1.3E+8	2.2E+8	3.8E+8	6.1E+8	9.6E+8	14.2E+8	20.1E+8
	1.1	20.2E+6	35.0E+6	66.1E+6	1.2E+8	2.2E+8	4.0E+8	6.7E+8	10.7E+8	16.3E+8	23.4E+8
	0.9	17.8E+6	32.4E+6	60.4E+6	1.2E+8	2.3E+8	4.2E+8	7.4E+8	12.3E+8	19.2E+8	28.2E+8
	0.7	14.6E+6	27.7E+6	55.7E+6	1.1E+8	2.3E+8	4.5E+8	8.5E+8	14.7E+8	23.6E+8	35.6E+8
	0.5	11.9E+6	23.3E+6	48.5E+6	1.0E+8	2.3E+8	5.0E+8	10.0E+8	18.5E+8	31.2E+8	48.7E+8
	0.3	7.9E+6	17.1E+6	38.1E+6	91.9E+6	2.3E+8	5.6E+8	12.8E+8	26.2E+8	47.6E+8	78.2E+8
	0.1	3.6E+6	8.7E+6	22.4E+6	64.2E+6	2.0E+8	6.6E+8	20.3E+8	53.6E+8	1.2E+10	2.2E+10

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	7843.1	9686.1	12476	15725	20035	25111	30999	37458	44475	51920
	1.7	7100.3	9041.1	11383	14831	19146	24180	30139	36756	43978	51678
	1.5	6156.8	7926.6	10436	13838	18046	23130	29220	36034	43482	51424
	1.3	5377.9	7217.4	9608.8	12798	16825	22112	28279	35267	42941	51131
	1.1	4729.7	6225.7	8548.2	11613	15665	20888	27168	34373	42317	50810
	0.9	4023.6	5421.1	7396.2	10272	14301	19502	25889	33333	41571	50435
	0.7	3215.8	4418.7	6265.7	8830.9	12663	17858	24372	32080	40690	49971
	0.5	2456.6	3433.5	4943	7259.5	10737	15776	22424	30447	39533	49370
	0.3	1552.3	2279.3	3399.7	5270.6	8299.3	12985	19641	28069	37821	48474
	0.1	609.7	944.06	1509.6	2550.2	4500.1	8120.3	14273	23193	34240	46594

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	1.7	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	1.5	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	1.3	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	1.1	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	0.9	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	0.7	59800	59800	59800	59800	59800	59800	59800	59800	59800	59800
	0.5	59801	59801	59801	59801	59801	59801	59801	59801	59801	59801
	0.3	59801	59801	59801	59801	59801	59801	59801	59801	59801	59801
	0.1	59801	59801	59801	59801	59801	59801	59801	59801	59801	59801

		Interference Elasticity									
Varieties		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	268	310	365	430	500	580	667	757	848	940
	1.7	289	339	400	471	558	652	755	861	970	1080
	1.5	310	370	440	524	625	740	861	990	1122	1254
	1.3	340	403	487	589	710	847	998	1156	1317	1480
	1.1	370	450	546	669	816	987	1176	1375	1579	1783
	0.9	414	504	620	770	959	1179	1423	1683	1950	2217
	0.7	470	580	728	920	1162	1459	1795	2154	2523	2893
	0.5	559	698	888	1146	1490	1920	2424	2971	3534	4097
	0.3	710	903	1180	1573	2126	2862	3768	4778	5826	6872
	0.1	1160	1537	2100	2970	4330	6400	9305	12850	16648	20435

Colors indicate whether the value is LOWER or HIGHER than its counterpart under the alternative management regime

Figure 16: Cournot Equilibria when Native SNR is 30dB

**Cournot Equilibria when native SNR is 40dB**

COMMONS

LICENSING

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	67.5E+6	1.0E+8	1.5E+8	2.3E+8	3.3E+8	4.6E+8	6.2E+8	8.2E+8	10.6E+8	13.4E+8
	1.7	63.9E+6	1.0E+8	1.5E+8	2.3E+8	3.4E+8	4.8E+8	6.7E+8	8.9E+8	11.7E+8	14.9E+8
	1.5	61.6E+6	97.9E+6	1.5E+8	2.4E+8	3.6E+8	5.2E+8	7.3E+8	9.9E+8	13.0E+8	16.8E+8
	1.3	56.4E+6	94.5E+6	1.5E+8	2.5E+8	3.8E+8	5.6E+8	8.0E+8	11.0E+8	14.7E+8	19.2E+8
	1.1	51.4E+6	89.9E+6	1.5E+8	2.5E+8	4.0E+8	6.1E+8	9.0E+8	12.6E+8	17.1E+8	22.4E+8
	0.9	47.5E+6	85.6E+6	1.5E+8	2.6E+8	4.3E+8	6.9E+8	10.3E+8	14.7E+8	20.3E+8	27.1E+8
	0.7	42.4E+6	78.5E+6	1.5E+8	2.7E+8	4.8E+8	7.8E+8	12.2E+8	18.0E+8	25.4E+8	34.4E+8
	0.5	35.4E+6	70.5E+6	1.4E+8	2.8E+8	5.3E+8	9.3E+8	15.3E+8	23.5E+8	34.1E+8	47.4E+8
	0.3	25.7E+6	56.9E+6	1.3E+8	2.9E+8	6.1E+8	12.0E+8	21.5E+8	35.1E+8	53.5E+8	76.9E+8
	0.1	12.6E+6	32.6E+6	90.1E+6	2.6E+8	7.4E+8	19.2E+8	42.8E+8	82.3E+8	1.4E+10	2.2E+10

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8	13.6E+8
	1.7	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8	14.8E+8
	1.5	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8
	1.3	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8	17.7E+8
	1.1	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8	19.6E+8
	0.9	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8	21.9E+8
	0.7	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8	24.9E+8
	0.5	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8	28.6E+8
	0.3	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8	33.3E+8
	0.1	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8	38.4E+8

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	24.4E-4	21.1E-4	18.4E-4	16.2E-4	14.4E-4	12.9E-4	11.7E-4	10.6E-4	9.8E-4	9.0E-4
	1.7	25.0E-4	21.4E-4	18.6E-4	16.3E-4	14.3E-4	12.8E-4	11.5E-4	10.4E-4	9.5E-4	8.8E-4
	1.5	25.6E-4	22.0E-4	18.9E-4	16.3E-4	14.3E-4	12.6E-4	11.3E-4	10.2E-4	9.3E-4	8.6E-4
	1.3	26.9E-4	22.7E-4	19.3E-4	16.5E-4	14.4E-4	12.6E-4	11.2E-4	10.1E-4	9.1E-4	8.4E-4
	1.1	28.3E-4	23.8E-4	19.9E-4	16.9E-4	14.5E-4	12.6E-4	11.1E-4	9.9E-4	9.0E-4	8.2E-4
	0.9	30.7E-4	25.3E-4	20.9E-4	17.5E-4	14.8E-4	12.7E-4	11.1E-4	9.9E-4	8.9E-4	8.1E-4
	0.7	33.9E-4	27.6E-4	22.4E-4	18.4E-4	15.3E-4	12.9E-4	11.2E-4	9.8E-4	8.8E-4	7.9E-4
	0.5	39.4E-4	31.4E-4	24.9E-4	19.9E-4	16.1E-4	13.4E-4	11.4E-4	9.9E-4	8.7E-4	7.8E-4
	0.3	50.4E-4	39.1E-4	30.0E-4	23.0E-4	17.9E-4	14.3E-4	11.8E-4	10.0E-4	8.7E-4	7.7E-4
	0.1	89.3E-4	66.0E-4	47.4E-4	33.5E-4	23.8E-4	17.3E-4	13.3E-4	10.7E-4	9.0E-4	7.7E-4

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	1.4E+8	2.1E+8	3.1E+8	4.6E+8	6.6E+8	9.2E+8	12.4E+8	16.4E+8	21.2E+8	26.9E+8
	1.7	1.3E+8	2.0E+8	3.1E+8	4.7E+8	6.8E+8	9.7E+8	13.4E+8	17.9E+8	23.3E+8	29.8E+8
	1.5	1.2E+8	2.0E+8	3.1E+8	4.8E+8	7.2E+8	10.4E+8	14.6E+8	19.7E+8	26.0E+8	33.5E+8
	1.3	1.1E+8	1.9E+8	3.1E+8	4.9E+8	7.6E+8	11.2E+8	16.0E+8	22.1E+8	29.5E+8	38.4E+8
	1.1	1.0E+8	1.8E+8	3.0E+8	5.1E+8	8.1E+8	12.3E+8	17.9E+8	25.2E+8	34.1E+8	44.9E+8
	0.9	95.2E+6	1.7E+8	3.0E+8	5.2E+8	8.7E+8	13.7E+8	20.6E+8	29.5E+8	40.7E+8	54.3E+8
	0.7	85.1E+6	1.6E+8	3.0E+8	5.4E+8	9.5E+8	15.7E+8	24.4E+8	36.0E+8	50.8E+8	68.9E+8
	0.5	71.0E+6	1.4E+8	2.9E+8	5.6E+8	10.6E+8	18.7E+8	30.7E+8	47.0E+8	68.3E+8	94.8E+8
	0.3	51.7E+6	1.1E+8	2.6E+8	5.7E+8	12.3E+8	24.1E+8	43.0E+8	70.4E+8	1.1E+10	1.5E+10
	0.1	25.5E+6	65.7E+6	1.8E+8	5.2E+8	14.8E+8	38.4E+8	85.8E+8	1.6E+10	2.8E+10	4.4E+10

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8	3.0E+8
	1.7	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8	3.3E+8
	1.5	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8	3.6E+8
	1.3	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8	4.0E+8
	1.1	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8	4.6E+8
	0.9	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8	5.3E+8
	0.7	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8	6.2E+8
	0.5	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8	7.5E+8
	0.3	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8	9.5E+8
	0.1	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8	13.0E+8

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	16059	19970	24248	29588	35421	41728	48567	55895	63520	71465
	1.7	14777	18618	22951	28210	34093	40634	47702	55201	63035	71226
	1.5	13636	17183	21602	26841	32812	39500	46754	54427	62537	70959
	1.3	12144	15720	20077	25373	31390	38214	45665	53615	61953	70670
	1.1	10675	14106	18330	23649	29821	36797	44467	52680	61314	70341
	0.9	9280	12454	16499	21723	27993	35183	43093	51579	60574	69961
	0.7	7740.2	10522	14416	19531	25858	33191	41388	50264	59665	69493
	0.5	5978.9	8435.3	11983	16821	23086	30629	39195	48533	58477	68885
	0.3	3955.3	5877.2	8807.2	13157	19230	26935	35965	45984	56716	67983
	0.1	1611	2579.1	4274.4	7240	12196	19630	29331	40646	53010	66082

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	1.7	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	1.5	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	1.3	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	1.1	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	0.9	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	0.7	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	0.5	79724	79724	79724	79724	79724	79724	79724	79724	79724	79724
	0.3	79725	79725	79725	79725	79725	79725	79725	79725	79725	79725
	0.1	79725	79725	79725	79725	79725	79725	79725	79725	79725	79725

		Interference Elasticity									
Varieties		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	431	498	570	647	729	813	900	988	1076	1163
	1.7	470	548	630	722	820	920	1024	1129	1233	1337
	1.5	519	605	705	814	930	1053	1177	1303	1429	1554
	1.3	570	675	795	928	1070	1220	1372	1527	1681	1835
	1.1	640	761	910	1072	1251	1440	1632	1826	2020	2214
	0.9	720	875	1060	1270	1500	1745				





Cournot Equilibria when native SNR is 70dB

COMMONS

LICENSING

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	9.3E+8	11.5E+8	14.2E+8	17.2E+8	20.6E+8	24.5E+8	28.9E+8	33.7E+8	39.0E+8	44.8E+8
	1.7	9.7E+8	12.1E+8	15.1E+8	18.5E+8	22.3E+8	26.8E+8	31.7E+8	37.2E+8	43.2E+8	49.9E+8
	1.5	10.1E+8	12.9E+8	16.2E+8	20.0E+8	24.5E+8	29.5E+8	35.2E+8	41.5E+8	48.6E+8	56.3E+8
	1.3	10.6E+8	13.8E+8	17.6E+8	22.0E+8	27.2E+8	33.1E+8	39.7E+8	47.2E+8	55.5E+8	64.7E+8
	1.1	11.3E+8	15.0E+8	19.4E+8	24.6E+8	30.8E+8	37.8E+8	45.8E+8	54.8E+8	64.9E+8	76.0E+8
	0.9	12.2E+8	16.5E+8	21.8E+8	28.1E+8	35.7E+8	44.4E+8	54.3E+8	65.6E+8	78.3E+8	92.4E+8
	0.7	13.3E+8	18.5E+8	25.2E+8	33.2E+8	42.9E+8	54.2E+8	67.3E+8	82.3E+8	99.1E+8	1.2E+10
	0.5	14.7E+8	21.5E+8	30.3E+8	41.4E+8	54.9E+8	71.0E+8	89.7E+8	1.1E+10	1.4E+10	1.6E+10
	0.3	16.6E+8	26.3E+8	39.8E+8	57.4E+8	79.6E+8	1.1E+10	1.4E+10	1.8E+10	2.2E+10	2.7E+10
	0.1	17.8E+8	35.9E+8	65.8E+8	1.1E+10	1.7E+10	2.5E+10	3.5E+10	4.7E+10	6.2E+10	7.8E+10

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8	41.8E+8
	1.7	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8	45.2E+8
	1.5	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8	49.3E+8
	1.3	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8	54.1E+8
	1.1	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8	59.9E+8
	0.9	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8	67.1E+8
	0.7	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8	76.1E+8
	0.5	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8	87.6E+8
	0.3	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10	1.0E+10
	0.1	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10	1.2E+10

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	10.2E-4	9.5E-4	8.9E-4	8.3E-4	7.8E-4	7.4E-4	7.0E-4	6.7E-4	6.3E-4	6.1E-4
	1.7	10.1E-4	9.4E-4	8.8E-4	8.2E-4	7.7E-4	7.2E-4	6.8E-4	6.5E-4	6.2E-4	5.9E-4
	1.5	10.1E-4	9.3E-4	8.7E-4	8.1E-4	7.6E-4	7.1E-4	6.7E-4	6.3E-4	6.0E-4	5.7E-4
	1.3	10.2E-4	9.3E-4	8.6E-4	8.0E-4	7.5E-4	7.0E-4	6.6E-4	6.2E-4	5.9E-4	5.6E-4
	1.1	10.3E-4	9.4E-4	8.6E-4	8.0E-4	7.4E-4	6.9E-4	6.5E-4	6.1E-4	5.8E-4	5.5E-4
	0.9	10.5E-4	9.5E-4	8.7E-4	8.0E-4	7.4E-4	6.8E-4	6.4E-4	6.0E-4	5.7E-4	5.4E-4
	0.7	10.9E-4	9.7E-4	8.8E-4	8.0E-4	7.4E-4	6.8E-4	6.3E-4	5.9E-4	5.6E-4	5.3E-4
	0.5	11.5E-4	10.1E-4	9.1E-4	8.2E-4	7.4E-4	6.8E-4	6.3E-4	5.9E-4	5.5E-4	5.2E-4
	0.3	12.9E-4	11.0E-4	9.6E-4	8.5E-4	7.6E-4	6.9E-4	6.4E-4	5.9E-4	5.5E-4	5.1E-4
	0.1	17.8E-4	14.1E-4	11.5E-4	9.7E-4	8.4E-4	7.4E-4	6.6E-4	6.0E-4	5.5E-4	5.1E-4

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	18.6E+8	23.1E+8	28.3E+8	34.4E+8	41.3E+8	49.1E+8	57.7E+8	67.4E+8	78.0E+8	89.7E+8
	1.7	19.3E+8	24.3E+8	30.2E+8	37.0E+8	44.7E+8	53.5E+8	63.4E+8	74.4E+8	86.5E+8	99.8E+8
	1.5	20.2E+8	25.8E+8	32.5E+8	40.1E+8	49.0E+8	59.1E+8	70.4E+8	83.1E+8	97.2E+8	1.1E+10
	1.3	21.3E+8	27.7E+8	35.2E+8	44.1E+8	54.4E+8	66.2E+8	79.5E+8	94.5E+8	1.1E+10	1.3E+10
	1.1	22.7E+8	30.0E+8	38.8E+8	49.3E+8	61.6E+8	75.6E+8	91.6E+8	1.1E+10	1.3E+10	1.5E+10
	0.9	24.4E+8	33.0E+8	43.6E+8	56.3E+8	71.4E+8	88.8E+8	1.1E+10	1.3E+10	1.6E+10	1.8E+10
	0.7	26.5E+8	37.1E+8	50.4E+8	66.5E+8	85.9E+8	1.1E+10	1.3E+10	1.6E+10	2.0E+10	2.4E+10
	0.5	29.4E+8	43.1E+8	60.8E+8	82.9E+8	1.1E+10	1.4E+10	1.8E+10	2.2E+10	2.7E+10	3.3E+10
	0.3	33.2E+8	52.7E+8	79.7E+8	1.1E+10	1.6E+10	2.1E+10	2.8E+10	3.5E+10	4.4E+10	5.4E+10
	0.1	35.7E+8	71.8E+8	1.3E+10	2.2E+10	3.4E+10	5.0E+10	7.0E+10	9.5E+10	#####	#####

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8	9.2E+8
	1.7	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8	10.1E+8
	1.5	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8	11.1E+8
	1.3	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8	12.4E+8
	1.1	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8	14.0E+8
	0.9	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8	16.1E+8
	0.7	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8	18.9E+8
	0.5	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8	22.9E+8
	0.3	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8	29.1E+8
	0.1	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8	39.7E+8

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	59429	66253	73415	80877	88609	96585	104784	113188	121801	130558
	1.7	57353	64261	71652	79341	87229	95442	103874	112479	121310	130313
	1.5	55100	62274	69815	77606	85773	94198	102822	111705	120784	130045
	1.3	52643	60030	67718	75768	84136	92789	101728	110860	120206	129748
	1.1	49997	57494	65362	73684	82334	91242	100453	109901	119553	129419
	0.9	46868	54563	62685	71237	80194	89435	98966	108771	118793	129035
	0.7	43140	51000	59424	68283	77582	87207	97164	107393	117867	128566
	0.5	38369	46441	55159	64337	74179	84317	94810	105600	116655	127952
	0.3	31576	39818	48955	58767	69168	80038	91313	102935	114856	127042
	0.1	18932	26842	36331	47061	58709	71060	83968	97330	111067	125125

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	139517	139517	139517	139517	139517	139517	139517	139517	139517	139517
	1.7	139517	139517	139517	139517	139517	139517	139517	139517	139517	139517
	1.5	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	1.3	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	1.1	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	0.9	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	0.7	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	0.5	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	0.3	139518	139518	139518	139518	139518	139518	139518	139518	139518	139518
	0.1	139519	139519	139519	139519	139519	139519	139519	139519	139519	139519

		Interference Elasticity									
Varieties		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	1029	1106	1184	1263	1342	1421	1501	1580	1659	1737
	1.7	1158	1250	1342	1437	1530	1625	1719	1812	1906	1999
	1.5	1313	1425	1537	1650	1763					

**Cournot Equilibria when native SNR is 80dB**

COMMONS

LICENSING

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	15.3E+8	18.4E+8	21.9E+8	25.9E+8	30.2E+8	35.0E+8	40.4E+8	46.2E+8	52.5E+8	59.4E+8
	1.7	16.2E+8	19.6E+8	23.6E+8	28.0E+8	32.9E+8	38.4E+8	44.4E+8	51.1E+8	58.3E+8	66.2E+8
	1.5	17.2E+8	21.1E+8	25.6E+8	30.6E+8	36.3E+8	42.6E+8	49.5E+8	57.2E+8	65.6E+8	74.7E+8
	1.3	18.5E+8	23.0E+8	28.1E+8	33.9E+8	40.5E+8	47.9E+8	56.1E+8	65.1E+8	75.1E+8	85.9E+8
	1.1	20.1E+8	25.3E+8	31.3E+8	38.3E+8	46.1E+8	55.0E+8	64.9E+8	75.9E+8	87.9E+8	1.0E+10
	0.9	22.2E+8	28.4E+8	35.8E+8	44.3E+8	54.0E+8	65.0E+8	77.4E+8	91.1E+8	1.1E+10	1.2E+10
	0.7	25.1E+8	32.9E+8	42.2E+8	53.1E+8	65.8E+8	80.1E+8	96.4E+8	1.1E+10	1.3E+10	1.6E+10
	0.5	29.3E+8	39.8E+8	52.5E+8	67.7E+8	85.6E+8	1.1E+10	1.3E+10	1.6E+10	1.9E+10	2.2E+10
	0.3	36.3E+8	52.4E+8	72.6E+8	97.5E+8	1.3E+10	1.6E+10	2.0E+10	2.5E+10	3.0E+10	3.6E+10
	0.1	51.0E+8	87.2E+8	1.4E+10	2.1E+10	2.9E+10	4.0E+10	5.3E+10	6.8E+10	8.5E+10	#####

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8	54.5E+8
	1.7	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8	59.0E+8
	1.5	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8	64.3E+8
	1.3	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8	70.7E+8
	1.1	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8	78.3E+8
	0.9	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8	87.7E+8
	0.7	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8	99.5E+8
	0.5	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10
	0.3	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10
	0.1	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10	1.5E+10

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	8.6E-4	8.1E-4	7.7E-4	7.3E-4	6.9E-4	6.6E-4	6.3E-4	6.0E-4	5.7E-4	5.5E-4
	1.7	8.5E-4	8.0E-4	7.5E-4	7.1E-4	6.8E-4	6.4E-4	6.1E-4	5.8E-4	5.6E-4	5.4E-4
	1.5	8.5E-4	7.9E-4	7.4E-4	7.0E-4	6.6E-4	6.3E-4	6.0E-4	5.7E-4	5.4E-4	5.2E-4
	1.3	8.5E-4	7.9E-4	7.4E-4	6.9E-4	6.5E-4	6.2E-4	5.9E-4	5.6E-4	5.3E-4	5.1E-4
	1.1	8.5E-4	7.9E-4	7.3E-4	6.9E-4	6.5E-4	6.1E-4	5.8E-4	5.5E-4	5.2E-4	5.0E-4
	0.9	8.6E-4	7.9E-4	7.3E-4	6.8E-4	6.4E-4	6.0E-4	5.7E-4	5.4E-4	5.1E-4	4.9E-4
	0.7	8.8E-4	8.0E-4	7.4E-4	6.9E-4	6.4E-4	6.0E-4	5.6E-4	5.3E-4	5.0E-4	4.8E-4
	0.5	9.2E-4	8.3E-4	7.5E-4	6.9E-4	6.4E-4	6.0E-4	5.6E-4	5.3E-4	5.0E-4	4.7E-4
	0.3	9.9E-4	8.8E-4	7.9E-4	7.1E-4	6.5E-4	6.0E-4	5.6E-4	5.2E-4	4.9E-4	4.6E-4
	0.1	12.5E-4	10.5E-4	9.0E-4	7.9E-4	7.0E-4	6.3E-4	5.8E-4	5.3E-4	4.9E-4	4.6E-4

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	30.7E+8	36.8E+8	43.8E+8	51.7E+8	60.5E+8	70.1E+8	80.8E+8	92.5E+8	1.1E+10	1.2E+10
	1.7	32.4E+8	39.3E+8	47.1E+8	56.0E+8	65.8E+8	76.8E+8	88.9E+8	1.0E+10	1.2E+10	1.3E+10
	1.5	34.5E+8	42.3E+8	51.2E+8	61.2E+8	72.6E+8	85.2E+8	99.1E+8	1.1E+10	1.3E+10	1.5E+10
	1.3	37.0E+8	46.0E+8	56.2E+8	67.9E+8	81.1E+8	95.9E+8	1.1E+10	1.3E+10	1.5E+10	1.7E+10
	1.1	40.3E+8	50.6E+8	62.7E+8	76.6E+8	92.4E+8	1.1E+10	1.3E+10	1.5E+10	1.8E+10	2.0E+10
	0.9	44.4E+8	56.9E+8	71.6E+8	88.6E+8	1.1E+10	1.3E+10	1.5E+10	1.8E+10	2.1E+10	2.5E+10
	0.7	50.2E+8	65.8E+8	84.5E+8	1.1E+10	1.3E+10	1.6E+10	1.9E+10	2.3E+10	2.7E+10	3.1E+10
	0.5	58.7E+8	79.7E+8	1.1E+10	1.4E+10	1.7E+10	2.1E+10	2.6E+10	3.1E+10	3.7E+10	4.4E+10
	0.3	72.8E+8	1.0E+10	1.5E+10	2.0E+10	2.5E+10	3.2E+10	4.1E+10	5.0E+10	6.0E+10	7.2E+10
	0.1	1.0E+10	1.7E+10	2.8E+10	4.1E+10	5.9E+10	8.0E+10	#####	#####	#####	#####

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	1.7	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	1.5	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	1.3	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	1.1	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	0.9	159449	159449	159449	159449	159449	159449	159449	159449	159449	159449
	0.7	159450	159450	159450	159450	159450	159450	159450	159450	159450	159450
	0.5	159450	159450	159450	159450	159450	159450	159450	159450	159450	159450
	0.3	159450	159450	159450	159450	159450	159450	159450	159450	159450	159450
	0.1	159450	159450	159450	159450	159450	159450	159450	159450	159450	159450

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	76393	83696	91275	99187	107228	115481	123970	132586	141372	150325
	1.7	74241	81765	89563	97609	105822	114316	123001	131864	140893	150076
	1.5	71991	79670	87645	95888	104373	113081	121994	131095	140371	149810
	1.3	69417	77333	85486	93990	102696	111681	120844	130232	139794	149515
	1.1	66599	74680	83104	91834	100837	110089	119566	129267	139137	149184
	0.9	63281	71614	80319	89350	98674	108260	118085	128127	138370	148800
	0.7	59340	67917	76955	86320	96035	106013	116266	126749	137442	148331
	0.5	54207	63153	72554	82376	92579	103096	113894	124943	136226	147716
	0.3	46766	56127	66086	76557	87471	98754	110366	122267	134424	146804
	0.1	32001	41831	52721	64423	76772	89649	102957	116632	130622	144884

		Interference Elasticity									
Varieties		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	1216	1292	1370	1447	1524	1601	1678	1755	1832	1908
	1.7	1375	1466	1557	1649	1740	1832	1924	2015	2106	2196
	1.5	1569	1679	1789	1899	2009	2119	2229	2339	2447	2556
	1.3	1813	1948	2083	2218	2353	2488	2622	2756	2889	3022
	1.1	2135	2304	2474	2644	2814	2983	3151	3319	3486	3652
	0.9	2580	2800	3022	3244	3466	3686	3906	4124	4341	4556
	0.7	3243	3550	3856	4162	4468	4772	5075	5375	5673	5968
	0.5	4360	4826	5293	5760	6226	6688	7147	7601	8051	8497
	0.3	6710	7576	8446	9314	10178	11034	11882	12720	13549	14368
	0.1	15916	19020	22185	25351	28490	31588	34637	37636	40584	43484

Colors indicate whether the value is LOWER or HIGHER than its counterpart under the alternative management regime

Figure 21: Cournot Equilibria when Native SNR is 80dB

**Cournot Equilibria when native SNR is 90dB**

COMMONS

LICENSING

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	23.2E+8	27.1E+8	31.6E+8	36.4E+8	41.8E+8	47.6E+8	53.9E+8	60.8E+8	68.1E+8	76.1E+8
	1.7	24.7E+8	29.1E+8	34.1E+8	39.6E+8	45.6E+8	52.2E+8	59.5E+8	67.3E+8	75.7E+8	84.8E+8
	1.5	26.6E+8	31.6E+8	37.3E+8	43.5E+8	50.5E+8	58.1E+8	66.4E+8	75.4E+8	85.2E+8	95.8E+8
	1.3	28.9E+8	34.7E+8	41.3E+8	48.5E+8	56.7E+8	65.6E+8	75.4E+8	86.1E+8	97.6E+8	1.1E+10
	1.1	31.9E+8	38.7E+8	46.5E+8	55.2E+8	64.9E+8	75.7E+8	87.5E+8	1.0E+10	1.1E+10	1.3E+10
	0.9	35.9E+8	44.2E+8	53.7E+8	64.4E+8	76.4E+8	89.8E+8	1.0E+10	1.2E+10	1.4E+10	1.6E+10
	0.7	41.6E+8	52.1E+8	64.3E+8	78.2E+8	93.9E+8	1.1E+10	1.3E+10	1.5E+10	1.8E+10	2.0E+10
	0.5	50.4E+8	64.7E+8	81.6E+8	1.0E+10	1.2E+10	1.5E+10	1.8E+10	2.1E+10	2.4E+10	2.8E+10
	0.3	66.4E+8	89.3E+8	1.2E+10	1.5E+10	1.9E+10	2.3E+10	2.8E+10	3.3E+10	3.9E+10	4.6E+10
	0.1	1.1E+10	1.7E+10	2.4E+10	3.4E+10	4.5E+10	5.9E+10	7.4E+10	9.2E+10	#####	#####

		Interference Elasticity									
Consumer Surplus		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8	69.0E+8
	1.7	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8	74.7E+8
	1.5	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8	81.4E+8
	1.3	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8	89.4E+8
	1.1	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8	99.1E+8
	0.9	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10	1.1E+10
	0.7	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10	1.3E+10
	0.5	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10	1.4E+10
	0.3	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10	1.7E+10
	0.1	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10	1.9E+10

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	7.5E-4	7.2E-4	6.8E-4	6.5E-4	6.2E-4	5.9E-4	5.7E-4	5.5E-4	5.3E-4	5.1E-4
	1.7	7.4E-4	7.0E-4	6.7E-4	6.4E-4	6.1E-4	5.8E-4	5.5E-4	5.3E-4	5.1E-4	4.9E-4
	1.5	7.4E-4	6.9E-4	6.6E-4	6.2E-4	5.9E-4	5.7E-4	5.4E-4	5.2E-4	5.0E-4	4.8E-4
	1.3	7.3E-4	6.9E-4	6.5E-4	6.1E-4	5.8E-4	5.6E-4	5.3E-4	5.1E-4	4.9E-4	4.7E-4
	1.1	7.3E-4	6.8E-4	6.4E-4	6.1E-4	5.8E-4	5.5E-4	5.2E-4	5.0E-4	4.8E-4	4.6E-4
	0.9	7.3E-4	6.8E-4	6.4E-4	6.1E-4	5.8E-4	5.5E-4	5.2E-4	5.0E-4	4.8E-4	4.6E-4
	0.7	7.4E-4	6.9E-4	6.4E-4	6.0E-4	5.7E-4	5.4E-4	5.1E-4	4.8E-4	4.6E-4	4.4E-4
	0.5	7.7E-4	7.0E-4	6.5E-4	6.1E-4	5.7E-4	5.3E-4	5.0E-4	4.7E-4	4.5E-4	4.3E-4
	0.3	8.1E-4	7.4E-4	6.7E-4	6.2E-4	5.8E-4	5.4E-4	5.0E-4	4.7E-4	4.5E-4	4.3E-4
	0.1	9.7E-4	8.4E-4	7.5E-4	6.7E-4	6.1E-4	5.6E-4	5.1E-4	4.8E-4	4.5E-4	4.2E-4

		Interference Elasticity									
Prices		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948	0.0948
	1.7	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036	0.1036
	1.5	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143	0.1143
	1.3	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274	0.1274
	1.1	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439	0.1439
	0.9	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653
	0.7	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942	0.1942
	0.5	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353	0.2353
	0.3	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985	0.2985
	0.1	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	46.4E+8	54.3E+8	63.2E+8	72.8E+8	83.6E+8	95.2E+8	1.1E+10	1.2E+10	1.4E+10	1.5E+10
	1.7	49.4E+8	58.3E+8	68.2E+8	79.2E+8	91.3E+8	1.0E+10	1.2E+10	1.3E+10	1.5E+10	1.7E+10
	1.5	53.2E+8	63.2E+8	74.6E+8	87.1E+8	1.0E+10	1.2E+10	1.3E+10	1.5E+10	1.7E+10	1.9E+10
	1.3	57.9E+8	69.5E+8	82.6E+8	97.1E+8	1.1E+10	1.3E+10	1.5E+10	1.7E+10	2.0E+10	2.2E+10
	1.1	63.9E+8	77.5E+8	93.0E+8	1.1E+10	1.3E+10	1.5E+10	1.8E+10	2.0E+10	2.3E+10	2.6E+10
	0.9	71.9E+8	88.4E+8	1.1E+10	1.3E+10	1.5E+10	1.8E+10	2.1E+10	2.4E+10	2.8E+10	3.2E+10
	0.7	83.2E+8	1.0E+10	1.3E+10	1.6E+10	1.9E+10	2.2E+10	2.6E+10	3.1E+10	3.5E+10	4.0E+10
	0.5	1.0E+10	1.3E+10	1.6E+10	2.0E+10	2.5E+10	3.0E+10	3.5E+10	4.2E+10	4.9E+10	5.6E+10
	0.3	1.3E+10	1.8E+10	2.3E+10	3.0E+10	3.7E+10	4.6E+10	5.6E+10	6.7E+10	7.9E+10	9.2E+10
	0.1	2.2E+10	3.4E+10	4.9E+10	6.8E+10	9.1E+10	#####	#####	#####	#####	#####

		Interference Elasticity									
Quantities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8	15.2E+8
	1.7	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8	16.7E+8
	1.5	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8	18.4E+8
	1.3	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8	20.5E+8
	1.1	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8	23.1E+8
	0.9	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8	26.6E+8
	0.7	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8	31.2E+8
	0.5	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8	37.9E+8
	0.3	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8	48.0E+8
	0.1	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8	65.7E+8

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	93920	101589	109584	117689	126057	134536	143228	152034	161012	170106
	1.7	91700	99600	107732	116078	124621	133348	142277	151325	160523	169862
	1.5	89345	97424	105822	114368	123125	132078	141240	150535	159988	169597
	1.3	86771	95090	103665	112425	121456	130684	140094	149674	159413	169301
	1.1	83890	92400	101197	110294	119580	129109	138805	148702	158761	168973
	0.9	80478	89266	98403	107769	117386	127254	137322	147564	157994	168584
	0.7	76364	85484	94938	104692	114714	124997	135494	146180	157062	168115
	0.5	71037	80545	90439	100690	111235	122046	133104	144377	155847	167500
	0.3	63210	73278	83814	94763	106059	117669	129555	141686	154038	166586
	0.1	47258	58264	70016	82365	95210	108476	122098	136029	150228	164665

		Interference Elasticity									
Qualities		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	1.7	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	1.5	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	1.3	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	1.1	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	0.9	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	0.7	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	0.5	179381	179381	179381	179381	179381	179381	179381	179381	179381	179381
	0.3	179382	179382	179382	179382	179382	179382	179382	179382	179382	179382
	0.1	179382	179382	179382	179382	179382	179382	179382	179382	179382	179382

		Interference Elasticity									
Varieties		2	1.8	1.6	1.4	1.2	1	0.8	0.6	0.4	0.2
Substitutability	1.9	1395	1470	1546	1621	1697	1772	1848	1923	1998	2072
	1.7	1582	1671	1761	1851	1940	2030	2119	2208	2297	2385
	1.5	1812	1920	2028	2135	2243	2350	2458	2564	2670	2776
	1.3	2103									