Online Supplemental Appendix to
Online Shopping Intermediaries:
The Strategic Design of Search Environments

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In this online supplemental appendix, we provide arguments for two claims made in the main article.

SA.1. Evaluation Costs Quadratic in Breadth

The consumer’s evaluation cost function in section 2 specified that search costs are linear in the breadth \( b \) but quadratic in depth \( d \). We can show that the main result in Proposition 1 can continue to hold even for a search cost specification that is quadratic in both dimensions.

**Proposition SA.1** Suppose the consumer’s evaluations cost is \( f(b, d; s) = \tau \left( \frac{n}{e} \right)^2 d^2 \). Let \( \tau > \mu(n/e)^2 \). If \( n > e \) then \( s^* = 1 - \frac{\mu(n/e)^2}{2\tau} \) < 1.

**Proof:** Let \( s \in [0,1] \) be an arbitrary search environment and the consumer’s evaluation objective be a modified version of (B1). Then the consumer’s optimal evaluation plan is

\[
\hat{b}(s) = \begin{cases} 
\frac{e}{\sqrt{2(1-s)\tau}} & \text{if } s < \bar{s} \\
\frac{\mu n^2}{2(1-s)re^2} & \text{if } s \geq \bar{s}
\end{cases}
\quad \text{and} \quad
\hat{d}(s) = \begin{cases} 
\frac{\mu n^2}{2(1-s)re^2} & \text{if } s < \bar{s} \\
1 & \text{if } s \geq \bar{s}
\end{cases}
\]

where \( \bar{s} = 1 - \frac{\mu n^2}{2re^2} \). Equilibrium prices \( \hat{p}(s) = \frac{\mu \hat{d}(s)}{1 - \frac{\mu}{1 - \sqrt{2(1-s)\tau}}} \) are clearly decreasing in \( s > \bar{s} \) and therefore \( s^* \) cannot exceed \( \bar{s} \). When \( s < \bar{s} \), equilibrium prices \( \hat{p}(s) = \frac{\mu \hat{d}(s)}{1 - \frac{\mu}{1 - \sqrt{2(1-s)re^2}}} \) are increasing in \( s \). This implies that the optimal level of search aids \( s^* = \bar{s} \). ■
SA.2. Competing Intermediaries

The two intermediaries are located at the two ends of a Hotelling line. We show that the main result in Proposition 1 remains when the transportation cost \( t \) is sufficiently large. However, when the transportation cost is small, we show that it is optimal for the intermediary to provide full aids \( (s^* = 1) \).

**Proposition SA. 2**

(i) When intermediaries are relatively differentiated \( (e_0 \geq e_0^- \equiv \frac{n}{n-1} \ln(n) - \frac{n}{n-1} + \frac{1}{e_0^2-1}) \), the equilibrium outcomes are the same as in Proposition 1.

(ii) Otherwise, in equilibrium, the intermediary minimizes search costs in the search environment \( (s^* = 1) \). The symmetric equilibrium price is \( \mu \left( \frac{n}{n-1} \right) \).

**Proof:** We claim that there is a symmetric equilibrium in which both intermediaries set

\[
 s^*_j = \begin{cases} 
 1 - \frac{\mu}{e^2} & t \geq \bar{t} \\
 1 & 0 \leq t < \bar{t}, 
\end{cases}
\]

where \( \bar{t} \equiv \frac{n(e_0^2-1)}{n-e^2} \left[ \ln(n) - \frac{n}{n-1} + \frac{1}{e_0^2-1} \right] \mu \).

To prove this claim we demonstrate directly that one intermediary, intermediary 1, cannot be more profitable by deviating from \( s^*_1 \) given that the other intermediary, intermediary 2, chooses \( s^*_2 \).

(i) Suppose \( t \geq \bar{t} \) and consider any deviation \( s_1 \neq s^*_1 = s^*_2 = 1 - \frac{\mu}{e^2} \), with the corresponding profits denoted by \( \bar{\pi}_1(s_1) \). Any deviation \( s_1 \in \left[ 0, 1 - \frac{\mu}{e^2} \right] \) leads to profits

\[
 \bar{\pi}_1(s_1) = \frac{1}{2t} \left( \frac{1}{e^2-1} \right) \rho \mu^2 \left( t - \frac{\mu^2}{e^2(e^2-1)(1-s_1)^2} + \frac{\mu}{e^2-1} \right),
\]

which we show is increasing on this interval. Specifically, \( \frac{\partial \bar{\pi}_1}{\partial s_1} > 0 \) as long as

\[
 t > \left( \frac{2\mu}{e^2(e^2-1)(1-s_1)^2} - 1 \right) \frac{1}{e^2-1} \mu.
\]

for all \( s_1 \). We have,

\[
 t \geq \bar{t} = \frac{n(e_0^2-1)}{n-e^2} \left[ \ln(n) - \frac{n}{n-1} + \frac{1}{e_0^2-1} \right] \mu > \frac{\mu}{e^2-1} > \left( \frac{2\mu}{e^2(e^2-1)(1-s_1)^2} - 1 \right) \frac{1}{e^2-1} \mu.
\]

where the first inequality holds by assumption, the second since \( n > e^2 \), and the third for \( s_1 \in \left[ 0, 1 - \frac{\mu}{e^2} \right] \). Therefore, any deviation \( s_1 < s^*_1 = 1 - \frac{\mu}{e^2} \) is not profitable.

Any deviation \( s_1 \in \left( 1 - \frac{\mu}{e^2}, 1 - \frac{\mu}{n^2} \right) \) leads to

\[
 \bar{\pi}_1(s_1) = \left( \frac{1}{2t} \right) \rho \mu^2 \left( t - \frac{\mu^2}{\mu-(1-s_1)^2} + \mu \ln \left( \frac{\mu}{(1-s_1)^2} \right) - \mu - \left( -\frac{\mu}{1-e^2} + \mu \right) \right).
\]
This deviation is not profitable if \( \frac{\partial \pi_1}{\partial s_1} < 0 \), which requires
\[
t > \mu f(s_1) \equiv \left( \frac{\mu}{1-s_1} \right) + \frac{2\mu}{\mu - (1-s_1)} - \ln \left( \frac{\mu}{1-s_1} \right) - \frac{1}{e^{2-1}} \mu.
\]
Since \( t > \frac{n(e^{-1})}{n-e^2} \left[ n - \frac{n^{-1}}{e^{2-1}} \right] \mu > (e^2 + \frac{1}{e^{2-1}})\mu = \mu f \left( 1 - \frac{\mu}{e^{-2}} \right) \), profits are decreasing near (and to the right of) \( s_1^* \). Note that the function \( f(s_1) \) is strictly increasing in \( s_1 \). Therefore, the condition \( \pi_1^* > \tilde{\pi}(s_1) \) at \( s_1 = 1 - \frac{\mu}{n\tau} \), the right endpoint of the interval, is sufficient for \( \pi_1^* > \tilde{\pi}(s_1) \) at any \( s_1 \in \left( 1 - \frac{\mu}{e^{-2}}, 1 - \mu \right) \). This condition is
\[
\pi_1^* = \frac{\rho \mu}{2} \left( \frac{e^2}{e^{2-1}} \right) > \left( \frac{\rho \mu}{2t} \right) \left( \frac{n}{n-1} \right) \left[ t + \mu \ln(n) - \frac{n^{-1}}{n} \mu + \frac{e^2}{e^{2-1}} \mu - 2\mu \right] = \tilde{\pi}_1 \left( 1 - \frac{\mu}{n\tau} \right),
\]
which holds by our assumption \( t > \bar{t} = \frac{n(e^{-1})}{n-e^2} \left[ n - \frac{n^{-1}}{e^{2-1}} + \frac{1}{e^{2-1}} \right] \mu \).
Any deviation \( s_1 \in \left( 1 - \frac{\mu}{n\tau}, 1 \right) \) leads to a profit of
\[
\tilde{\pi}_1(s_1) = \left( \frac{\rho \mu}{2t} \right) \left( \frac{n}{n-1} \right) \left[ t + \mu \ln(n) - \frac{n^{-1}}{n} \mu - (1 - s_1)n\tau + \frac{\mu}{e^{2-1}} \right],
\]
which is increasing in \( s_1 \) and therefore bounded above by \( \tilde{\pi}(s_1 = 1) \). The condition \( t > \bar{t} = \frac{n(e^{-1})}{n-e^2} \left[ n - \frac{n^{-1}}{e^{2-1}} + \frac{1}{e^{2-1}} \right] \mu \) directly implies that
\[
\pi_1^* = \frac{\rho \mu}{2} \left( \frac{e^2}{e^{2-1}} \right) > \left( \frac{\rho \mu}{2t} \right) \left( \frac{n}{n-1} \right) \left[ t + \mu \ln(n) - \frac{n^{-1}}{n} \mu + \frac{\mu}{e^{2-1}} \right] = \tilde{\pi}_1(1) \geq \tilde{\pi}_1(s_1),
\]
for all \( s_1 \in \left( 1 - \frac{\mu}{n\tau}, 1 \right) \).

(ii) Now suppose \( t \leq \bar{t} = \frac{n(e^{-1})}{n-e^2} \left[ n - \frac{n^{-1}}{e^{2-1}} + \frac{1}{e^{2-1}} \right] \mu \). We first consider any deviation \( s_1 \in \left[ 0, 1 - \frac{\mu}{e^{-2}} \right] \). This leads to a deviation profit of
\[
\tilde{\pi}_1(s_1) = \frac{\rho \mu^2}{2t^2} \left[ \frac{1}{e^{2-1}} \right] \left[ t - \frac{\mu^2}{e^{-2}} \left( \frac{n}{n-1} \right) \left( \frac{n^{-1}}{e^{2-1}} \right) + \mu \left[ \frac{n}{n-1} - \ln(n) \right] \right].
\]
Characterizing the shape of this deviation profit function depends on the level of \( t \). We argue that \( \tilde{\pi}_1(s_1) \leq \pi_1^* \) for different three levels of \( t \).

For \( 0 < t < \left[ \ln(n) - \frac{n^{-1}}{n-1} \right] \mu \), the expression for the demand at intermediaries 1,
\[
\tilde{D}_1 = \frac{1}{2t} \left( t - \frac{\mu^2}{e^{-2}} \left( \frac{n}{n-1} \right) \left[ \frac{n^{-1}}{e^{2-1}} \right] + \mu \left[ \frac{n}{n-1} - \ln(n) \right] \right) < 0.
\]
So any deviation under this condition is not profitable.

For \( \left[ \ln(n) - \frac{n^{-1}}{n-1} \right] \mu \leq t \leq \left[ \ln(n) - \frac{n^{-1}}{n-1} + \frac{2}{e^{2-1}} \right] \mu \), the derivative \( \partial \tilde{\pi}_1 / \partial s_1 \) has the following property:
\[
\frac{\partial \pi_1}{\partial s_1} \bigg|_{s_1=1-\frac{\mu}{r}e^{-2}} < 0 < \frac{\partial \pi_1}{\partial s_1} \bigg|_{s_1=0}.
\]

Since the derivative is continuous, it means that any maximizer, \( \hat{s}_1 \), of \( \bar{\pi}_1(s_1) \) in \( [0, 1 - \frac{\mu}{r} e^{-2}] \) must solve \( \frac{\partial \bar{\pi}_1}{\partial s_1} = 0 \). This solution is expressed \( \hat{s}_1 = 1 - \frac{\mu}{r} \left\{ e^2 (e^2 - 1) \left[ \frac{1}{\mu} - \ln(n) + \frac{n}{n-1} \right] \right\}^{-1} \) and leads to profits

\[
\bar{\pi}_1(\hat{s}_1) = \left( \frac{\tau \rho \mu}{n} \right) \left[ \frac{t}{\mu} - \ln(n) + \frac{n}{n-1} \right] \left\{ 1 - \frac{\mu}{r} \left[ \ln(n) - \frac{n}{n-1} \right] \right\}.
\]

Under the condition that \( t \leq \left[ \ln(n) - \frac{n}{n-1} + \frac{2}{e^2 - 1} \right] \mu \),

\[
\bar{\pi}_1(\hat{s}_1) \leq \frac{1}{4} \left( \frac{\rho \mu}{e^2 - 1} \right) \left( \frac{2}{e^2 - 1} \right) \left( \ln(n) - \frac{n}{n-1} + \frac{1}{e^2 - 1} \right) \mu < \frac{\rho \mu}{2(1-1/n)},
\]

where the last term is the profit \( \pi_1^* \) the intermediary earns by sticking to \( s_1^* = 1 \). Hence, no deviation \( s_1 \in [0, 1 - \frac{\mu}{r} e^{-2}] \) is profitable.

For \( [\ln(n) - \frac{n}{n-1} + \frac{2}{e^2 - 1}] \mu < t \leq \bar{t} = \frac{n(e^2 - 1)}{n-e^2} \left[ \ln(n) - \frac{n}{n-1} + \frac{1}{e^2 - 1} \right] \mu \), we have \( \frac{\partial \bar{\pi}_1}{\partial s_1} > 0 \) for all \( s_1 \in [0, 1 - \frac{\mu}{r} e^{-2}] \). Thus,

\[
\bar{\pi}_1(s_1) \leq \bar{\pi}_1 \left( 1 - \frac{\mu}{r} e^{-2} \right) = \frac{\rho \mu}{2t} \left( \frac{e^2}{e^2 - 1} \left[ t - \mu \left( 1 - \frac{e^2 - 1}{n-1} \right) + \ln(n) \right] \right) \leq \pi_1^*,
\]

for all \( s_1 \in [0, 1 - \frac{\mu}{r} e^{-2}] \). Hence, for any \( t \leq \bar{t} = \frac{n(e^2 - 1)}{n-e^2} \left[ \ln(n) - \frac{n}{n-1} + \frac{1}{e^2 - 1} \right] \mu \), there is no profitable deviation for any \( s_1 \in [0, 1 - \frac{\mu}{r} e^{-2}] \).

Now consider deviations \( s_1 \in \left( 1 - \frac{\mu}{r} e^{-2}, 1 - \frac{\mu}{n^2} \right) \) when \( s_2^* = 1 \).

Intermediary 1’s profit is given by

\[
\bar{\pi}_1(s_1) = \frac{\rho \mu^2}{\mu - (1-s_1) r} \left( \frac{1}{2t} \right) \left\{ t + \mu \left[ \ln \left( \frac{\mu}{\mu - (1-s_1) r} \right) - \frac{\mu}{\mu - (1-s_1) r} - 1 - \ln(n) + \frac{n}{n-1} \right] \right\}.
\]

It can be shown that

\[
\frac{\partial \pi_1}{\partial s_1} > 0 \iff t < \left[ \frac{\mu}{(1-s_1) r} + \frac{2\mu}{\mu - (1-s_1) r} - \ln \left( \frac{\mu}{\mu - (1-s_1) r} \right) - \frac{n}{n-1} + \ln(n) \right] \mu \equiv \mu f(s_1).
\]

where \( f(s_1) > 0 \) is increasing on \( \left( 1 - \frac{\mu}{r} e^{-2}, 1 - \frac{\mu}{n^2} \right) \).

Suppose \( 0 \leq t \leq \left( e^2 + \frac{2}{e^2 - 1} - \frac{n}{n-1} + \ln(n) \right) \mu = \mu f \left( 1 - \frac{\mu}{r} e^{-2} \right) \). Then \( \frac{\partial \pi_1}{\partial s_1} > 0 \) for all \( s_1 \in \left( 1 - \frac{\mu}{r} e^{-2}, 1 - \frac{\mu}{n^2} \right) \). Thus,

\[
\bar{\pi}_1(s_1) \leq \bar{\pi}_1 \left( 1 - \frac{\mu}{n^2} \right) = \frac{\rho \mu}{2} \left( \frac{n}{n-1} \right) \left( 1 - \frac{\mu}{t} \right)
\].
for all $s_1 \in \left(1 - \frac{\mu}{\tau} e^{-2}, 1 - \frac{\mu}{n\tau}\right)$. However, by choosing $s_1^* = 1$, intermediary 1 earns $\pi_1^* = \frac{\rho\mu}{2(1-1/n)}$, which exceeds $\bar{\pi}_1 \left(1 - \frac{\mu}{n\tau}\right)$.

Suppose $[e^2 + \frac{2}{\sigma^2-1} - \frac{n}{n-1} + \ln(n)]\mu < t < \bar{t} = \frac{n(e^2-1)}{n-e^2} \left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right] \mu$. Then $\frac{\partial \pi_1}{\partial s_1} < 0$ near $s_1 = 1 - \frac{\mu}{\tau} e^{-2}$. In this case, $\bar{\pi}_1 (s_1)$ is bounded by either $\bar{\pi}_1 \left(s_1 = 1 - \frac{\mu}{\tau} e^{-2}\right)$ or $\bar{\pi}_1 \left(s_1 = 1 - \frac{\mu}{n\tau}\right)$. We know from above that both of these values are exceeded by the profit $\pi_1^*$. Hence there is no profitable deviation $s_1 \in \left(1 - \frac{\mu}{\tau} e^{-2}, 1 - \frac{\mu}{n\tau}\right)$.

Finally consider any deviation $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right)$. This leads to profits given by

$$\bar{\pi}_1 (s_1) = \frac{\rho\mu}{2t} \left(\frac{n}{n-1}\right) \left[t - (1-s_1)\frac{n\tau}{t}\right],$$

which is obviously increasing in $s_1$. Therefore, choosing $s_1^* = 1$ gives intermediary 1 more profit than any in $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right)$.