Problem 8.19 from the textbook.

Problem 8.27 from the textbook.

Problem 8.38 from the textbook.

Write the VERTEX COVER Problem (minimize the number of nodes such that each edge is covered) as a Linear Program with integrality constraints, i.e., you are allowed to restrict your variables to take on only integer values. (In this way, you will have proved that INTEGER LINEAR PROGRAMMING is NP-hard.) Now get rid of the integrality constraints and compute the dual of your linear program. Come up with an interpretation of the meaning of the dual variables and the dual constraints. (In your interpretation, you may also treat the dual variables as integers.)

Remember the multi-commodity flow problem from class. For both versions (where you maximize the sum of all flow values and where you maximize the minimum of all flow values), write a linear program based on paths instead of edges (similar to what we did for regular flows). Then, compute the dual and interpret the meaning of the dual for both programs. Again, you get to treat the dual variables as integers if that helps you in your interpretation.

Chocolate Problem (2 chocolate bars): Remember the chocolate problem for Homework 2. We had a long highway (path), and drivers starting from nodes $s_i$, going to the same sink $t$, each with a budget $b_i$. The goal was to price the edges so as to maximize the total profit from those drivers, where a driver only drives if he can afford the total price of all edges he needs.

Now, let’s modify this slightly. We still have just one highway, but different drivers may have different destinations. In other words, for each $i$, there is a source $s_i$, sink $t_i$, and budget $b_i$. We still get to price the edges individually, and a driver will drive from $s_i$ to $t_i$ if he can afford the price of all edges he needs to use.

Prove that this new version of the problem is NP-complete.