(1) Suppose you want to schedule jobs you perform in order to maximize your profit. In each week $i$, you have a choice between not doing any job, doing a “lightweight” job with value $\ell_i \geq 0$, or doing a “heavy” job with value $h_i \geq 0$. In order to do a heavy job, you must rest the previous week, i.e., if you do a heavy job in week $i$, then you must have done no job in week $i-1$. If you do a lightweight job in week $i$, you are free to do any kind of job (or no job at all) in week $i-1$.

(a) A simple greedy algorithm for this problem would be to compare $h_{i+1}$ with $\ell_i + \ell_{i+1}$. If it is larger, rest in week $i$ and do the heavy job in week $i+1$; otherwise, do two light jobs. Then move on to $i+2$. Give an example input on which this algorithm does not find the best solution.

(b) Design and analyze a polynomial-time algorithm to determine the maximum total value you can obtain over the course of $n$ weeks, given the values $\ell_i$ and $h_i$ for each week. First derive and justify a recurrence relation (be sure to be clear about what exactly you denote by OPT etc.). Then turn it into a bottom-up solution.

(2) Problem 15-2 from the textbook.

(3) Problem 15-4 from the textbook.