

# CS303 (Spring 2008) — Assignment 12

Due: 04/23/2008

- (1)
  - (a) What is an efficient certifier?
  - (b) How is the class NP defined formally?
  - (c) Give an informal description of what it means for a problem to be in NP.
  - (d) What do we mean when we write  $X \leq_p Y$ .
  - (e) What does it mean for a problem to be NP-hard?
  - (f) What does it mean for a problem to be NP-complete?
- (2) Prove that if  $X \leq_p Y$  and  $Y \leq_p Z$ , then also  $X \leq_p Z$ . (That is, reductions are transitive.)
- (3) Prove that if  $X$  is any problem in NP, then  $X$  can be decided by an algorithm with running time  $O(2^{p(n)})$ , for some polynomial  $p$ .
- (4) For each of the following three problems, first formulate them as an essentially equivalent decision problem, and then prove that the decision problem you formulated is in NP.
  - (a) Given a graph  $G$  with distances  $d_e \geq 0$  on the edges, find the tour of shortest total length visiting each vertex at least once.
  - (b) Given a graph representing friendships between  $2n$  people, represented by edges. Find a way to group them together in pairs such that as many people as possible are partnered with a friend.
  - (c) Given a collection of items  $i$  with values  $v_i \geq 0$  and weights  $w_i \geq 0$ . Find a subcollection  $S$  of these items with the property that  $\sum_{i \in S} w_i \leq W$ , for some target weight  $W$ , and  $\sum_{i \in S} v_i$  is maximized subject to the weight constraint.
- (5) Describe (intuitively, but precisely) a computer science problem with the following properties:
  - (a) It is derived from an area of computer science that you personally care about (e.g., games, robotics, networks, or whatever else).
  - (b) The only way you know to solve it uses exhaustive search, and takes exponential time in the input size.
  - (c) When you formulate it as a decision problem (which you should do), the problem is in NP.

Provide the decision formalization, and prove that your problem is in NP. Also, explain briefly why you think the problem is interesting.