(1) (a) What does it mean formally when we say that a set $X$ “respects functions”?
(b) What does it mean intuitively?
(c) In general, if $f$ is a function, what does it mean for $x$ to be a fixpoint of $f$?
(d) In the context of recursion theory, what does it mean, assuming $f$ is a total and recursive function?

(2) Prove that the following sets of programs are undecidable. You can use whatever method you would like to prove it.

(a) $\text{USC} = \{ P | \text{Program } P \text{ outputs 'USC' for each even number } x \}$.
(b) $\text{MST} = \{ P | \text{Program } P \text{ always returns the MST of its input graph } G \}$.
(c) $\text{HALT}^+ = \{ P | \text{Program } P \text{ halts when run on } P, \text{ and contains the statement “}y=x+y+z\text{” somewhere} \}$.

(3) Prove that for every two sets $X, Y$, there is a third set $Z$ such that $X \leq_m Z$ and $Y \leq_m Z$.

(4) (a) Prove that there is a program $P$ which, when given an input number $k$, prints its own source code $k$ times.
(b) Prove that there are programs $P, P'$ such that $P$ prints the source code of $P'$, and $P'$ prints the source code of $P$. 