Practice Midterm 1

Last Name: ________________________________
First Name: ________________________________
Signature: ________________________________
Start Time of your Lecture (11AM or 12PM): __________

Instructions:
Please show all of your work and reasoning. No other notes, books, calculators, electronic devices, or other memory aids are allowed for use during the test. Numerical answers should be left in calculator-ready form, unless otherwise indicated. If you need more space than what is provided, please use the back of the previous page, indicating clearly where the solution is continued.

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1
1. [10.3-10.5 Dot Products, Cross Products, Lines and Planes]

(a) Consider the line \( \ell_1 \) that is given by the symmetric equations:

\[
-x + 2 = y - 4 = \frac{z - 3}{2}
\]

and the point \( P = (1, 4, 1) \).

(i) Find the equation of the plane that contains both \( P \) and \( \ell_1 \).

(ii) Consider the line \( \ell_2 \) that passes through \( P \) and intersects \( \ell_1 \) at a right angle. Find symmetric equations for \( \ell_2 \).

(iii) Find the distance of \( P \) from \( \ell_1 \).

2. [10.6 Cylinders and Quadric Surfaces]

(a) Sketch the surface

\[
-x^2 + 4y^2 - z^2 = 4.
\]

(Be sure to show your work and label any important features in your sketch.)

(b) Match each equation with its graph. Write the corresponding number below the appropriate graph.

1. \(-y^2 + (z - 2)^2 - x + 1 = 0\)
2. \(y^2 - (z - 2)^2 - x + 1 = 0\)
3. \(-y^2 - (z - 2)^2 - x + 1 = 0\)
4. \(y^2 + (z - 2)^2 - x + 1 = 0\)

3. [10.7-10.8 Vector Functions, Space curves, and Arc Length]

(a) Consider the vector-valued function

\[
r(t) = (1 - e^t, 3e^t, 2e^t)
\]

i) Describe the curve \( C \) that is traced out by \( r \).

ii) Find the length of \( C \) between the points given by \( t = 0 \) and \( t = 2 \).

iii) Reparameterize the curve by arc-length starting from \( t = 0 \) and going in the direction of increasing \( t \).

4. [11.1-11.2 Functions of Several Variables and Limits]

(a) Prove that the limit

\[
\lim_{(x,y) \to (0,0)} \frac{3xy^2}{x^3 + y^3}
\]

does not exist.
(b) Use the “Squeeze Theorem” to show that

\[ \lim_{{(x,y) \to (0,0)}} \frac{x^2 y}{\sqrt{x^2 + y^2}} = 0. \]

5. [11.3 Partial Derivatives]

(a) Let \( f(x, y) = x^y \). Find \( f_x \) and \( f_y \).

(b) If \( f(x, y) = \ln(x + \sqrt{x^2 + y^2}) \), find \( f_x(3, 4) \).

(c) Shown below is a contour map of the function \( f \).

(i) Estimate \( f_y(1, 0) \).

(ii) Is \( f_{yy}(1, 0) \) positive or negative? Explain.

(iii) Is \( f_{yx}(1, 0) \) positive or negative? Explain.