Ant colony.
$z = A.$
$t = \text{time} \geq 10$

meteor shower?

Spring morning

$y(t) = \frac{1}{2} t - 5$
$x(t) = t^3 + 1$

Temperature = $x = \sin t$
Time = $t$
$z = \text{chance of survival}$
$y = \# \text{ of bears awake}$
\[ z = \text{amount of ivory}. \]
\[ x(t) = \# \text{ of elephants killed at time } t. \]
\[ y(t) = \# \text{ of rhinos}. \]
\[ t = \text{time}. \]

\[ z' = \text{speed of buzz over calf's blood flow, temperature.} \]
\[ x(t) = \text{energy spent during warm-up}. \]
\[ y = \text{how tired after warming up}. \]
\[ t = \text{time of warm-up}. \]

\[ \oint z = z(x, y) = z(t) \]
\[ = (z(x(t), y(t))). \]

\[ z = f(x, y). \]

\[ \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f_x = D_x f = \frac{\partial}{\partial x} f. \]
Math 226: Lecture #15: Chain Rule.

\[ z = f(x(t), y(t)) \]

〜5min 1. Write a short story, draw a picture, or describe a scenario in which a relationship between variables of the above form applies.

〜5min 2. Share your pictures/stories/descriptions in groups of 2 or 3.

〜2min 3. Share with full class.

〜2min Q: What does \( \frac{dz}{dt} \) represent in your story/picture? (Discuss in groups).

Suppose we have lots of information about \( x(t), y(t) \), and \( f \). Let's find a formula for \( \frac{dz}{dt} \).

Note: \( z = f(x, y) \) and \( x = x(t), y = y(t) \).

From last class:

\[ \frac{\Delta z}{\Delta t} \approx \frac{\partial f}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y} \frac{\Delta y}{\Delta t} \]

\[ \Delta x = \frac{dx}{dt} \Delta t \quad \Delta y = \frac{dy}{dt} \Delta t \]
\[
\Delta z = \frac{\partial z}{\partial x} \frac{dx}{dt} \Delta t + \frac{\partial z}{\partial y} \frac{dy}{dt} \Delta t
\]

\[\Rightarrow \frac{\Delta z}{\Delta t} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}.\]

Chain Rule.

Example 1.

An ant crawls on a hot plate.

The temperature of the hot plate at position \((x,y)\) is:

\[z = f(x,y) = e^{-y^2-x^4}\]

The position of the ant as a function of time is:

\[\langle x(t), y(t) \rangle = \langle \cos(t), \sin(2t) \rangle\]

At what rate does the ant's perceived temperature change with respect to time?

\[
\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}.
\]

\[
= -4x^3 (e^{-y^2-x^4})(-\sin t) - 2y e^{-y^2-x^4} (2 \cos(2t))
\]

\[
= -4e^{-y^2-x^4}(x^3 \sin t - y \cos (2t))
\]

\[
= 4e^{-\sin^2(2t)-\cos^2(t)} (\cos^2 t \sin t - \sin (2t) \cos (2t))
\]
Example 2.

Same question except that you want to find $\frac{dz}{dt}$ when $t = 0$ and all you know about $f, x, y$ is:

$f(1, 3) = 2$

$f_x(1, 3) = -5$

$f_y(1, 3) = 4$

and that $x = e^t$

$y = 3 \cos t$
\[ z = f(x(s, t), y(s, t)) = g(s, t) \]

\[ \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}. \]