Office hours on Wed cancelled.
Substitute for Friday's class.

Quick note about parametrizations.

\[ \mathbf{r} = (x, y, z) \]

\[ \begin{align*}
  t &= -3 \\
  s &= 6, t = -2.7 \\
  s &= \text{arc length} \\
  \mathbf{R} &= (x, y, z)
\end{align*} \]

\[ \begin{align*}
  s &\rightarrow \mathbf{R} \\
  t &\rightarrow \mathbf{R}
\end{align*} \]

Variable function.

\[ \mathbf{R}(s) = \mathbf{r}_1(-2.7) \]

Last day, we had \( \mathbf{r}_1(t) \).
We found \( t = f(s) \).

\[ \mathbf{R} = \mathbf{r}(s) = \mathbf{r}_1(t) = \mathbf{r}_1(f(s)) \]
Motivation: Optimization problems that are not so contrived.

If 1200 cm$^2$ of material is available to make a box with an open top, find the largest possible volume of the box.

\[ V = xyz \]

square base: \( x = y \)

surface area: \( 1200 = xy + 2yz + 2xz \)

\[ V = f(x, y, z) = xyz \]

function of 3 variables.

We are now interested in functions

\[ f: D \rightarrow \mathbb{R} \]

where the domain \( D \) is (part of) \( \mathbb{R}^2 \) (or \( \mathbb{R}^3 \)).
Graph of a function \( f : \mathbb{R}^2 \to \mathbb{R} \).

The graph forms a surface in \( \mathbb{R}^3 \).

**LEVEL CURVES**

- These are the "\( z \)-traces" of the surface \( z = f(x,y) \); i.e., the curves
  \[ f(x,y) = k \]
  for some constant \( k \).

**Example 1: Mountain Terrain**

"Contour map"
Example 2: \( f(x, y) = e^{-x^2-y^2} \).

What shapes are the level curves?

A) paraboloid
B) bell curves
C) circles
D) ellipses
E) other.

"Level curves" \( z = f(x, y) = k \) for some constant\( e^{-x^2-y^2} = k \).

\( \Rightarrow -x^2 - y^2 = \ln(k) \), \( \Rightarrow x^2 + y^2 = -\ln(k) = \text{null} \), \( \Rightarrow x^2 + y^2 = \frac{1}{k} \) (a constant)

\( 0 < k \leq 1 \)

\( x = 0 \Rightarrow -y^2 \)

\( z = e^{-y^2} \).

\( f(x, y) = g(x^2 + y^2) \)

will always level curves that are circles.

(in this case \( g(u) = e^{-u} \))