Problem Set 8  
Math 226 - Sections 39583 and 39587

The following problem set is split into three parts: the quiz practice problems, which form the basis for the upcoming quiz, and the homework problems, which will be handed in, and the extra practice, which will not be handed in.

Quiz Practice Problems
The problems on the quiz (Thursday, March 10) will be selected from the following problem set, but may be slightly modified for the quiz. During the quiz, no notes, books, calculators or other aides will be allowed (only writing utensils). Leave any numerical responses in calculator-ready form.

- Section 11.6 # 15, 17, 24, 28, 30, 31, 33, 35, 39
- Section 11.7 # 1, 2, 3, 5, 9, 11, 13
- True or False: The gradient vector $\nabla f(a, b)$ is orthogonal to the tangent plane to the graph $z = f(x, y)$ at the point $(a, b, f(a, b))$. Explain your answer.

Homework Problems
The solutions to the following problems are due to be handed in at the beginning of your lecture on Friday, March 11 or your discussion session on Thursday March 10. Please don’t forget to write your name and lecture time at the top of the page, and staple it.

- Section 11.6 # 26(c), 36, 44, 48
- Section 11.7 # 2, 8, 12

1. Consider the sphere $x^2 + y^2 + z^2 = 9$.
   The goal of this problem is to find the equation of the tangent plane $P$ to this sphere at the point $(1, 2, 2)$ in three different ways. (The important thing here is to see the connections between these three ideas.)
   (a) On the upper hemisphere, i.e. the part of the sphere with $z \geq 0$, solve for $z$ as a function of $x$ and $y$. Use the methods of Section 11.4 to find an equation for $P$.
   (b) Note that this sphere is a level surface of the function $F(x, y, z) = x^2 + y^2 + z^2$. Use the gradient vector of $F$ to find an equation of $P$.
   (c) In a previous homework problem, you showed that for any parametrized curve on a sphere, the tangent vector $\vec{r}'(t)$ is orthogonal to the vector $\vec{r}(t)$. Use this idea to find the equation of $P$.

2. For a given function $f$ of two variables, let $F(x, y, z) = z - f(x, y)$.
   (a) Briefly, show why the graph of $f$ is a level surface for $F$.
   (b) Consider the tangent plane at a point $(x_0, y_0, z_0)$ on the graph of $f$. Use the gradient vector $\nabla F(x_0, y_0, z_0)$ to write down the equation for this tangent plane. Simplify this equation as needed to show that it is the same as Equation 2 on Page 628, in Section 11.4.

Extra Practice
At minimum, we recommend working through the quiz practice problems and homework problems each week. If you feel like you need extra practice, you may draw on problems from the set below, choosing problems based on topics that you are struggling with most. These are not to be handed in, unless they are also listed above.
• Section 11.6: # 15-54
• Section 11.7: # 1-18, 29, 31