Average rate of change of $f(x)$ as $x$ changes from $a$ to $b$:

\[
\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}
\]

Graphically, this is the \underline{slope} of the line that passes through \((a, f(a))\) and \((b, f(b))\). This line is called a \textit{"SECANT LINE"}.

**Example:**

The height (in metres) of a coconut falling from a tree at time $t$ (in s) is

\[h(t) = 20 - 9.8t^2\]

Find the average velocity of the apple as $t$ changes from 0 to 1.

\[
\frac{\Delta h}{\Delta t} = \frac{h(1) - h(0)}{1 - 0}
\]

\[= \frac{(20 - 9.8(1)^2) - (20 - 9.8(0)^2)}{1} \text{ m/s}
\]

\[= -9.8 \text{ m/s}.
\]
Q. Why is \( \frac{\Delta h}{\Delta t} \) negative?

Because \( h \) decreases as \( t \) increases.

Would you expect each of the following to be positive or negative?

1. The rate of change of the volume of a balloon with respect to its radius.
   \( \frac{\Delta V}{\Delta r} > 0 \); positive.

2. The rate of change of the amount of money in your retirement savings account with respect to time.
   \( M \)
   \( t_r \) = time of retirement when \( t < t_r \), \( \frac{\Delta M}{\Delta t} > 0 \)
   \( t \) when \( t > t_r \), \( \frac{\Delta M}{\Delta t} < 0 \)

3. The rate of change of the volume of a tumour with respect to time, if it is being treated with chemotherapy.
   \( \Delta V \) shrinking \( \Rightarrow \frac{\Delta V}{\Delta t} < 0 \).
Back to two car example.

\[ t = 0, \quad x = 0 \]  

(in mi) \[ \sqrt{\cdot} \]  

Let \( x(t) \) be the position of the car relative to the tree at time \( t \) since it passed the tree.  

(in h)  

\[ x \]  

what is the velocity of the car at \( t = 1 \)?  

(as read on the speedometer.)  

\[ x(t) = t^2. \]  

So far, we know how to find average velocities.

E.g. If \( x(t) = t^2 \), the average velocity as \( t \) changes from 1 to 2 is

\[ \frac{\Delta x}{\Delta t} = \frac{x(2) - x(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = 3. \]
The average velocity as \( t \) changes from 1 to 1.5 is
\[
\frac{\Delta x}{\Delta t} = \frac{x(1.5) - x(1)}{1.5 - 1} = \frac{(3/2)^2 - 1^2}{\sqrt{2}} = \frac{9/4 - 1}{\sqrt{2}}
\]
\[
= 2 \left( \frac{5}{4} \right) = \frac{5}{2} = 2.5 \text{ m/s}.
\]

We would like the velocity exactly at \( t = 1 \), but we cannot take \( \Delta t = 0 \) because we can't divide by zero!

However, we can look at the average velocity as \( t \) changes from 1 to \( 1 + h \) where \( h \) is very small.

\[
\frac{\Delta x}{\Delta t} = \frac{x(1+h) - x(1)}{1+h - 1}
\]
\[
= \frac{x(1+h) - x(1)}{h}.
\]
\[
= \frac{(1+h)^2 - (1)^2}{h}.
\]
\[
= \frac{1 + 2h + h^2 - 1}{h}.
\]
\[
= \frac{2h + h^2}{h}.
\]
\[
= 2 + h.
\]
Definition:
The DIFFERENCE QUOTIENT of a function \( f(x) \) is
\[
\frac{f(x+h) - f(x)}{h}
\]

Example:
Find and simplify the difference quotient for the function
\[
f(x) = \frac{1}{x}
\]