E1) Example: A car traveling at speed 50 mi/h down a straight road, then stops.

Sketch a graph of the distance the car has traveled as a function of time t.

What is the car doing for t > 2h?
A) braking
B) going back to where it came from

E2) Example: Car with another distance vs. time graph for a car.

a) Write a story about the person driving the car.
b) What is the velocity of the car at...

7AM - 8AM?
0

8AM - 9AM?

\[ \text{velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta x}{\Delta t} = 50 \text{ mi/h} \]

= 50 mi/h.

3:30 - 4PM?

\[ \text{velocity} = \frac{\Delta x}{\Delta t} = -\frac{30 \text{ mi}}{\frac{1}{2} \text{ h}} = (20 - 50) \text{ mi} \]

= -60 mi/h.

a) Tell a story about the car.

b) How would you describe the velocity of the car? How would you calculate it?
DEFINITION

The average rate of change of \( f(x) \) with respect to \( x \) as \( x \) changes from \( a \) to \( b \) is

\[
\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}
\]

Examples:

1. Find the average rate of change of \( f(x) = \sqrt{x+1} \) as \( x \) changes from 0 to 8.

\[
\frac{f(8) - f(0)}{8 - 0} = \frac{\sqrt{8+1} - \sqrt{0+1}}{8} = \frac{3 - 1}{8} = \frac{2}{8} = \frac{1}{4}
\]

2. Find the average velocity of the car with position given by \( x(t) = 40t^2 \) as time \( t \) changes from \( \frac{1}{2} \) to 1.

\[
\frac{\Delta x}{\Delta t} = \frac{x(1) - x\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = \frac{40(1)^2 - 40\left(\frac{1}{2}\right)^2}{\frac{1}{2}} \text{ mi/h}
\]

\[
= 2(40 - 10) \text{ mi/h} = 60 \text{ mi/h}.
\]
3. Find the rate of change of the volume $V$ of a balloon with respect to its radius $r$, as $r$ changes from 5 cm to 10 cm.

$$ V = \frac{4}{3} \pi r^3 $$

$$ \frac{\Delta V}{\Delta r} $$

**Graphical Interpretation of average rate of change**