Step-by-step prescription for completing the square.

- Summary of graph transformations for functions $y = f(x)$.
- Circles and their tangent lines

Steps of completing the square.

$$f(x) = 3x^2 - 5x + 7$$

1. Factor out the coefficient of $x^2$ from the two terms involving $x$.

$$f(x) = 3 \left( x^2 - \frac{5}{3}x \right) + 7$$

2. Add and subtract \((\frac{b}{2})^2\)

$$f(x) = 3 \left( x^2 - \frac{5}{3}x + \left(\frac{\frac{5}{6}}{2}\right)^2 - \left(\frac{\frac{5}{6}}{2}\right)^2 \right) + 7$$

3. Factorize & simplify.

$$f(x) = 3 \left( \left( x - \frac{5/6}{2} \right)^2 - \left(\frac{5}{6}\right)^2 \right) + 7$$

$$= 3 \left( x - \frac{5}{6} \right)^2 - \frac{25}{12} + 7$$

$$= 3 \left( x - \frac{5}{6} \right)^2 + \frac{21}{12}$$
Summary of Graph Transformations

(1) Horizontal Translations

\[ y = f(x - c) \]

(c > 0 means a shift to the right)

(c < 0 means a shift to the left)

(2) Vertical Translations

\[ y = f(x) + c \]
3. Vertical Expansions

\[ y = c f(x) \]

- \( c = 2 \)
  \[ y = 2f(x) \]
- \( c = \sqrt{2} \)
  \[ y = \frac{1}{2} f(x) \]

4. Reflections

a) Vertical
b) Horizontal reflections.

\[ y = f(-x) = g(x) \]

\[ e.g. \ g(1) = f(-1) \]

Examples.

\( g(x) = \sqrt{-x} \)

Sketch the graph of \( g(x) = \sqrt{-x} \)
Ex 2) Sketch the graph of \( h(x) = -|x-3| + 1 \).

Ex 3.

\( y = x^3 + 1 \)

\( y = \frac{1}{2} x^3 + 1 \)

\( y = \frac{1}{2} x^3 \)