M108: Lecture 11.

(see attached worksheet).

- We can graph any function of the form:
  \[ f(x) = a(x-b)^2 + c \]
  with any constants.

- What about other quadratic functions, such as:
  \[ g(x) = x^2 - (ex + 2) \]

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**Goal:** Take any function of the form
\[ f(x) = ax^2 + bx + c \]
and express it in the form \( \ast \) above.

**Note:**

\[ (x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9. \]

In general, for any \( u \),

\[ (x+u)^2 = (x+u)(x+u) = x^2 + 2ux + u^2. \]
Examples.

1. \( h(x) = x^2 - 6x + 9 \)  
   \[ u = -3 \]
   - Make this \( 2ux = -6x \)
   - \( u = -3 \)
   - Then \( u^2 = 9 \)

2. \( g(x) = x^2 - 6x + 11 \)

Thoughts:

\( g(x) = x^2 - 6x + 11 \)

\( = x^2 - 6x + 9 + 2 \)

\( h(x) = (x-3)^2 \)

\( \text{"completing the square"} \)

\( g(x) = (x-3)^2 + 2 \)

\( f(x) = x(x-6) \)

\( h(x) = x^2 \)

Shift to the right by 6.

\( l(x) = (x-6)^2 \)

\( y = g(x) \)

\( y = x^2 \)

\( y = (x-3)^2 \)
3. \[ f(x) = 3x^2 - 6x + 8 \]
\[ = 3 \left( x^2 - 2x + \frac{8}{3} \right) \]
\[ = 3 \left( \left( x^2 - 2x + 1 \right) + \frac{5}{3} \right) \]
\[ = 3 \left( (x-1)^2 + \frac{5}{3} \right) \]
\[ = 3(x-1)^2 + 5 \cdot \]

\[ (x-2)^2 = x^2 - 4x + 4 \]
\[ (x-1)^2 = x^2 - 2x + 1 \]

\[ y = x^2 \]
\[ y = 3x^2 \]
\[ y = 3(x-1)^2 + 5 \]

\[ f(x) = -x^2 + -x + - \]
1. Use the axes to sketch the graphs of the functions below.

\[ f_1(x) = x^2 \]

\[ f_2(x) = (x - 2)^2 \]

\[ f_3(x) = -(x - 2)^2 \]

\[ f_4(x) = -\frac{1}{2} (x - 2)^2 \]

\[ f_5(x) = -\frac{1}{2} (x - 2)^2 + 3 \]
2. Fill in the blanks with any numbers you like.

\[ f(x) = \_ \_ (x - \_ \_ \_ )^2 + \_ \_ \_ \]

Exchange papers and sketch graphs of each other's functions.
3. Find an equation for the function $f$ graphed below.

\[ y = f(x) \]

\[ f(x) = \]

4. [Time permitting] Sketch the graph of $g(x) = 2x^2 + 4x + 1$. 