Math 108: Lecture 10

- Quiz cancelled for next week. (due to Midterm)
- Homework due Wed. (Updated)

Chapter 1, Chapter 3

Functions (cont'd)

Summary so far:

\[ x \rightarrow f \rightarrow f(x). \]

Domain = set of inputs

Range = set of outputs

"rule"

Notation

\[ f(x) = y. \]

input \quad \text{output}

Example:

C1) If \( f(x) = x^2 - 1 \), what is \( f(x+1) \)?

A) \( x^2 \)
B) \( x^2 + 2 \)
C) \( x^2 + 2x \)
D) Other.

\[
\begin{align*}
\text{f(x+1)} &= (x+1)^2 - 1 \\
&= x^2 + 2x + 1 - 1 \\
&= x^2 + 2x.
\end{align*}
\]

\[
\begin{align*}
f(x+1) &= f(x) + 1 \\
f(1) &= f(1) + f(1)
\end{align*}
\]
In this case, \[ y = f(x) = x^2. \]

What is the domain of this function?

55  
A. \( \mathbb{R} \)  
B. \([0, \infty)\)  
C. \((0, \infty)\)  
D. More information needed  
E. Other: ___

*When we don’t specify the domain, we assume that it is the largest set of numbers for which the function is defined (makes sense).

What is the range of \( f(x) = x^2 \)?

A. \( \mathbb{R} \)  
B. \([0, \infty)\)  
C. \((0, \infty)\)  
D. More info needed  
E. Other: ___
Example: \( x = y^2 \)

Graph:

Does this define a function \( y = f(x) \)?

No!

However, we can create a function by choosing only the upper branch:

\[
y = g(x) = \sqrt{x} \quad \text{non-negative} \quad \Rightarrow \quad \text{the positive square root of } x
\]

(3) What is the domain of \( g \)?

- A all real numbers, \( \mathbb{R} \)
- B \([0, \infty)\)
- C \((0, \infty)\)
- D other

\( \sqrt{0} = 0 \).

\( 0 \cdot 0 = 0 \)

(4) What is the range of \( g \)? \([0, \infty)\)
(A) \( h(x) = \frac{1}{x^2 - 1} \)

what is the domain of \( h \)?

A) \( \mathbb{R} \)

B) \( \{ x \neq 0 \} \)

C) \( \{ x \neq \pm 1 \} \)

D) Other: \( \{ x \neq \pm 1 \} \).  

we can't have 0 in the denominator:

\[
x^2 - 1 = 0 \\
\Rightarrow x^2 = 1 \\
\Rightarrow x = \pm 1
\]

Piecewise-defined Functions.

Last class, we saw an example that looked something like this.

\[ y = f(x) \]

Can we represent this function symbolically?

\[
f(x) = \begin{cases} 
  x - 1 & \text{if } x < 2, \\
  2 & \text{if } x \geq 2.
\end{cases}
\]

E.g. \( f(3) = 2 \), \( f(-1) = (-1) - 1 = -2 \)
Examples:

1. Sketch the graph of the function:

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 2 \\
  3-x & \text{if } 2 < x \leq 4 \\
  1 & \text{otherwise} 
\end{cases} \]

2. A country charges 15% tax on the first $60,000 of yearly income and 40% on all income over $60,000. Express the total tax T as a function of income M.