Computer simulation of impact-induced particle breakage

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Abstract

The breakage induced in single circular particles that impact on solid plates has been studied using a two-dimensional simulation of solid fracture. The simulation allows the computer 'experimenter' to vary independently material properties such as Young's modulus, Poisson's ratio and work of fracture, flexibility that is unavailable in direct experimentation. Where comparison is possible, the simulation appears to mimic experimental results accurately. This study shows that the size distributions are, as would be expected, most strongly dependent on the collisional energy. Of secondary importance is the ratio of the impact velocity to the sound speed within the solid material. Finally, the size distributions show little effect of Poisson's ratio.

Keywords: Breakage; Fragmentation; Impact

1. Introduction

At the most fundamental level, all industrial grinding reduces to the breakage of individual particles that occurs through contact with other particles or with the solid walls of the mill. As a result, much attention has been paid to single-particle breakage from both experimental and theoretical points of view [1-8]. Experimentally, data are usually presented as particle size distributions (often in Rosin–Rammler or Schuhmann–Gaudin form), perhaps combined with photographs and other qualitative post-mortem descriptions of the fragments; collectively, these provide very little insight into the dynamics of the fracture process. Moreover, there is always some uncertainty about the material properties of the samples. On the other hand, theoretical investigations are limited to either very simple cases (due to the complexity of handling the simultaneous growth of multiple fractures) or to global or statistical descriptions that ignore the details of the fragmentation process.

The current study uses a simulation of solid fracture that allows precise control over the material properties and allows the fracture process to be examined in detail. The technique is referred to as a 'simulation', as it performs the fracture on a simulated material. The advantage of this technique is that literally everything about the simulated material is known and accessible to the computer 'experimenter'.

2. Description of the model

A complete description of the simulation technique can be found elsewhere [10–12] and only a brief reprise will be given here. At the present time, the simulation is limited to two dimensions. A three-dimensional counterpart has been developed, but is too computationally inefficient to be put to any extensive use with currently available workstation-class computers.

This technique is an outgrowth of the discrete particle simulations originally developed for the study of the granular flows of unbreakable particles (see the review in Ref. [9]). Here, a breakable solid material is created by 'gluing' together unbreakable and nondeformable solid elements. For computational efficiency of the contact search algorithms, the elements must be convex polygons (triangles were used in this study), although the assembled particles need not be convex. As for the particles in most soft-particle granular simulations, the elements do not deform on contact with a wall or other elements, but, instead, the contacting surfaces overlap slightly and produce a restoring force that is proportional to the overlap. This particular simulation was built around the polygonal code of Hopkins [13], which
assumes that the restoring force is proportional to the area of overlap. A glued joint that connects the edges of two adjacent polygons can be thought of as a set of elastic fibers that connect initially coincident points on the two edges. Each such 'fiber' has stiffness $K_n$ in the direction normal to the joint, so that the normal force generated inside the 'fiber' (i.e., the force per unit length of joint) is $K_n$ times the length of the projection of the fiber onto the direction normal to the element surfaces. Similarly, the force in the direction tangential to the surfaces is the tangential projection of the fiber multiplied by the tangential stiffness, $K_t$. In addition to the elastic stress, a small viscous resistance acts to eliminate vibrations induced by numerical errors; the strength of the viscous force is proportional to the rate of deformation of the fiber and acts in a direction parallel to the fiber. (The viscous force is so small that it is responsible for, at most, 0.2% of the energy dissipation in the most violent collision [11].) Thus, due to the compliance of the joints, the composite body is deformable, even though its constituent elements are not. Furthermore, as shown in Refs. [10,11], Young's modulus and Poisson's ratio of the composite depend on $K_n$ and $K_t$ in a predictable manner.

The composite material may fracture along the joints between its constituent elements. It is assumed that a glued joint can withstand normal tensile stress up to some limit $\sigma_{\text{ens}}$. If the tensile stress on any portion of the joint exceeds $\sigma_{\text{ens}}$, the 'glue' along that portion breaks and can no longer support tensile stresses; in other words, a crack forms, but only along that portion of the joint for which the tensile strength is exceeded. The corresponding energy release is equal to the potential energy stored in that portion of the joint, which is at least the energy stored in the normal displacement of the joint, $\sigma_{\text{ens}}^2/2K_n$ per unit length. (This is the minimum energy lost, as energy may also be stored in the tangential distortion of the joint.)

The original boundaries of the particle and those portions of glued joints that have broken experience 'collisional' contacts when they come into contact with other solid material. (Glued and collisional contacts are schematically illustrated in Fig. 1.) Collisional contacts are handled according to the methods outlined in Ref. [13]. Collisional contacts cannot withstand tensile forces and occur only if there is some overlap of the contacting elements. The normal elastic force on the contact is proportional to the product of the overlapping contact area and $K_n$. (The same constant used for glued contacts, $K_n$, is used here so that the force will be continuous during joint breakage in which the contact changes from glued to collisional.) As for the glued contacts, a small viscous force, proportional to the rate of change of overlapping area, acts parallel to the elastic force. The force tangential to the contact is elastic and proportional to the relative displacement of contacting elements with the coefficient $K_tL_p$, where $L_p$ is length of the particle side; tangential forces may be supported up to a frictional limit corresponding to a friction coefficient, $\mu$.

Using the above procedure, all the forces and moments that result from every glued and collisional contact on each element are calculated. The subsequent motion of the elements (and thus of the composite particles) are determined by integrating appropriate equations of motion using second-order numerical integration (as is typically done for soft-particle granular simulations). In essence, time in the simulation progresses from timestep to timestep of the numerical integration.

Three element shapes, equilateral triangles, Delaunay triangles and Voronoi polygons, were investigated [10-12]. The two triangular elements were shown to have similar properties, but Delaunay triangles are preferable as their random orientation eliminates the nearly crystalline arrangement of equilateral triangles. Delaunay triangular elements are used throughout this paper.

This technique is similar to that used for agglomerate breakage studies by Thornton and his colleagues [14-16]. However, the agglomerates are simulated using discs or spheres joined by point contacts rather than the continuous contacts employed here.

As pointed out elsewhere [10,11], there are some interpretation problems involved in applying these two-dimensional results to the three-dimensional world. In particular, neither the deformation nor the stress state is specified or even considered for the direction out of the plane. Thus, it is possible that the simulated systems be assumed to operate in plane-stress mode, plane-strain mode, or anything in between. This is a moot point from a numerical point of view, as the assumption does not effect the progress of the simulation, but only affects the relationship between the stiffnesses, $K_n$ and $K_t$, and the perceived elastic prop-
The most obvious dimensionless quantity should relate the strength of the material to the violence of the impact. As the greatest historical concern in comminution has been energy consumption, it seems appropriate to express this as a ratio of the kinetic energy of the impact

$$E_{\text{kin}} = \frac{\rho \pi L_0^2 V_0^2}{8}$$

where $L_0$ is the diameter of the circular particle, to the minimum energy lost in creating a crack that spans the particle

$$E_{\text{cr}} = (\sigma_{\text{tens}}^2 L_0)/(2K_n)$$

Then

$$\frac{E_{\text{kin}}}{E_{\text{cr}}} = (\pi \rho L_0 V_0^2 K_n)/(4\sigma_{\text{tens}}^2)$$

Obviously, the larger $E_{\text{kin}}/E_{\text{cr}}$, the greater the damage the particle should experience. Note, however, that to create a crack that actually spans the body, the impact energy must be considerably greater than $E_{\text{cr}}$, as a great deal of work must be put into the elastic deformation of the body, most of which will not be lost in breakage but be transformed into the kinetic energy of the fragments.

Charles [3] showed long ago that large impact velocities produced a different product than a slowly applied collision of the same overall energy. He argued that a high-speed impact concentrated its energy in a narrow elastic wave. Thus, an additional parameter should derive from the impact velocity, $V_0$, which is conveniently scaled by the internal sound speed, $C$. Here $C$ is the speed of longitudinal sound waves, which is $[E/\rho(1 - \nu^2)]^{1/2}$ for the plane-stress case and $[E(1 - \nu^2)/(\rho(1 + \nu)(1-2\nu))]^{1/2}$ for the plane-strain case. (Substituting Eqs. (1) and (2) into the appropriate expressions would yield identical expressions for $C$ in terms of $K_n L_p$ and $K_n/L_n$ so that, as would be hoped, there is only one sound speed in the simulated material.) In essence, $V_0/C$ represents the ratio of the rate at which the energy of the impact is applied at the contact point to the rate at which it is carried away by elastic waves; values of $V_0/C$ approaching unity imply that the impact energy is concentrated within a narrow elastic wave. Small values of $V_0/C$ correspond to nearly quasistatic loading.

Note that due to the dependence of $C$ on $(E/\rho)^{1/2}$, $V_0/C$ may also be interpreted as a dimensionless Young’s modulus. As such, if the particle does not break, $V_0/C$ may be seen to represent a characteristic particle deformation. That is, if one assumes that the kinetic energy of the impact is transformed into a strain, $\varepsilon$, distributed over a two-dimensional body of area $A$: $\frac{1}{2} \rho A V_0^2 \varepsilon = \frac{1}{2} E \varepsilon A$. Rearranging yields $\varepsilon = V_0^2/(E/\rho)^{1/2}$, which, for some function of Poisson’s ratio $\nu$, is essentially $V_0/C$. From this point of view, $V_0/C$ may be understood as a parameter that governs the magnitude
of the tensile forces, while $E_{\text{kin}}/E_{\text{cr}}$ governs the energy available to propagate the cracks.

The resulting parameter space that will be searched in this paper is governed by the three dimensionless parameters: $E_{\text{kin}}/E_{\text{cr}}$, $V_0/C$, and $\nu$.

4. A representative collision

This paper will only concern itself with normal impacts between a round two-dimensional particle and a plate. For all of these simulations, the particle is subdivided into 2043 Delaunay triangles as shown in Fig. 2. (It was shown in Ref. [11] that the results are relatively independent of the element size.) Now the parameter space, outlined in the last section, will be searched by varying one of the three dimensionless groups about standard conditions. The standard collision is shown as an example in Fig. 3. It corresponds to the parameters $E_{\text{kin}}/E_{\text{cr}} = 1.84 \times 10^3$, $V_0/C = 5 \times 10^{-4}$. The value of Poisson's ratio depends on whether the calculation was made under plane-stress (Eq. (1)) or plane-strain conditions (Eq. (2)). Both values will be given in all of the subsequent simulations: here $\nu' = 0.2$ ($\nu'$ represents plane-stress conditions) and $\nu'' = 0.167$ ($\nu''$ represents plane-strain conditions). Initially the particle is moving vertically downward.

Fig. 3(a) shows a time history of the development of cracks within the particle. Time in this Figure is given in terms of $L_0/C$, or, roughly, in units of the time it takes an elastic wave to cross the particle. Initially, the cracks form on lines fanning outward from the point of contact. Physically, this might be expected, as the maximum compressive forces generated by a Hertzian contact follow such lines [17] and, consequently, any tensile stresses must work in the direction normal to these lines. (As the Hertz analysis does not take particle shape into account, it does not predict tensile stresses.) For spheres, the three-dimensional geometry precludes this exact pattern of breakage, but the same arguments would result in the separation of the sphere into large fragments that resemble the segments of oranges, such as was observed in Ref. [4]. However, once the division into orange segments is complete,
fanlike fracture patterns can be observed on longitudinal
cuts through the sphere [4], which are generated by
exactly the same mechanisms discussed here. Also
similar to Ref. [4], these two-dimensional results show
a wedge of highly fractured material that surrounds
the point of contact. By the final picture, taken at
about 13.9 L o/C, all of the breakage has ceased, and
subsequently, the fragments will simply fly apart.

Since the majority of experimental data on com-
mination are presented in the form of
Schuhmann–Gaudin cumulative size distributions, the
simulation data will be presented in the same form.

Fig. 3(b) shows the distribution for the simulation shown
in Fig. 3(a). Here, the ‘size’ of the fragment is taken
to be the square root of the fragment area. A vertical
dashed line is drawn through this Figure at a location
corresponding to about two of the average element
dimensions, L o. Above that size, the data plot nearly
onto a straight line, as is commonly found in experiments.

The slope of the line is about 1.1, which corresponds
to the majority of experimental data [3,4]. In fact, this
coincidence of the slope is quite encouraging given the
two-dimensional character of this model and the some-
what different patterns of breakage (although resulting
from the same physical mechanisms) observed in three
dimensions. This slope will be discussed in detail in
the next section concerning the influence of V o/C. To
the left of the dashed line, the slope becomes very
steep as the distribution becomes dominated by the
element size which, of course, determines the smallest
possible fragment size. (Note that there is still a size
distribution, even when the material is broken into
single elements, due to the slightly random sizes of the
Delaunay triangles.) Such behavior is characteristic of
materials that possess a well-defined grain size [18],
and, in some respects, it is comforting that the simulation
reproduces it. However, it is not the intention of this
simulation technique to model such materials, and
attention should be restricted to fragments significantly
larger than an element size.

Finally, the reader should understand that each of
the results presented here is a single event, while most
commutation data reflect many thousands or millions
of events. In particular, this means that there may be
small variations in the statistical distributions from
otherwise identical collisions using particles with dif-
ferent tessellations. Most importantly, one should not
apply a great deal of importance to the maximum
fragment size, or similar items, that may only reflect
single observations and thus can represent statistical
aberrations.

4.1. Dependence on V o/C

The parameter V o/C characterizes the rate at which
the kinetic energy of the collision is transferred to the
strain energy of the body. For small V o/C, the loading
is close to quasistatic and it is natural to expect that
the breakage will be nearly independent of this pa-
parameter. In most experiments on single-particle break-
age this parameter is usually very small (about 0.01%),
since most were performed simply by dropping the
bodies. Experiments done by Charles [3] extend this
parameter to a value of about 1%, by firing a bullet
into a glass specimen. However, larger values (3–4%) are
possible in high-speed pinmills. No detailed in-
formation about single-particle impacts at larger V o/C
is known to the authors.

All of these simulations were made for parameters
ν’ = 0.2 (ν’ = 0.167) and E kin/E0 = 1.84 × 10^4. The first
set of computations has been made for V o/C ranging
from 0.0001 to 0.0115, for which experimental data are
available. Fig. 4(a) shows examples of the final fracture
patterns, while cumulative size distributions are pre-
seated in Fig. 4(b). While, at first glance, the fragment-
size distributions are rather close, there is a small but
distinct increase in slope with larger V o/C, rising from
about 1.1 to about 1.4. Nearly the same slope change
was observed experimentally by Charles [3]. It is com-
forting that these effects are present even in this two-
dimensional simulation.) Charles also observed qualita-
tively that the fragments at large V o/C are more
elongated. To examine that observation quantitatively,
we define an ersatz average aspect ratio for the frag-
ments. In its common definition, the aspect ratio is
defined as the ratio of the maximum and minimum
dimensions of the fragments. However, for a randomly
shaped particle the minimum dimension is not a well-
defined quantity; e.g., if the fragments contain a point,
then the actual minimum dimension is something like
twice the radius of curvature of the point which, for
the polygons used in this simulation, is zero. Here, as
shown in Fig. 5(a), the aspect ratio L 1/L 2 is defined
as the maximum possible length of intersection be-
tween a line and the fragment, L 1, and the maximum
length in the direction perpendicular to that line, L 2.
Consequently, for a rectangle, this reduces to the con-
ventional definition of the aspect ratio. Furthermore, it
will have unit value for both squares and circles. The
average aspect ratio for all fragments consisting of more
than four elements is shown in Fig. 5(b). Again, in
agreement with Charles’ observations, the aspect ratio
can be seen to increase from about 1.87 at small V o/
C to about 2.3 at V o/C = 0.03, indicating that, indeed,
the fragments are becoming more elongated for higher
impact velocities. The physics behind this change in
shape remain elusive.

However, some speculation is possible, based on the
simulated results. Fig. 6 shows the time history of a
collision at large V o/C = 0.0115. The time spacing of
the panels was chosen to correspond exactly to those
shown for small V o/C in Fig. 3(a). By comparing the
results panel by panel, it is easy to see that the radial fan of cracks is formed much faster for larger $V_0/C$. (Note that this would be difficult to observe experimentally, and is thus an illustration of the utility of this simulation technique.) The fragments that initially form between the fan lines are, of course, elongated; perhaps the fragments remain elongated as, once the fan-like pattern of cracks has formed, the internal tensile stresses that cause breakage relax somewhat, resulting in smaller subsequent fragmentation. Such a relaxation could also explain Charles' [3] observation that smaller fragments are possible for high-energy impacts at small $V_0/C$. The early formation of the fan may be related to both interpretations of $V_0/C$. Charles speculated that large $V_0/C$ implies that it is stronger elastic waves that carry the tensile stresses resulting from the impact.
through the particle and could lead to earlier breakage. However, this effect must be small, as the observed changes occur at such small values of \( V_\theta/C = 10^{-3} \) that the loading may be considered to be quasistatic. From another point of view, a large \( V_\theta/C \) implies larger deformation about the impact point, which in turn results in the generation of larger tensile stresses that accelerate the formation of the fan-like crack pattern.

A further simulation was run for a larger \( V_\theta/C = 0.0490 \). The fracture history is shown in Fig. 7(a) and the cumulative size distribution (which, for comparison, is plotted alongside that for \( V_\theta/C = 0.0115 \)) is shown in Fig. 7(b). Here the two distributions have approximately the same slope at the larger fragment sizes, but the distribution for \( V_\theta/C = 0.0490 \) lies above that for \( V_\theta/C = 0.0115 \), indicating a larger degree of fragmentation. The reason for the change can be seen in Fig. 7(a). At such large \( V_\theta/C \), the fan-like pattern develops so quickly that the fragments are still moving towards the plate at the time of breakoff. These fragments then impact the plate and are shattered nearly into single elements. (This can be seen in the final, time = 6.414, panel of Fig. 7(a), in which nearly all of the fragments adjacent to the plate are completely broken.) This secondary breakage accounts for the vertical shift in the size distribution. It also indicates that, at such large velocities, the impact energy is more efficiently converted into fracture. It would be interesting to see what would happen at even higher \( V_\theta/C \), even though it is unlikely that such impact velocities could be practically realized. However, as large \( V_\theta/C \) corresponds to large particle deformation, they may not be accurately described by the current model, as the rigid elements used here cannot conform to the highly deformed shape of the particles.

4.2. Dependence upon Poisson's ratio

This simulation technique is useful in testing the sensitivity to material properties, such as Young's modulus \( E \) and Poisson's ratio \( \nu \), as it is possible to vary them independently; on the other hand, in actual ex-
Fig. 8. (a) Final breakage patterns for collisions at small $V_0/C = 5 \times 10^{-4}$ with different Poisson's ratio. The energy is $E_{kin}/E_{cr} = 1.84 \times 10^3$. Note that there is a very little difference of the overall picture of breakage. (b) The corresponding cumulative size distributions.

Fig. 9. (a) Final breakage pattern for impacts at large $V_0/C = 4.9 \times 10^{-3}$, with different Poisson's ratio. The energy is $E_{kin}/E_{cr} = 1.84 \times 10^3$. Once again, there is very little difference of the overall picture. (b) The corresponding cumulative size distributions. There is a very little difference between the distributions, although there does appear to be a definite increase in the breakage with increasing Poisson's ratio.

Experiments, the variability is limited to the available materials. As $V_0/C$ represents a dimensionless Young's modulus, the effects of varying $E$ are included in the last section, leaving only the effects of Poisson's ratio to be studied. This simulation is particularly useful for evaluating any possible influence of $v$, as it may be freely varied in the simulation, while the majority of materials are confined to the narrow band $0.15 < v < 0.35$. (Note that, under general loadings, values of $v'$ between $-1$ and 0 are thermodynamically permissible, but, under plane-stress loading, values of $v'$ as large as 1.0 are possible due to the ability of the material to expand into the third dimension without performing work; see the discussion in Refs. [10,11].) As shown in Refs. [10,11], the simulation permits variation of plane-stress $v'$ from $-1/3$ to 1.0 and plane-strain $v''$ from $-0.5$ to 0.5, thus extending the investigation even into the range of physically unrealistic negative Poisson's ratios.

In this study, Poisson's ratio is varied by changing $K_s/K_n$ through the range between 0.05 and 2.0, which covers the range $-0.143 < v' < 0.980$ ($-0.167 < v'' < 0.495$), while holding $E_{kin}/E_{cr} = 1.84 \times 10^3$. Examples of the final fracture patterns for small $V_0/C = 5 \times 10^{-4}$ are shown in Fig. 8(a), with the corresponding size distributions shown in Fig. 8(b); the same information for
the large $V_0/C = 4.9 \times 10^{-3}$ is presented in Fig. 9. It is evident that changing Poisson's ratio has little effect on impact-induced breakage for both small and large $V_0/C$. What variation there is at small $V_0/C$ (Fig. 9(b)) is inconsistent, with both large positive and negative Poisson's ratios occupying the highest positions. A more-or-less consistent trend from negative Poisson's ratios at the lowest position and the large positive values at the highest positions can be observed for large $V_0/C$ in Fig. 9(b), but still, the effect is small. This indicates that Poisson's ratio has little effect on tensile stress generation with round two-dimensional particles. (Indeed two-dimensional Hertzian theory finds that the stresses are independent of $\nu$ [17].) As tensile stresses depend on particle shape, it is possible that $\nu$ may have a stronger effect for different particle shapes.

4.3 Dependence upon $E_{\text{kin}}/E_{\text{cr}}$

The degree of breakage has long been known to depend largely on the kinetic energy of the impacting particle. In our dimensional analysis, this is represented dimensionlessly as the ratio of the initial kinetic energy, $E_{\text{kin}}$, to the minimum energy released during creation of the crack that spans the particle, $E_{\text{cr}}$. Fig. 10 dem-
onstrates the strong effect of $E_{\text{kin}}/E_{\text{cr}}$ on the final fracture patterns and the corresponding cumulative size distributions, respectively, for small $V_o/C=5\times 10^{-4}$, while Fig. 11 shows the same functions for large $V_o/C=4.9\times 10^{-3}$. All of these simulations were performed for $\nu'=0.2 (\nu''=0.167)$. Note that as most of the kinetic energy will go into elastic potential energy and not into breakage, typical values of $E_{\text{kin}}/E_{\text{cr}}$ will be much larger than one. The progressively finer grinding reflected in the vertical shift of the size distributions with increasing $E_{\text{kin}}/E_{\text{cr}}$ is expected, but the effect of $V_o/C$, seen by comparing Fig. 10(b) and Fig. 11(b), is not so obvious. These Figures are in accord with the experimental observations of Charles [3] that it is easier to break the particle with large $V_o/C$, but that, at large impact energies, finer fragments are obtained with small $V_o/C$. In particular, the size distributions for small $V_o/C$ are spread over a wider area than for large $V_o/C$, even though exactly the same ranges of $E_{\text{kin}}/E_{\text{cr}}$ are covered. At large $V_o/C$ (Fig. 11(b)) the size distribution for $E_{\text{kin}}/E_{\text{cr}}=205$ lies above that for small $V_o/C$ (Fig. 10(b)), indicating a larger degree of breakage. On the other hand, the high impact energy, $E_{\text{kin}}/E_{\text{cr}}=7410$, distribution for small $V_o/C$ (Fig. 10(b)), generally lies above that for large $V_o/C$ (Fig. 11(b)), also indicating a larger percentage of fines. However, these observations can be accounted for by the speculations about the effects of $V_o/C$ that surround the discussion of Fig. 6 in Section 4.1.

5. Conclusions

This paper describes a two-dimensional computer-simulation study of impact-induced breakage in single circular particles. A dimensional analysis showed that the problem may be described by three parameters: $\nu$, $V_o/C$ and $E_{\text{kin}}/E_{\text{cr}}$. The results showed little dependence on Poisson's ratio, $\nu$, for the parameters studied here.

The second parameter, $V_o/C$, may be interpreted either as an indication of the strength of the elastic wave generated by the impact, or as a characteristic strain of the particle. Charles [3] showed that relatively small values of $V_o/C (0.5\%)$ generated an increase in the slope of the size distribution from about 1 to about 1.4. This shift is also observed in the simulation. Charles had also made the qualitative observation that the fragments were more elongated at large $V_o/C$; this was somewhat quantified in the current analysis through an ersatz aspect ratio that was found to increase with $V_o/C$. It is noted that $V_o/C$ may be interpreted as a characteristic strain of the particle, to which the particle responds with larger tensile forces that accelerate the formation of the fanlike crack pattern. For the largest $V_o/C$ studied, the fragments are released while the particle is still approaching the plate and subsequently shatter in a secondary breakage process that leads to a larger percentage of fines in the debris. The early development of the fanlike crack pattern at large $V_o/C$ can also explain the large aspect ratio of the fragments. In particular, the internal relaxation of the tensile stresses that must accompany the completion of the crack pattern would both resist further breakage and thus freeze the fragments in the elongated form of the fan segments. These observations would be extremely difficult to reproduce experimentally and are a clear indication of the value of the simulation technique in providing insight into particle fracture.

Finally, the greatest change in the size distribution is seen by varying the impact energy, represented through the dimensionless term $E_{\text{kin}}/E_{\text{cr}}$. Also in accord with Charles [3], a larger degree of breakage was observed for large $V_o/C$ and small $E_{\text{kin}}/E_{\text{cr}}$, but at large $E_{\text{kin}}/E_{\text{cr}}$, finer grinding might be had for small $V_o/C$.

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