The breakage induced by a single grinding ball dropped onto a randomly packed particle bed

Alexander V. Potapov, Charles S. Campbell

Department of Mechanical Engineering, School of Engineering, University of Southern California, Los Angeles, CA 90089-1453, USA

Received 1 August 1998; received in revised form 29 April 1999; accepted 19 May 1999

Abstract

This paper describes some computer simulations of the breakage induced in a two-dimensional particle bed by the descent of a single large grinding ball. These are approximate experiments related to ball-milling that are tractable by the simulation technique (i.e., involve relatively few particles). As a result of the impact of the grinding ball, the particles in the bed are broken and/or scattered away from the grinding ball as it falls. Simulations were performed for four different bed depths and three different frictions. Generally, the deeper the bed, the less the total amount of breakage. The results show also show the dual role that friction plays in the breakage process. On one hand, the majority of the grinding ball’s energy is lost to friction so that increasing the friction increases this energy loss. On the other hand, the friction holds the bed together and the larger the friction, the longer the bed stays in place for the grinding ball to do its work. It appears that this latter effect is the stronger of the two in that increasing the particles’ coefficients of surface friction greatly increases the amount of breakage; that extra energy for breakage appears to come from a reduced kinetic energy of the scattered fragments. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Breakage; Grinding ball; Particle bed

1. Introduction

The research presented in this paper utilizes a computer simulation technique capable of studying large-scale particle breakage [1–4]. The technique has been used to study single particle breakage [5,6] but was particularly designed to examine particle breakage in flowing [7] and process systems such as mills. Ball-mills, being the most commonly used, are an obvious candidate for investigation. However, this type of simulation is still too inefficient to model a fully functioning ball-mill. Instead, we chose to model a single grinding ball dropped onto a packed bed of particles. This is similar to the experiments [8] performed at the University of Utah in which a single grinding ball is dropped onto a stationary bed of particles mounted atop an Ultra-Fast Load Cell, which measures the forces applied by the ball, and transmitted through the particle bed to the anvil underneath. (Similar experiments have been performed on single particle impacts [9–11].) The big advantage of studying these problems with a computer simulation is that it is possible to assess details that are inaccessible to experiments such as how the impact energy is distributed into the three categories of breakage, frictional losses and scattering of the bed particles. An early example of this simulation can be found in Ref. [2].

2. Description of the model

The model used for this simulation is exactly that described in detail in Ref. [1] and used in Refs. [2,5–7]. Here, we shall just briefly outline the principles behind the model.

The particles to be broken are created by ‘‘gluing’’ together convex polygonal elements. In all the simulations presented here, Delaunay triangles are used for the element shapes. The elements themselves are assumed to be rigid and the compliance of the body is due solely to the compliance of the glued joints. The ‘‘glue’’ that connects the elements acts as a collection of elastic fibers; each such ‘‘fiber’’ has stiffness $K_n$ in the direction normal to the contacting side, and $K_t$ in the direction tangential to the contacting side. To stabilize the numerical technique, a small viscous force also acts on each such fiber, whose
magnitude is proportional to the rate of fiber deformation with coefficient $K_n$. This coefficient is chosen large enough to ensure the stability of the calculations but small enough to not cause any major contribution to the energy loss.

Edges of the elements which are free from glued contacts can experience "collisional" contacts with the edges of other elements that have become unglued or with the rigid plate.

Unlike glued contacts, collisional contacts cannot withstand tensile forces. As the elements are rigid, the edges will overlap slightly when exposed to compressive forces. The elastic restoring force in this contact is proportional to the area of the particles’ overlap with the coefficient $K_n$. That force is applied at the center of gravity of the overlapping area and is normal to a "contact plane." The definition of the contact plane is discussed in detail in Ref. [1]. In addition to the elastic restoring force, a small viscous force also acts in the same direction whose magnitude is proportional to the rate of change of the overlapping area. The other force acting on collisional contacts is frictional, acts parallel to the contact plane and is also applied at the center of gravity of the overlapping area. Initially, there is no slip between the contacting element surfaces and a force is generated which is proportional to the total relative displacement parallel to the contact plane that has occurred since the contact was initiated with

Fig. 1. The initial positions of the particles in the box at the start of the preparation phase of the ball drop simulation. A five-particle layer case is shown here.

Fig. 2. Positions of the particles in the box at the initiation of the drop ball simulation. A five-particle layer case is shown here.

Fig. 3. A schematic of the drop ball simulation with five layers of particles. The large grinding ball is dropped vertically downward with initial velocity $V_0$.

Fig. 4. Snapshots at the initiation of the simulation for two (a), three (b), four (c) and five (d) layers of particles. The ball is situated so that the lowest point of the ball is at the height of the uppermost reach of the particles in the bed.
coefficient of proportionality $K, L_p$ (where $L_p$ is the mean length of the element sides). However, if the magnitude of this force exceeds $\mu |F_n|$, where $\mu$ is coefficient of friction and $F_n$ is the normal force on the contact, the element surfaces are allowed to slip and the magnitude of the tangential force is set to $\mu |F_n|$ with preservation of direction.

Using the procedure described above, the forces acting on all elements are calculated at each time-step and are numerically integrated using a second-order Runge–Kutta integration technique to yield the motion of all of the individual elements and thus, of the macroscopic body as a whole.

Fracture in this model may be thought of as a transition from glued to collisional contacts. A glued contact can only withstand a normal tensile stress up to a specified limit $\sigma_{\text{tens}}$. If the normal tensile stress on the contact exceeds this limit, the glue breaks; i.e., from that point forward, only the portion of the joint for which the tensile strength is exceeded may participate in collisional contacts. This process creates a material with a work of fracture equal to $\sigma_{\text{tens}}^2/(2 K_n)$. As shown in Ref. [1], the values of Young’s modulus $E$, and Poisson’s ratio $\nu$, will be different depending on whether the two-dimensional stresses are assumed to be plane-stress, plane-strain or something in between (although that assumption does not, in any way, affect the method or the progress of the simulation). Here, we shall regard all cases to be plane-stress and present the corresponding value of $\nu$ in all following descriptions. These are related to the primitive element properties by the formulae:

$$E = \frac{(4K_n L_p) K_r/K_n}{\sqrt{3} (3K_r/K_n + 1)}$$

$$\nu = \frac{1 - K_r/K_n}{3K_r/K_n + 1}$$

3. Problem definition

The first step in all these simulations is to build a bed of particles onto which to drop the grinding ball. For these purposes, several layers of round particles of diameter $d_{\text{sm}}$.

Fig. 5. A time sequence of events for a two-layer simulation with a coefficient of friction, $\mu = 0.3$. The time is given in the units of $d_{\text{sm}}/C$, i.e., roughly in the units of time necessary for a sound wave to cross the particle.
were placed loosely into an imaginary rectangular box which is used to confine the particles only while the initial configuration is built (Fig. 1). Then gravity is switched on to settle the particles into a stable packing (Fig. 2). After this stage, the walls of the box are removed, leaving just the bed of particles. To save computer time during the bed assembly, these particles are not yet subdivided into elements but are instead treated as unbreakable solid discs interacting through linear viscous–elastic contacts. During this phase of the simulation, the stiffness of contacts between particles is set very high to avoid large overlaps. Once the assembly is complete, the particles are subdivided into Delaunay triangles with an average side length \( L \). The subdivision of every particle into Delaunay triangles is the same, but the orientation of particles is arbitrary. Once the division is complete, the grinding ball, a large round particle with diameter \( D_b \), is dropped from above (Fig. 3). At the start of the simulation, the ball is placed so that the lowest point of the ball is at the level of the highest point of the particle bed. The history of events that occur as the ball progresses through the particle bed is recorded. The simulation is stopped once the ball starts to move upward even though in a real experiment, the ball can bounce several times before finally coming to a rest.

The ball drop simulation can be described by the dimensionless parameters used for single particle breakage [5] plus some additional parameters that are particular to this problem. We have chosen the following parameters to define these simulations: the plane-strain Poisson’s ratio of particles \( \nu \), the ratio of the initial kinetic energy of the ball to the energy dissipated in the process of splitting of particle into two equal parts \( E_{\text{kin}}/E_{\text{cut}} \), the ratio of the diameter of the ball to the diameter of a bed particle \( D_b/d_{\text{min}} \), the friction coefficient \( \mu \), the ratio of the initial velocity of the ball to the speed of sound of the material of particles \( V_0/C \), the number of particle layers \( N_{\text{in}} \), a dimensionless form of the coefficient of viscosity \( K_{\nu} \), \( K_{\nu}/(\rho g d_{\text{min}}^2/C) \) (here, \( \rho \) is density of the material of particles), the ratio of the mean size of the sides of the triangular elements to the diameter of the particle \( L_n/d_{\text{min}} \) and finally, the gravity \( g \) is scaled by the impact velocity yielding \( g d_{\text{min}}/V_0^2 \). The gravity does not play any important role in the energy balance since the work done by gravity is negligible both in the experiment and in the simulation (this is reflected in a small value of the inverse Froude number \( g d_{\text{min}}/V_0^2 \). But during some early simulations, a case was observed in which the grinding ball was so slowed by friction within the particle bed that, although it continued to move towards the plate, it would not make

![Fig. 6](image)
contact until nearly 6 months of computer time had elapsed. Realizing that in actual experiment, gravity would prevent such an occurrence even when applied at levels that would insignificantly affect the overall energetics of the system. Consequently, a small amount of gravity was included for the current work simply to bring it closure to the simulation.

The list of dimensionless parameters contains nine values, and to vary all of them is a very time-consuming task especially considering that a complete simulation requires about 2 weeks of CPU time on an HP-715. Thus, based on preliminary results, the only parameters that will be varied are the depth of the particle bed (represented by the number of bed layers $N_b$) and the particle surface friction $\mu$. Simulations were performed with $N_b = 2, 3, 4, 5$ and with friction coefficients $\mu = 0.1, 0.3$ and $0.6$. (Throughout each simulation, the surface friction is assumed to be a constant. This may not be totally realistic as, in a real material, the surface friction may vary somewhat over the surface of a particle, over different particles, and would most likely be different for freshly cut fragments, than for the parent particle.) The rest of the parameters were fixed to the following values: $\nu = 0.2$, $E_{\text{kin}}/E_w = 9.77 \times 10^3$, $D_0/d_{\text{sm}} = 2.67 \times 10^3$, $V_0/C = 4.32 \times 10^{-3}$, $g/(V_0^2/d_{\text{sm}}) = 6.7 \times 10^{-3}$, $L_p/d_{\text{sm}} = 0.12$ and $K_s \sqrt{pgd_{\text{sm}}^2}/C = 4.71 \times 10$. These parameters were chosen for various reasons. Some such as $V_0/C$, $\nu$, $D_0/d_{\text{sm}}$ and $g/(V_0^2/d_{\text{sm}})$ were chosen to correspond to the appropriate dimensionless values in the Utah experiment. The value of the interelement viscosity $K_s \sqrt{pgd_{\text{sm}}^2}/C$ has been chosen to be small enough not to cause any significant influence on the energy balance in the system yet large enough to assure numerical stability. The dimensionless element size, $L_p/d_{\text{sm}}$, was determined largely by the limits of the 64 MB computer memory in the workstation-class computers used for these studies. Finally, the value of $E_{\text{kin}}/E_w$ was based on the results of the single particle breakage calculations presented in Refs. [3,7]. The “standard” case investigated there corresponds to the value $E_{\text{kin}}/E_w = 1.84 \times 10^3$ which causes pervasive fracturing of single particles and results in fragments with a wide distribution of sizes ranging from single elements up to fragments composed of hundreds of elements and thus allowed reasonable size.

Fig. 7. The scattering of particles in the bed for the two-layer simulation with friction coefficients of $\mu = 0.1$ (a), $\mu = 0.3$ (b) and $\mu = 0.6$ (c). The pictures presented correspond to the moment when the grinding ball has just started its rebound. Time is given in the units of $d_{\text{sm}}/C$.

Fig. 8. The same as Fig. 7, but for three layers of particles.
distribution statistics to be gathered. But as many particles can be expected to break in these ball drop simulations, the value of \( E_{\text{kin}}/E_{\text{cr}} = 9.77 \times 10^3 \) was simply chosen to be several times the single particle value.

Snapshots taken at the initiation of the ball drop simulation are shown in Fig. 4 for different numbers of particle layers. Note that the actual number of layers in the settled bed does not really correspond to \( N_s \); instead, \( N_s \) is the number of layers initially placed in the box (Fig. 1). Since particles are placed in the box at some distance from each other to allow settling of the bed, the actual number of layers that remain in the bed at the time of initiation is slightly smaller than \( N_s \). (See, e.g., Fig. 4d where \( N_s = 5 \), while the actual number of layers in the settled bed is about 4 1/2.)

4. Results of the simulation

The impact of the unbreakable grinding ball will cause breakage of the particles, but will also set up a flow within the bed which eventually leads to scattering of the both fragments and unbroken particles. Figs. 5 and 6 show a time sequence of events for a two-layer simulation (\( N_s = 2 \)) with \( \mu = 0.3 \). Both figures show the same events. Fig. 5 shows the details of the breakage beneath the grinding ball while Fig. 6 shows a more global view of the scattering of the particle bed. Time on these figures is given in the units of \( d_{\text{min}}/C \), or the time required for the sound wave to cross a small particle. The last frame of both figures corresponds to the time when the ball starts to rebound from the plate. This shows the process that was observed in the preliminary simulations. Fracture of the particles begins near the base, almost immediately after the contact with the grinding ball. However, there is no scattering of particles until all of the particles which experience substantial fracture have been broken. What appears to happen is that friction between the grinding ball and the wall freezes the particles at the center into position. When those particles break, they release much of their elastic energy explosively and it is that energy release which breaks apart the bed. It is evident that it is the explosive release of the elastic energy that leads to the scattering of the particles since the particles are scattered with velocities that are significantly
Fig. 11. The fraction of the initial kinetic energy of the ball $E_{kin}$ spent for friction, kinetic energy of the fragments and breakage as a function of the friction coefficient $\mu$ for two (a), three (b), four (c) and five (d) layers of particles.
larger than the rate at which the grinding ball descends. Thus, it is reasonable to conclude that such a process is strongly dependent on the particle surface friction.

The importance of the friction in two-layer simulations can also be seen in Fig. 7a–c, which show the scattering of particles for different values of interparticle friction. All the frames in Fig. 7 correspond to the moment of rebound; note that the rebound time is about the same for $\mu = 0.1$ and $\mu = 0.3$, but is significantly later for $\mu = 0.6$, presumably due to slowing of the grinding ball by interparticle friction. The effect of friction is also evident in the fact that the scattering of particles in Fig. 7a ($\mu = 0.1$) is much more evident than in Fig. 7b or c — this in spite of the fact that the time corresponding to Fig. 7c is larger than that for Fig. 7a which should allow the particles more time to scatter. This indicates that the larger the friction, the smaller the scattering velocity or, in other words, the larger the friction, the larger the amount of impact energy that is lost either to frictional work or particle breakage. Figs. 8–10 show the scattering of particles at the moment of rebound for three, four and five layers of particles and all values of the friction coefficient. Unlike Fig. 7, there seems to be no clear connection between the characteristic length of particles scattering and the friction coefficient on these figures. This indicates that for these deeper beds, the surface friction slows the descent of the grinding ball and the rate of particle scattering at roughly the same rate so that roughly the same degree of scattering is observed at the instance that rebound begins.

Of particular interest are the cases presented in Fig. 10, for five-layer deep beds. In this case, the time from the beginning of the simulation up to the start of rebound is larger for $\mu = 0.3$ than for $\mu = 0.6$, which seems to contradict the general tendency seen in the other cases. The reason for this can be understood by looking at the number of unbroken particles between the ball and plate for the case $\mu = 0.6$. For all other cases, the fragments that remain in this gap are reduced to the single elements before rebound; however, for $N_b = 5$ and $\mu = 0.6$, rather large fragments are left between the ball and the plate. This is an indication of the fact that dissipation due to friction is so large, that the ball no longer possesses the energy necessary to break all the particles. Furthermore, the large friction can “freeze” large number of particles in the gap between the ball and the plate so that they put up a united front to the falling ball forcing an early rebound.

The qualitative observations made above are confirmed quantitatively by the information presented in Fig. 11. This figure shows the fraction of the initial kinetic energy of the ball dissipated due to friction, spent on breakage process and transformed into the kinetic energy of the scattering particles as functions of the number of particle layers and friction coefficient. (Note that these fractions need not add up to one since the kinetic energy of the grinding ball is not included in the figure. Note also, that in a true ball-mill energy is effectively lost as fragments may agglomerate over time [12], but as these simulations only cover a short time single breakage event, there is no time for agglomer-

![Fig. 12. The dependence of dimensionless total crack length scaled by the grinding ball diameter on the initial number of bed layers and the particle surface friction.](image-url)
...tion to occur.) As we can expect from the speculations presented above, the energy dissipated to friction and the energy dissipated in breakage increase with the friction coefficient, while the energy transformed to kinetic energy of the scattering particles decreases with the friction coefficient. This indicates that to some extent, friction is very important in the breakage process as it freezes particles in position and without it, the particles would simply scatter away beneath the grinding ball; consequently, the larger the friction, the larger the energy lost to breakage. The counter-example appears to be the five-layer \(\mu = 0.6\) case shown in Fig. 11d. All the three energy loss mechanisms seem to decrease slightly between the \(\mu = 0.3\) and the \(\mu = 0.6\) cases, which indicates that more energy is left in the kinetic energy of the rebounding grinding ball. This might be understood if taken in context with the reduced breakage apparent in the snapshot shown in Fig. 10d. The large friction will tend to lock the particles together and not allow them to scatter as was observed for the two-layer bed. Now if the energy of the grinding ball is too small, it would simply bounce off this packed bed, causing little breakage and retaining most of its kinetic energy. Fig. 11d appears to be a transitional case where the kinetic energy of the grinding ball is not large enough to cause complete fragmentation of the trapped portion of the particle bed and results in larger fragments and in the retention of more kinetic energy by the grinding ball.

Fig. 12 shows the importance on the bed depth and the interparticle friction on the total amount of breakage induced within the bed. This plots the total crack length (normalized by the radius of the grinding ball) as a function of \(\mu\) and \(N_p\). The strong effect of the particle surface friction is somewhat surprising. Examining the data for the two-layer simulation, one finds that there is close to twice the breakage for \(\mu = 0.6\) than for \(\mu = 0.1\) even though Fig. 7 indicates that approximately the same number of bed particles experiences substantial breakage in both cases (in the sense that 23 particles are scattered with, at most, minor damage for \(\mu = 0.1\) while 22 such particles are scattered for \(\mu = 0.6\)). In other words, the particles are broken into finer fragments for \(\mu = 0.6\). Furthermore, this trend continues for deeper beds. Also note that for \(\mu = 0.1\) and \(\mu = 0.3\), there is a sharp drop in the crack length between the two- and three-layer beds, while for \(\mu = 0.6\), that drop is delayed until after the three-layer bed. Both observations may be explained by the fact that the larger friction holds the bed together longer and allows the grinding ball to induce a greater degree of breakage. This also shows the importance of the induced flow in the grinding process. The flow wastes energy in two ways. It both converts the kinetic energy of the ball into kinetic energy of unbroken particles and it also sweeps particles and fragments out of the way, reducing the opportunity of the ball to induce breakage. Eliminating this flow leaves the particles in position so the grinding ball can do its work, leading to a larger degree of breakage.

5. Conclusions

This paper describes some computer simulation results related to ball-milling. As it is currently not possible to realistically simulate a working ball-mill, these simulations consisted of a single grinding ball dropped onto a prepared bed of breakable particles. The impact causes both the breakage of particles within the bed and the scattering of unbroken and partially broken particles. An optimized system would convert as much of the particle’s kinetic energy as possible into breakage, and minimize the losses to other mechanisms (although a truly efficient system, one for which the majority of energy goes into breakage is clearly not possible).

These simulations dramatically illustrate the complex role that friction plays in ball-milling. All of the results show that the majority of the ball’s impact energy are lost to frictional dissipation. From that point of view, one would think that to reduce friction would leave more energy available for breakage and would thus be a desirable thing. But that discounts the role that friction plays in influencing the flow of particles around the grinding ball. Increasing the friction delays the flow of bed particles around the grinding ball, allowing more time for the grinding ball to do its work. As a result, a larger friction results in more breakage, even though more energy is lost to frictional dissipation. The extra energy for breakage appears to come for a reduced kinetic energy of the scattering particles. The larger frictional dissipation appears to simply leave less energy in the rebounding grinding ball. It should be noted that for irregular particles, the same function may be accomplished by the irregular shape of the particles which will cause the bed to lock together and be roughly independent of the actual surface friction.

Acknowledgements

This work was supported by the International Fine Particle Research Institute for which the authors are truly grateful.

References