Optimal choice of characteristics for a non-excludable good

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In this model a principal decides whether to produce one indivisible good and which characteristics it contains. Agents are differentiated along two substitutable dimensions: a vertical parameter that captures their valuation for the good, and a horizontal parameter that captures their disutility when the characteristics are distant from their preferred ones. When valuations are private information, the principal produces a good with characteristics more on the lines of the preferences of the agent with the lowest valuation. Under asymmetric information on the horizontal dimension, the principal biases the decision in favor of the agent who incurs the highest disutility.

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1 Introduction

Consider the following problem. A principal needs to decide whether to produce one indivisible good and, if she does, which characteristics it contains. Production affects positively the utility of two agents who are differentiated along two dimensions: a vertical parameter captures their valuations for the good and, a horizontal parameter captures their differences in preferences for the characteristics of the good. For example, a privately interested investor is deciding whether to construct a football stadium and where to locate it. Two neighboring cities are interested in the project. The vertical differentiation parameter is the cities' demand for football. The horizontal differentiation is the physical location of the stadium. Each city prefers to have the stadium located between the two cities or even in the neighboring city rather than no stadium at all. Naturally, horizontal differentiation can also account for differences in tastes.

It seems natural to conclude that the stadium should be built in the city that values it the most. Interestingly, this is not always the case in practice. In this paper, we determine the conditions under which this can occur. More generally, we characterize the optimal contract between the principal and the agents when there is asymmetric information either on the vertical dimension or on the horizontal dimension. We assume in the first case that the intrinsic valuations for the good are unknown. In the second, we suppose that agents face privately known "transportation costs" for each unit of distance between their preferred characteristics and the actual characteristics of the good. Optimal contracting with asymmetric information and positive externalities has already been studied in the literature. Yet, previous studies suppose that the characteristics of the good are given prior to the contracting stage.[1] To the best of our knowledge, the endogenous choice of characteristics has been overlooked.

The literature studying contracting problems with positive externalities can be divided into two branches. The first branch analyzes situations where the principal contracts separately with several agents and the contract between the principal and one agent generates positive externalities on other agents. This problem has been studied in a general setting in Segal (1999) and Segal and Whinston (2003).[2] It has also been investigated in specific settings by Cornelli (1996) and Lockwood (2000) among others.[3] In these articles, the item to be contracted upon is generally excludable and the principal can contract only with a subset of agents if she finds

[1] We mean by characteristic a feature of the good on which agents disagree because their tastes differ (in the Industrial Organization jargon, a "horizontal differentiation" parameter).

[2] Segal (1999) studies the nature of inefficiencies depending on whether contracts are observable or not. Segal and Whinston (2003) considers a larger family of games of contracting where contracts between the principal and one agent are not observed by other agents. The paper analyses general properties of equilibrium outcomes that must be satisfied by all equilibria of all games considered.

[3] In Cornelli (1996), the firm has a high fixed cost of production. Positive externalities arise between consumers, since purchasing the good affects positively the probability that the firm finds it profitable to produce it. In, Lockwood (2000), the agents' marginal cost of effort is private information and the output of an agent is affected positively by his effort and that of his co-workers.

it optimal. In our model the principal cannot produce one good for each agent (i.e., she cannot serve both agents separately, specify a different output level for each of them, or exclude one of them). Instead, she must determine the characteristics of the good. One could reinterpret "producing the good most preferred by one agent" as "selling the good to that agent". Thus, our setting resembles an auction, where the good can be allocated to one of the agents but the other one still enjoys a positive utility when this happens. However, unlike in the auction of an indivisible good, the principal is not forced to produce a good with the characteristics most liked by one agent. Instead, she chooses from a wide array of combinations, ranging from most preferred by one agent to equally liked by both.

The second branch studies the optimal contract between a principal and several agents when the item that is contracted upon affects the payoff of all agents. The literature on the provision of public goods in the tradition of Clarke (1971), Groves (1973) and D'Aspremont and Gérard-Varet (1979) discusses mechanisms to implement the socially optimal level of public good with or without budget balance for the government.[4] As in the present paper, the good is non-excludable and all agents benefit from its provision. However, our focus departs in two respects. First, the situations we have in mind are not necessarily decisions to produce public goods and we do not impose budget balance. Instead, we want to ensure that all parties participate and contribute. Second and more importantly, in these analyses, the principal decides over the *quantity* to be produced, and more quantity is always preferred by agents. By contrast, in our model, the principal decides over an *attribute* on which agents disagree since the best characteristic for one agent is also the worst for the other. This generates a new trade-off for the contract designer.

Last, even though the focus is different from the current analysis, we should mention the literature on quality. For instance, Che (1993) studies competition in a procurement environment where agents bid on the price of a product and its quality. Quality is distorted downwards under asymmetric information in order to diminish the rent left to agents. The author examines how two-dimensional auctions in which bids (on price and quality) are evaluated by a scoring rule perform to implement this second best. Incentives to provide quality have been studied also in Lewis and Sappington (1988) and (1991).[5] The present analysis focuses on horizontal differentiation instead of quality. As discussed above, the crucial difference is that agents disagree on the characteristics on the horizontal dimension although they concur on quality. Also, there are no externalities between agents in Che (1993) whereas it is a main ingredient in our analysis. These two differences make the contracting problem of a very different nature.

[4] An important literature also discusses from a positive point of view how local public goods should be financed by residents and land owners. It addresses the issue of which type of tax should be used taking into account how land prices affect location decisions as well as the size of the jurisdictions. See Scotchmer (2002) for a review.

[5] See also Laffont and Tirole (1993), chapter 4 (and the literature there-in) for a detailed analysis of the regulation of quality.

The main features of the optimal contract are the following. We assume that the vertical and horizontal dimensions are substitutable, in the sense that the marginal importance attached by an agent to the characteristics of the good decreases as his valuation increases. Introducing a horizontal dimension generates a qualitative departure only when this assumption is satisfied. In that case, the principal always produces the good under full information. Besides, keeping the transportation cost equal and constant for both agents, she prefers to favor the agent with lowest valuation, that is, to offer a good with characteristics more on the lines of his preferences than on the lines of the preferences of the other agent. Given the substitutability of the vertical and horizontal differentiation parameters, the loss in the revenue extracted from the high-valuation agent under this strategy is smaller than the gain in the revenue extracted from the low-valuation agent. Alternatively, agents with high transportation costs are relatively more sensitive to distance. Therefore, keeping the valuation constant and equal for both agents, it is optimal under full information to bias the decision in favor of the agent with the highest cost.

Asymmetric information on the vertical dimension induces two distortions in the optimal contract, one for each agent. In fact, since production of the good affects the utility of the two agents, the optimal contract is such that the principal demands payments and grants informational rents to both of them. Interestingly, under incomplete information the principal favors *even more* the agent with lowest valuation than under full information. The idea is that the principal distorts the characteristics of the good offered in order to reduce the rents left to agents. Due to substitutability of characteristics and valuation, marginal rents are greatest for the lowest valuation agent. Therefore, it is relatively more interesting to reduce the rents of this agent, which is achieved by selecting characteristics that are closer to his favorite ones. To sum up, positive externalities together with the capacity to extract payments from both agents induces the principal to select a convex combination of characteristics, with a tendency to favor the agent with lowest valuation. Asymmetric information exacerbates this bias.

When the principal does not fully observe the preferences on the horizontal dimension, two opposite effects are at work. First, given high cost agents are relatively more sensitive to distance, it is beneficial to bias the decision in favor of the agent with the highest cost. Second, if the good possesses the preferred characteristics of one agent, then that agent does not incur any cost. Then, *the principal can increase or decrease the amount of asymmetric information with each agent by choosing the characteristics.* Given a low cost agent has relatively less incentives to reveal truthfully his information and must be granted higher rents, the principal can minimize them by choosing a characteristic closer to the preferred characteristic of the agent who turns out to have the lowest cost. Overall, the bias obtained under complete information can be increased or decreased depending on which of these two effects dominates.

The plan of the paper is the following. The model and the basic properties of the optimal mechanism are presented in section 2. We solve for the case of asymmetric information on the vertical dimension and the horizontal dimension in sections 3 and 4, respectively. In section 5, we characterize the optimal contract when agents at different locations also have different distributions of valuations. Moreover, we analyze situations in which the good can be located at different places over time. In section 6, we study the mechanism when the principal maximizes welfare instead of revenue. Also, we determine the properties of the contract if one agent is also the producer of the good. Concluding remarks are collected in section 7. All proofs can be found in the Appendix.

2 The model

Basic ingredients

We consider two agents A and B indexed by i and j. Each agent ("he") is located at one extreme of a Hotelling line of measure N. Denoting by y_i the location of agent i, we have $y_A = 0$ and $y_B = N$. An indivisible good can be produced and located somewhere on the line.[6] We denote by θ_i agent i's intrinsic valuation for the good (also referred to as "type") and we assume that $\theta_i \in [\underline{\theta}, \overline{\theta}]$. Valuations are private information and they are independently drawn from a common knowledge distribution $F(\theta_i)$ with continuous and strictly positive density $f(\theta_i)$. It also satisfies the monotone hazard rate property: $d\left[\frac{1-F(\theta)}{f(\theta)}\right]/d\theta < 0$. Agents care about the location x of the good. We assume that x can take a finite but arbitrarily large number of locations, and we order them from closest to agent A to closest to agent B: $x \in \{0, 1, ..., N-1, N\}$. We denote by γ_i $(= |x - y_i|)$ the distance between the location of the good and the location of agent i. The payoff of agent i as a function of his valuation and distance takes the following form:

$$v(\theta_i, \gamma_i) = \pi(\theta_i - c\gamma_i) \tag{1}$$

where, following the Hotelling terminology, c is a positive "transportation cost", $\pi' > 0$, $\pi'' < 0$ and, for technical convenience, $\pi''' \geqslant 0$. According to this formalization, the payoff is increasing in the valuation $(\partial v/\partial\theta_i > 0)$ and decreasing in the distance $(\partial v/\partial\gamma_i < 0)$. Moreover, valuation is relatively more important the bigger the distance between the location of the agent and the location of the good $(\partial^2 v/\partial\theta_i\partial\gamma_i > 0)$. In other words, high type agents are relatively less sensitive to distance. Overall, agents are differentiated along two substitutable dimensions captured by two parameters, a vertical differentiation parameter (the valuation for the good) and a horizontal differentiation parameter (the distance between the good and the agent).

To be in the interesting case, the payoff of each agent when the good is produced is always greater than the payoff when it is not, which we normalize to 0 $(\pi(\underline{\theta} - cN) > 0)$. Our setting is

[6] It can be shown that, in equilibrium, the good will never be located outside $[0, N]$.

characterized by *positive and type-dependent externalities.* Each agent prefers to have the good produced and the payoff of agents increases with their valuation, independently of x.

We want to determine how the good is optimally located on the Hotelling line. We assume that the location decision is in the hands of a third party (from now on "principal" or "she"). Denote by $e = \emptyset$ the event "the principal does not produce the good" and by $e = x \in \{0, ..., N\}$ the event "the good is produced and located at x". In order to better concentrate on the inefficiencies of the allocation due to the asymmetry of information, we assume that producing the good is costless for the principal and generates no delay. Also, we concentrate in section 3 on the case in which the principal maximizes revenue. This assumption is relaxed in section 6.

Examples

The purpose of this subsection is to provide a few examples in which the ingredients of our theory are present and for which we believe our normative approach can be useful.[7]

Physical location of a non-excludable private or public good.

Agents A and B are two neighboring cities. The vertical differentiation parameter θ_i is the intrinsic demand for football of each city and the horizontal differentiation parameter is the distance between the city and the stadium. Also, c is simply a transportation cost. The payoff of each city when the stadium is built increases with its demand for football ($\partial v/\partial\theta_i > 0$) and decreases with the distance between the city and the stadium ($\partial v/\partial\gamma_i < 0$). Keeping c constant, inhabitants of a city supporting a football team are relatively more inclined to drive to attend an event ($\partial^2 v/\partial\theta_i\partial\gamma_i > 0$). Also, each city prefers a stadium located far away rather than no stadium at all (positive externalities) and, the utility of cities increases with their valuation, independently of the location (type-dependent externalities). The principal is an investor willing to build and manage a new stadium, and she maximizes revenue. Or, the principal is a local authority trying to make the two cities agree to finance a public stadium. The model can be applied to other decisions to locate a non-excludable good such as a shopping mall or a hospital.

Creation of a private school.

Agents A and B are two types of parents. The vertical differentiation parameter θ_i is the intrinsic willingness to pay for a new private school and the characteristics of the good is the emphasis of the school on languages vs. sciences. Given our assumptions, the payoff of a group of parents increases with their valuation for private education. Parents disagree on the emphasis and the payoff decreases with the distance between the actual emphasis of the school and the desired emphasis of each type of parent. The parameter c captures how sensitive parents are to

[7]Other forces might also be at work in some of the examples. For instance, the principal might not have as much bargaining power in real life and parties might bargain instead of resorting to take-it-or-leave-it offers. Our theory provides an upper bound on the payoff the principal can obtain in that situation.

a departure from their preferred emphasis. Our model corresponds to the case where parents with a high valuation for the new school are relatively more willing to compromise on emphasis.

As a special case, the good may be French education, where θ_A is the valuation of French parents located in a foreign country for a new French school in that country (i.e. , their willingness to pay to have the same education as in France[8]) and θ_B is the valuation of local citizens. The horizontal dimension captures for instance the emphasis on mathematics: French parents want to have the same curriculum as in France, however local citizens want part of the emphasis on mathematics replaced by local history and geography. Parents with high valuations are more likely to compromise on the curriculum because, for instance, there are few good alternatives to French education in the country considered. Also, the principal is an investor[9] or a parent willing to offer a personalized education to his own children and offering this new concept to other parents as well.[10] This special case is interesting because we observe that most French schools located in foreign countries do adapt the curriculum to the preferences of local citizens.

Services offered to club members.

The principal is the administrator of a private golf or tennis club and maximizes revenue or welfare of club members. The club accepts families (agent A) who enjoy other activities besides sports (e.g. socializing, using a restaurant) and individuals (agent B) who come mainly to practice. Then, θ_i captures the intrinsic demand for the club in group i and the horizontal dimension is the quantity of activities beyond sports. Given our assumptions, club members with high valuation for the club are relatively more willing to compromise on the services offered.

Development of a new product.

The principal is a monopolist deciding to develop a new product and maximizes profit. Agents A and B are two groups of consumers. The parameter θ_i is the demand for the new good in each group and γ_i is the difference between the preferred and the actual characteristics of the good for group i. The model captures the fact that consumers with a high valuation for the good are relatively more willing to compromise on characteristics. Also, each group prefers to have the possibility to buy the good even if its main characteristic is not the preferred one.

Choice of the program of an Opera season.

The principal is the general director of the Opera and maximizes either revenue or the welfare of attendants. Agents of type A represent the group of music lovers and agents of type B are

[8]These institutions aim at offering French education (and diploma) to French citizens located abroad. Parents who plan to come back to France or expect to travel from country to country in the future value highly the fact their children can get the same education at every location.

[9]Even though schools are public in France, most French Lycées in foreign countries are private institutions.

[10]This has been the case for instance of the Lycée International de Los Angeles combining a French education with an international component (http://www.lilaschool.com).

casual attendants or tourists. The parameter θ_i represents the willingness to pay for tickets in each group and γ_i is the difference between the preferred and the actual program offered at the Opera. Each group is better-off if the Opera offers performances the coming year. However, they differ in their preferences over the program: type B agents prefer to attend well known performances whereas type A agents prefer rare productions. The latter group is also relatively more willing to compromise. The same logic extends to goods such as theater performances or temporary exhibits in art museums.

In these examples, the principal is not fully informed about the preferences of the agents. The demand for football, the private school, or the Opera are generally unknown. Consequently, we assume that the principal does not observe θ. It also occurs that the principal does not observe how sensitive agents are to the choice of characteristics. We analyze this case in section 4.

First-best

From now on in this section, we assume that the principal maximizes her expected revenue. To have a benchmark for comparison, we denote by x_F the first-best location. It maximizes the payoff of the principal under full information. For any possible location x, the principal extracts all the surplus generated by the production of the good at that location. Formally, her total revenue is $\pi(\theta_A - cx) + \pi(\theta_B - c(N - x))$. Therefore, the good is located at x_F such that

$$x_F \;=\; \arg\max_{x} \quad \pi(\theta_A - cx) + \pi(\theta_B - c(N - x)) \tag{2}$$

Lemma 1 The optimal location is $x_F \gtreqless N/2$ when $\theta_A \gtreqless \theta_B$. It is increasing in θ_A and decreasing in θ_B.

The principal always produces the good. Given the substitutability of the vertical and horizontal differentiation parameters, she prefers to favor the agent with lowest valuation and to offer a good with characteristics more on the lines of his preferences. Formally, $x_F \gtreqless N/2$ when $\theta_A \gtreqless \theta_B$. The optimal location increases with θ_A (and becomes closer to B) and decreases with θ_B (and becomes closer to A). By doing so, the loss in the revenue extracted from the high-valuation agent is smaller than the gain in the revenue extracted from the low-valuation agent. In section 3 we study how the presence of asymmetric information on the valuations modifies this allocation. In particular we determine whether it exacerbates the bias or not.

3 Optimal location with unknown valuations

Properties of the mechanism under asymmetric information

The contract must specify an allocation rule and payments when both agents accept the contract but also when at least one refuses it. Indeed, given the presence of externalities, the outside option of each agent is *mechanism dependent*. Note that the objective of the principal is to extract as much payments as possible from both agents. Therefore, she benefits from designing a mechanism in which the outside option is the smallest possible. Following the standard contracting literature, we assume that the *principal can commit* to any mechanism offered to the agents and each agent accepts the contract when the utility of accepting is at least equal to the outside option. Then, it is optimal for the seller to commit not to produce the good if at least one agent refuses to participate in the contract. The idea is simply that given the positive externalities, the worst possible scenario for any agent who refuses to participate is the one in which the good is never produced. Note that this threat is only credible if the principal can commit. On the other hand, it is costless for her, as it is only made off-the-equilibrium path.

From the revelation principle, we know that we can restrict the attention to a direct revelation mechanism. The principal offers a menu of contracts that depends on the pair of announced valuations $(\tilde{\theta}_A, \tilde{\theta}_B)$. The menu specifies a probability $p_x(\tilde{\theta}_A, \tilde{\theta}_B)$ of production at each possible location x, and a transfer $t_i(\tilde{\theta}_A, \tilde{\theta}_B)$ from each agent to the principal. We also denote by $p_\emptyset(\tilde{\theta}_A, \tilde{\theta}_B)$ the probability of not producing the good. For notational convenience, let $\pi_i(\theta_i, x) \equiv \pi(\theta_i - c|x - y_i|)$ be agent i's payoff when the good is located at x. We have:

$$\pi_A(\theta_A, x) = \pi(\theta_A - c\,x) \qquad \text{and} \qquad \pi_B(\theta_B, x) = \pi(\theta_B - c\,(N - x)) \tag{3}$$

Also, let $u_i(\theta_i, \tilde{\theta}_i)$ be the *expected utility* of agent i when his valuation is θ_i, he announces $\tilde{\theta}_i$ and the other agent discloses his true valuation θ_j. We denote by $u_i(\theta_i) \equiv u_i(\theta_i, \theta_i)$ his expected utility under truthful revelation. We have:

$$u_i(\theta_i, \tilde{\theta}_i) = \int_{\underline{\theta}}^{\overline{\theta}} \Big(\Big[\sum_{x=0}^{N} \pi_i(\theta_i, x) p_x(\tilde{\theta}_i, \theta_j) \Big] - t_i(\tilde{\theta}_i, \theta_j) \Big) dF(\theta_j)$$

A mechanism $\{p_x(\cdot), t_i(\cdot)\}$ is optimal if and only if it maximizes the expected revenue R:

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[t_A(\theta_A, \theta_B) + t_B(\theta_A, \theta_B) \Big] dF(\theta_A) dF(\theta_B)$$

and satisfies three kinds of constraints. First, *incentive-compatibility*, which states that each agent must prefer to reveal his true valuation rather than any other one:

$$u_i(\theta_i) \geqslant u_i(\theta_i, \tilde{\theta}_i) \qquad \forall\, i, \theta_i, \tilde{\theta}_i$$

Second, *individual-rationality*, which implies that each agent must be willing to accept the contract offered by the principal (recall that in case of non-acceptance of the contract the good is never allocated, so the agent's reservation utility is zero):[11]

$$u_i(\theta_i) \geqslant 0 \qquad \forall\, i, \theta_i$$

Last, the allocation rule must be *feasible*:

$$p_x(\theta_A, \theta_B) \geqslant 0 \quad \forall\, x, \theta_i, \theta_j \qquad \text{and} \qquad \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \leqslant 1 \quad \forall\, \theta_i, \theta_j$$

Lemma 2 The optimal mechanism solves the following program $\mathcal{P}$:

$$\mathcal{P}: \max_{p_x(\theta_A,\theta_B)} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Big[\Phi_A(\theta_A, x) + \Phi_B(\theta_B, x) \Big] dF(\theta_A) dF(\theta_B)$$

$$\text{s. t.} \quad \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \pi_A}{\partial \theta_A} \times \frac{\partial p_x}{\partial \theta_A}\, dF(\theta_B) \geqslant 0 \quad \text{and} \quad \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \pi_B}{\partial \theta_B} \times \frac{\partial p_x}{\partial \theta_B}\, dF(\theta_A) \geqslant 0 \tag{M}$$

$$p_x(\theta_A, \theta_B) \geqslant 0 \quad \forall\, x \quad \text{and} \quad \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \leqslant 1 \tag{F}$$

$$\Phi_A(\theta_A, x) = \pi_A(\theta_A, x) - \frac{\partial \pi_A(\theta_A, x)}{\partial \theta_A} \frac{1 - F(\theta_A)}{f(\theta_A)} \tag{4}$$

$$\Phi_B(\theta_B, x) = \pi_B(\theta_B, x) - \frac{\partial \pi_B(\theta_B, x)}{\partial \theta_B} \frac{1 - F(\theta_B)}{f(\theta_B)} \tag{5}$$

In the optimal mechanism, the expected utility of agent $i = \{A, B\}$ is

$$u_i(\theta_i) = \int_{\underline{\theta}}^{\theta_i} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds.$$

The *net surplus* of agents A and B when the good is located at x are $\pi_A(\theta_A, x)$ and $\pi_B(\theta_B, x)$, respectively. Under complete information, this also corresponds to their willingness to pay and therefore to the maximum revenue that the principal can extract. Asymmetric information introduces a distortion in the agents' willingness to pay. This is the case because an agent with a high valuation can mimic an agent with a low valuation. To avoid mimicking, the seller must grant rents and the equilibrium utility of agents must increase in the valuation. Overall, the seller extracts only the *virtual surplus* $\Phi_A(\theta_A, x)$ and $\Phi_B(\theta_B, x)$ from agents A and B respectively when the good is located at x, that is the net surplus adjusted for the informational rents. She

[11] An agent cannot decide to produce the good on his own. Under this alternative assumption, the outside option would be type-dependent and countervailing incentives would arise (see Maggi and Rodriguez (1995) and Jullien (2000)). Besides, the decision of each agent to produce the good would also affect the outside option of the other agent. This analysis is out of the scope of the present paper. However, we analyze in Section 4.2. the case in which one agent designs the contract (that is, becomes the principal) and produces the good.

chooses the location that maximizes the virtual surplus under the standard monotonicity (M) and feasibility (F) constraints. Given the concavity of π and the monotone hazard rate property, the virtual surplus increase with the valuations of agents for all x: $\partial\Phi_A/\partial\theta_A > 0$ and $\partial\Phi_B/\partial\theta_B > 0$. The analysis of the allocation mechanism considered here is an adaptation of the procedure introduced by Myerson (1981) in the context of an auction.

Optimal contract with two possible locations

In this subsection, we assume that the good can only be located at $x = 0$ or $x = N$. For instance, a sports tournament needs to be located in one of two cities and the physical location of the sport facilities already exist. The intrinsic demand for the game is unknown and the organizer must elicit this information to raise funds and locate the event optimally. In fact, this problem is formally identical to the optimal auction of an indivisible good with two bidders (A and B), private valuations and positive type-dependent externalities. First, the principal may decide not to produce the good and both agents get utility 0. Second, she may produce the good and locate it at $x = 0$, then agent A gets utility $\pi(\theta_A)$ and agent B gets utility $\pi(\theta_B - cN)$. Third, she may produce the good and locate it at $x = N$, in which case agent A gets utility $\pi(\theta_A - cN)$ and agent B gets utility $\pi(\theta_B)$. Call $v_i(\theta_i) \equiv \pi(\theta_i)$ and $\alpha_i(\theta_i) \equiv \pi(\theta_i - cN)$ ($< v_i$ for all θ_i). Locating the good at $x = 0$ and at $x = N$ is formally equivalent to selling the good to agent A and to agent B respectively: the agent who purchases it gets utility v_i (increasing in his type θ_i) and the other one enjoys a positive externality α_j (also increasing in his type θ_j).

Using Lemma 2, equations (4)-(5) and ignoring for the moment constraint (M), it is immediate that in the optimal mechanism:

$$\begin{array}{llll} \text{If} \quad \Phi_A(\theta_A,0)+\Phi_B(\theta_B,0) & > & \max\{\,0,\Phi_A(\theta_A,N)+\Phi_B(\theta_B,N)\,\}, & \text{then} \quad p_0(\theta_A,\theta_B)=1 \\ \text{If} \quad \Phi_A(\theta_A,N)+\Phi_B(\theta_B,N) & > & \max\{\,0,\Phi_A(\theta_A,0)+\Phi_B(\theta_B,0)\,\}, & \text{then} \quad p_N(\theta_A,\theta_B)=1 \end{array}$$

Also, denote by $r_i(\theta_j, x)$ the value of θ_i such that $\Phi_i(r_i(\theta_j, x), x) + \Phi_j(\theta_j, x) = 0$.[12]

Proposition 1 With two possible locations $x \in \{0, N\}$, the optimal contract is such that:

$$\begin{cases} p_0(\theta_A,\theta_B)=1 & \text{if} \quad \theta_A<\theta_B \quad \text{and} \quad \theta_A>r_A(\theta_B,0) \\ p_N(\theta_A,\theta_B)=1 & \text{if} \quad \theta_B<\theta_A \quad \text{and} \quad \theta_B>r_B(\theta_A,N) \\ p_\varnothing(\theta_A,\theta_B)=1 & \text{otherwise} \end{cases}$$

When the principal decides where to locate the good, she compares the virtual surplus at each location. Since externalities are positive and type-dependent, the surplus depends on the valuations of both agents. First, the good will never be produced where the agent with highest

[12] If this expression is always positive (respectively negative), then $r_i(\theta_j, x) \equiv \underline{\theta}$ (respectively $r_i(\theta_j, x) \equiv \overline{\theta}$).

valuation is located (formally, $\theta_A > \theta_B \Rightarrow x \neq 0$ and $\theta_B > \theta_A \Rightarrow x \neq N$). This is due to the type-dependency of the externality and the substitutability between the vertical and horizontal dimensions ($\partial^2 v/\partial\theta_i\partial\gamma_i > 0$). The key issue is that, at any location, the principal extracts payments from *both agents*. So, suppose that $\theta_A > \theta_B$. By definition, A is willing to pay more than B to have the good at his own location. However, by locating the good at $x = N$ rather than at $x = 0$, the loss in the revenue extracted from agent A is smaller than the gain in the revenue extracted from agent B.

Second, the allocation is ex-post inefficient as in the standard literature. Even if the auctioneer's utility of keeping the good is smaller than the bidders' lowest valuation, the good may not be sold in order to decrease the rents (Myerson (1981)). Under positive externalities, this inefficiency is diminished but persists: for some pairs (θ_A, θ_B) the principal does not produce the good even though each agent has a positive utility under all locations.

Third, r_i is the analogue of a reserve price for bidder i in an auction mechanism. However, instead of being fixed, it depends negatively on the valuation of the other agent ($\partial r_i/\partial\theta_j < 0$). This new feature is due to the type-dependency of the externality. As the valuation of one agent increases, his willingness to pay at any given location also increases. Therefore, the minimum valuation of the other agent above which the principal finds it optimal to produce decreases.

Last, note that $\partial\pi(\theta_i - N)/\partial N < 0$ and $\partial\pi(\theta_i - c\gamma_i)/\partial c < 0$. The size of the externality is inversely related to the length of the Hotelling line and to the transportation cost. We show that $\partial r_i(\theta_j, x)/\partial N > 0$. As the externality increases (i.e. as N decreases) the regulator can extract more payoff from the agents. Therefore, the event $e = x \in \{0, N\}$ becomes relatively more profitable than the event $e = \emptyset$ (i.e. r_i decreases). Similarly, $\partial r_i(\theta_j, x)/\partial c > 0$. The project is less profitable as the transportation cost increases and the principal prefers to decrease the probability of producing in order to diminish the rent. Of course the allocation is more often ex post inefficient in that case. These results are depicted in Figure 1.

[INSERT FIGURE 1 HERE]

Remark 1. The reader might argue that sometimes high type agents are also relatively more concerned about distance, that is, the payoff function satisfies the assumption $\partial^2 v/\partial\theta_i\partial\gamma_i < 0$. Then, under both complete and asymmetric information, the good is located in 0 (resp. N) when $\theta_A > \theta_B$ (resp. $\theta_A < \theta_B$). The agent with the highest valuation is also the least willing to compromise on location, and the decision is biased towards that agent. Therefore, when $\partial^2 v/\partial\theta_i\partial\gamma_i < 0$, horizontal differentiation affects only quantitatively the optimal location compared to a model with vertical differentiation only. In both cases, the good is located closer to the agent with the highest valuation. Also, observing that a good is located close to the interest of the party who enjoys it the least cannot be reconciled with the assumption $\partial^2 v/\partial\theta_i\partial\gamma_i < 0$.

Optimal contract with several possible locations

We now analyze the case where the number of potential locations for the good is finite but arbitrarily large ($x \in \{0, 1, ..., N\}$). This formalization is more suitable to explain the optimal choice of characteristics of a new good or the optimal location of a new shopping mall to be built anywhere between two communities. Interestingly, this case cannot be reinterpreted as an auction of an indivisible good with externalities. Formally, it shares some features with the auction of a divisible good (Maskin and Riley (1989)): for example, locating the good at $x = N/2$ is similar to selling half of the good to one agent and half to the other one. However, there is a crucial difference between the two interpretations. In fact, not producing the good in our model ($e = \emptyset$) corresponds to not selling it in the auction case, and locating the good somewhere in the line ($e = x$) corresponds to splitting it entirely between the two bidders. Yet, in auctions of divisible goods there is a third possibility implicitly ruled out in our setting, which is to sell a fraction of the good and keep the rest.[13]

We denote by x_S the optimal second-best location. It maximizes the sum of the virtual surplus, that is the payoff of the principal given the asymmetry of information:

$$x_S = \arg\max_x \quad \Phi_A(\theta_A, x) + \Phi_B(\theta_B, x) \tag{6}$$

Proposition 2 When the set of locations is arbitrarily large, the optimal contract is such that:[14]

$$\begin{cases} p_{x_S}(\theta_A, \theta_B) = 1 & \text{if } \ \theta_B > r_B(\theta_A, x_S) \\ p_{\emptyset}(\theta_A, \theta_B) = 1 & \text{otherwise} \end{cases}$$

where $r_B(\theta_A, x_S)$ is such that $\Phi_A(\theta_A, x_S) + \Phi_B(r_B(\theta_A, x_S), x_S) = 0$. The reserve price $r_B(\theta_A, x_S)$ is such that $\frac{\partial r_B}{\partial \theta_A} < 0$. The location x_S is such that: $\frac{\partial x_S}{\partial \theta_A} > 0$, $\frac{\partial x_S}{\partial \theta_B} < 0$ and $x_S \gtreqless x_F \gtreqless N/2$ for all $\theta_A \gtreqless \theta_B$.

The location principle highlighted in Proposition 1 extends to the case of a large number of possible locations. The principal first determines which location x_S maximizes the virtual surplus, and then compares this total payoff with the payoff under no production. If both agents have the same valuation, then the good is located halfway between the two. As before, when types are different, the good is located closer to the agent with lowest valuation, although it is not necessarily at his exact location ($\theta_A \gtreqless \theta_B \Leftrightarrow x_S \gtreqless N/2$). Given informational rents, the principal may again decide not to produce the good ($e = \emptyset$). However, the ability to choose from a wider range of locations makes this event relatively less frequent than in Proposition 1. Moreover, the good is located more efficiently than in Proposition 1.

[13]In other words, our setting can be reinterpreted as the auction of a divisible good under the restriction that the auctioneer must either keep the good or allocate it entirely between the bidders.

[14]Note that the problem is formally different from the allocation of a good to one person. Therefore, the proof does not follow Myerson (1981) and needs to be adapted to our specific contracting problem.

Asymmetric information induces the principal to increase the distance between the location of the good and that of the agent who values it the most, relative to the socially optimal level. In fact, the principal has to manage simultaneously two distortions, $\frac{\partial \pi_A}{\partial \theta_A}\frac{1-F(\theta_A)}{f(\theta_A)}$ and $\frac{\partial \pi_B}{\partial \theta_B}\frac{1-F(\theta_B)}{f(\theta_B)}$, and both increase with the distance between the agent and the good. As the valuation θ_i of an agent increases, the distortion becomes less sensitive to the distance γ_i. To decrease the rents, it becomes relatively more interesting to bring the location of the good closer to the agent with lowest valuation. The properties of the optimal mechanism are represented in Figure 2.

[INSERT FIGURE 2 HERE]

Example 1. Let us assume $c = 1$, $\pi(\theta_i - \gamma_i) = 4(\theta_i - \gamma_i) - (\theta_i - \gamma_i)^2$ and $\theta_i \sim$ U$[1, 2]$. We also let $N = 1$ and we assume that $x \in [0, 1]$, so that $\theta_i - \gamma_i \in [0, 2]$. Using (2)-(3)-(4)-(5)-(6), the expressions for the optimal locations are:

$$x_F = \frac{1}{2} + \frac{1}{2}(\theta_A - \theta_B) \qquad \text{and} \qquad x_S = \begin{cases} 0 & \text{if} \quad \theta_A - \theta_B < -1/2 \\ \frac{1}{2} + (\theta_A - \theta_B) & \text{if} \quad \theta_A - \theta_B \in [-1/2, 1/2] \\ 1 & \text{if} \quad \theta_A - \theta_B > 1/2 \end{cases}$$

Under asymmetric information, the good is always located closer to the agent with lowest valuation than under full information. Moreover, as long as x_F and x_S are interior, the distortion increases as the difference in the valuations of the agents $|\theta_A - \theta_B|$ increases. Also, if the difference between valuations is sufficiently high ($|\theta_A - \theta_B| > 1/2$), then the agent with lowest valuation enjoys the good at his favorite location.

Last, note that it is sometimes not possible to commit ex ante to not supply the good. For example, the organizer of sports events may have difficulties in shutting down the competition. In that case, we have the following result.

Corollary 1 If the principal cannot commit to not supply the good, then the allocation is ex post efficient and the good is located at x_S.

In Proposition 2, the principal compares the surplus she extracts at each possible location x and selects the location that provides the highest payoff $x_S = \arg\max_x \; \Phi_A(\theta_A, x) + \Phi_B(\theta_B, x)$ provided this payoff is positive. Then, the good is not produced when $\Phi_A(\theta_A, x_S) + \Phi_B(\theta_B, x_S) < 0$. If she cannot commit not to supply the good, it is located at x_S even when $\Phi_A(\theta_A, x_S) + \Phi_B(\theta_B, x_S) < 0$. The inability to commit does not affect the optimal location. The allocation is suboptimal ex ante, but it is ex post efficient.

Remark 2. The analysis extends to situations with more than 2 agents. Suppose that there are m agents, indexed by $k \in \{1, 2, ..., m\}$. Agent k is located at $y_k \in [0, N]$, and let $\pi_k(\theta_k, x) \equiv \pi(\theta_k - c|x - y_k|)$ be the payoff of agent k when the good is located at x. For each vector of valuations

$\theta = (\theta_1, \theta_2, ..., \theta_m)$, the optimal location is $x_M = \arg\max_x \sum_{k=1}^m \Phi_k(\theta_k, x)$ and the optimal contract is such that $p_{x_M}(\theta) = 1$ if $\sum_{k=1}^m \Phi_k(\theta_k, x_M) > 0$ and $p_\emptyset(\theta) = 1$ otherwise. Production occurs if θ is above a given level. At location x, agent i faces the reserve price $r_i(\theta_{-i}, x)$ where θ_{-i} is the vector of valuations of other agents and $\Phi_i(r_i(\theta_{-i}, x), x) + \sum_{k \neq i} \Phi_k(\theta_k, x) = 0$. The seller finds profitable to produce at location x if the configuration of valuations is such that $\theta_i > r_i(\theta_{-i}, x)$ for all θ_i. For each θ such that it is profitable to produce at some x, she picks the location x_M that gives her the highest payoff. If agents are evenly distributed on the Hotelling line then, other things being equal, the good is located where agents have the lowest valuations.

4 Optimal location with unknown transportation costs

In some applications, the principal is not fully informed about the preferences for characteristics of the agents. Once a niche in a market is identified, it is sometimes relatively easy to determine the intrinsic demand for the new good but less obvious to know the preferences for characteristics. Suppose for instance that an entrepreneur wants to build a childcare center. The intrinsic demand θ_i represents the number of parents interested in the service and the horizontal differentiation parameter captures their preferences for extra care in the evening. It is difficult for the entrepreneur to know how flexible each parent is when she designs her service.

In this section, we analyze how the efficient location is affected when agents possess private information on the horizontal dimension and we assume that the transportation cost of each agent is unknown to the principal. Formally, agent i's transportation cost is $c_i \in [\underline{c}, \overline{c}]$ with $\underline{c} \geqslant 0$. Transportation costs are independently drawn from a common knowledge distribution $H(c_i)$ with continuous and strictly positive density $h(c_i)$ and satisfying the monotone hazard rate property $d\left[\frac{H(c)}{h(c)}\right]/dc > 0$. To concentrate on the effect of asymmetric information on the horizontal dimension, we assume $\theta_A = \theta_B = \theta$. Agent i's payoff is:

$$w(c_i, \gamma_i) = \pi(\theta - c_i \gamma_i)$$

The payoff is decreasing in the transportation cost ($\partial w / \partial c_i < 0$) and high cost agents are relatively more sensitive to distance ($\partial^2 w / \partial c_i \partial \gamma_i < 0$). From the perspective of a revenue maximizing principal, the first-best location x_F^H is such that:

$$x_F^H = \arg\max_x \; \pi(\theta - c_A x) + \pi(\theta - c_B(N - x)) \tag{7}$$

Lemma 3. The optimal location is $x_F^H \gtreqless N/2$ when $c_A \lesseqgtr c_B$. It is decreasing in c_A and increasing in c_B.

Again, the principal always produces the good under complete information. Also, she prefers to favor the agent with the highest transportation cost and the optimal location decreases with

c_A and increases with c_B. The increase in revenue extracted from the agent with the highest transportation cost by moving the optimal location closer to him offsets the loss of revenue from the agent with the smallest transportation cost. Overall, $x_F^H \gtreqqless N/2$ when $c_A \lesseqqgtr c_B$.

Under complete information, the same logic applies when the transportation costs or the valuations are unknown. In both cases, the agent with the lowest overall payoff (the lowest valuation θ_i in section 3 and the highest transportation cost c_i in the present section) is favored.

Under asymmetric information, the principal offers a menu of contracts contingent on the reports $\tilde{c}_A$ and $\tilde{c}_B$. Each contract specifies probabilities $p_x(\tilde{c}_A, \tilde{c}_B)$ of producing the good at location x and transfers $t_i(\tilde{c}_A, \tilde{c}_B)$ from agent i to the principal. As in section 3, the mechanism is optimal if it maximizes the revenue of the principal under the incentive compatibility, the individual rationality and the feasibility constraints. For notational convenience, let $\pi_A(c_A, x) = \pi(\theta - c_A x)$ and $\pi_B(c_B, x) = \pi(\theta - c_B(N - x))$. The counterpart of Lemma 2 is:

Lemma 4 The optimal mechanism solves the following program $\mathcal{P}_\mathcal{H}$:

$$\mathcal{P}_\mathcal{H} : \max_{p_x(c_A,c_B)} \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} \sum_{x=0}^{N} p_x(c_A, c_B) \Big[\Psi_A(c_A, x) + \Psi_B(c_B, x) \Big] dH(c_A) dH(c_B)$$

$$\text{s. t.} \quad \sum_{x=0}^{N} \int_{\underline{c}}^{\overline{c}} \frac{\partial \pi_A}{\partial c_A} \times \frac{\partial p_x}{\partial c_A} \, dH(c_B) \geqslant 0 \quad \text{and} \quad \sum_{x=0}^{N} \int_{\underline{c}}^{\overline{c}} \frac{\partial \pi_B}{\partial c_B} \times \frac{\partial p_x}{\partial c_B} \, dH(c_A) \geqslant 0 \tag{M}$$

$$p_x(c_A, c_B) \geqslant 0 \quad \forall x \quad \text{and} \quad \sum_{x=0}^{N} p_x(c_A, c_B) \leqslant 1 \tag{F}$$

$$\Psi_A(c_A, x) = \pi_A(c_A, x) + \frac{\partial \pi_A(c_A, x)}{\partial c_A} \frac{H(c_A)}{h(c_A)} \tag{8}$$

$$\Psi_B(c_B, x) = \pi_B(c_B, x) + \frac{\partial \pi_B(c_B, x)}{\partial c_B} \frac{H(c_B)}{h(c_B)} \tag{9}$$

In the optimal contract, the expected utility of agent $i = \{A, B\}$ is

$$u_i(c_i) = -\int_{c_i}^{\overline{c}} \int_{\underline{c}}^{\overline{c}} \sum_{x=0}^{N} p_x(s, c_j) \frac{\partial \pi_i}{\partial s}(s, x) dH(c_j) ds.$$

Under asymmetric information, low cost agents can mimic high cost agents and the equilibrium utility must be decreasing in the transportation cost. The main difference with Lemma 2 is that *by choosing a location, the seller implicitly chooses the degree of asymmetric information with each agent.* In particular, if the seller decides to locate the good at $x = 0$, there is no asymmetric information between the seller and agent A. In that case, the surplus of the agent is $\pi_A(c_A, 0) = \pi(\theta)$, it is known and can be fully extracted. More generally, the surplus that can be extracted by the principal from agents A and B at location x are respectively $\Psi(c_A, x)$ and $\Psi(c_B, x)$ where the distortions due to asymmetric information (second term in equations (8) and (9)) are proportional to the distance (formally, $\partial\pi_A/\partial c_A = -x\pi'(\theta - c_A x)$ and $\partial\pi_B/\partial c_B = -(N-x)\pi'(\theta - (N-x)c_B)$. The two surplus are decreasing in the transportation cost ($\partial\Psi_A/\partial c_A < 0$ and $\partial\Psi_B/\partial c_B < 0$). Let x_S^H be the optimal second-best location,

$$x_S^H = \arg\max_x \quad \Psi_A(c_A, x) + \Psi_B(c_B, x) \tag{10}$$

Proposition 3 The optimal contract is such that:

$$\begin{cases} p_{x_S^H}(c_A, c_B) = 1 & \text{if } \; c_B > r_B^H(c_A, x_S^H) \\ p_\emptyset(c_A, c_B) = 1 & \text{otherwise} \end{cases}$$

where $r_B^H(c_A, x_S^H)$ is such that $\Psi_A(c_A, x_S^H) + \Psi_B(r_B^H(c_A, x_S^H), x_S^H) = 0$. The reserve price $r_B^H(c_A, x_S^H)$ is such that $\frac{\partial r_B^H}{\partial c_A} < 0$. The location x_S^H is such that: $\frac{\partial x_S^H}{\partial c_A} < 0$, $\frac{\partial x_S^H}{\partial c_B} > 0$ and $x_S^H \gtreqless N/2$ for all $c_A \lesseqgtr c_B$. Furthermore, for all c_B, there exists $\hat{c}_A$ such that for all $c_A < \hat{c}_A$, $x_S^H > x_F^H$ and for all c_A, there exists $\hat{c}_B$ such that for all $c_B < \hat{c}_B$, $x_S^H < x_F^H$.

First, when $c_A = c_B$, it is optimal to locate the good at half distance. If c_A increases, the surplus that can be extracted from A decreases and given agent A becomes relatively more sensitive to distance, it is profitable to move the location closer to A. Therefore, when $c_A > c_B$, $x_S^H < \frac{N}{2}$. By the same token, when $c_A < c_B$, then $x_S^H > \frac{N}{2}$.

Second, compared to the solution obtained under complete information, there are two effects going in opposite directions. Suppose $c_A < c_B$, in which case $x_F^H > \frac{N}{2}$. Under asymmetric information and other things being equal, agent A must receive a higher informational rent. Therefore, the seller has an incentive to move the location closer to A to diminish the degree of asymmetric information with him. However, agent B is relatively more sensitive to distance. Then, it is also profitable to move the location closer to B. Overall, the bias obtained under complete information is decreased or increased, depending on which of the two effects dominates. This crucially depends on the shape of the utility function $\pi(\cdot)$ and the distribution $h(\cdot)$.

Third, as in the previous section, the good is not always produced: if the two costs are sufficiently high, the seller does not produce. Moreover, when c_A increases, the surplus that can be extracted decreases and the seller requires agent B to have a low enough transportation cost to carry the project at each possible location x. In other words, $r_B(c_A, x)$ decreases in c_A.

Last, the effects of asymmetric information in the present section can be contrasted with those obtained in section 3. In both sections, the principal has incentives to increase the bias obtained under complete information. Given the agent who is a priori more valuable (the agent with the highest θ in section 3 and the agent with the lowest c in this section) is also less sensitive to distance, moving the location away from him allows to capture extra rents from the other agent. However, it is now possible to *increase or decrease the amount of asymmetric information* with each agent by moving the location. This possibility generates a qualitative departure with respect to section 3. As a result, the overall bias might not be exacerbated.

Example 2. Suppose that $\pi(\theta - \gamma_i) = \log(\theta - c_i\gamma_i)$. Under complete information, we have

$$x_F^H = \frac{N}{2} + \frac{\theta}{2}\frac{(c_B - c_A)}{c_A c_B}$$

Under asymmetric information, $\Psi_A(c_A, x) + \Psi_B(c_B, x)$ is concave in x and

$$\frac{\partial}{\partial x}\Big(\Psi_A(c_A, x_F^H) + \Psi_B(c_B, x_F^H)\Big) = \frac{c_B^2}{(\theta - c_B(N - x_F^H))^2}\Big[\frac{H(c_B)}{h(c_B)}\frac{1}{c_B^2} - \frac{H(c_A)}{h(c_A)}\frac{1}{c_A^2}.\Big]$$

If the distribution is uniform on $[\underline{c}, \overline{c}]$, then $\frac{H(c)}{h(c)}\frac{1}{c^2}$ is concave with a maximum at $c^* = 2\underline{c}$. There are two cases. When the support is small enough ($\overline{c} < 2\underline{c}$), we have $x_F^H \lesseqgtr x_S^H$ when $c_A \lesseqgtr c_B$. Then, it is always optimal to move the good closer to the agent with the highest transportation cost and the bias is increased under asymmetric information. When the support is large ($\overline{c} > 2\underline{c}$), for all c_B, there exists $\tilde{c}_B$ such that $\frac{H(c_B)}{h(c_B)}\frac{1}{c_B^2} = \frac{H(\tilde{c}_B)}{h(\tilde{c}_B)}\frac{1}{\tilde{c}_B^2}$. When $c_A \leqslant \min(c_B, \tilde{c}_B)$, then $x_F^H \leqslant x_S^H$. Then, the bias in favor of the agent with the highest transportation cost is increased. When $c_A \in (\min(c_B, \tilde{c}_B), \max(c_B, \tilde{c}_B))$, the principal finds profitable to move the good closer to A and $x_F^H > x_S^H$. Last, when $c_A \geqslant \max(c_B, \tilde{c}_B)$, then $x_F^H \leqslant x_S^H$ and the bias is decreased.

The intuition is as follows. When the support is large enough, low values of the transportation cost are relatively less likely, but at the same time, a low cost agent has more possibilities to mimic a high cost agent. Then, the rents that must be granted to a low cost agent are relatively higher when the support is large. It becomes relatively more profitable to use the location to 'regulate' the amount of asymmetric information. When the difference between the transportation costs is substantial, the principal reduces the bias obtained under complete information by moving the good closer to the agent with the smallest cost.

5 Asymmetric preferences and dynamic choices

Optimal location when agents' preferences are not symmetric

In this section, we want to determine how the efficient location is modified when observing the preferences of the agent on the horizontal dimension convey relevant information on his preferences on the vertical dimension. For instance, suppose an entrepreneur wants to offer private education with primary emphasis on science. Some parents think that mathematics should be an important ingredient of the curriculum (group A) while other parents believe that a large part of mathematics and some of science should be replaced by arts and languages (group

B). If the public system emphasizes arts rather than mathematics and science, not only parents with a high valuation for education in science are expected to be more willing to compromise on the content in mathematics vs. arts ($\partial v/\partial\theta_i\partial\gamma_i > 0$), but also parents in group B should be willing to pay less for private education (high values of θ_B are less likely). Then, the intensity of preferences for private education is correlated to the preferences for the courses taught.

To capture this idea, we extend the model of section 2 and assume that the likelihood of a given valuation θ is $f_A(\theta)$ at location 0 and $f_B(\theta)$ at location N. The two distributions satisfy the Monotone Likelihood Ratio Property (MLRP)

$$\frac{d}{dv}\Big[\frac{f_A(\theta)}{f_B(\theta)}\Big] > 0$$

MLRP says that an agent in location 0 is relatively more likely to have a high type and relatively less likely to have a low type than an agent in location N. It implies that $(1-F_A(\theta))/f_A(\theta) > (1-F_B(\theta))/f_B(\theta)$ and $F_A(\theta) < F_B(\theta)$. The virtual surplus become:

$$\Phi_A^C(\theta_A, x) = \pi_A(\theta_A, x) - \frac{\partial\pi_A(\theta_A, x)}{\partial\theta_A}\frac{1-F_A(\theta_A)}{f_A(\theta_A)}$$

$$\Phi_B^C(\theta_B, x) = \pi_B(\theta_B, x) - \frac{\partial\pi_B(\theta_B, x)}{\partial\theta_B}\frac{1-F_B(\theta_B)}{f_B(\theta_B)}$$

and the optimal second best location is

$$x_S^C = \arg\max_x \quad \Phi_A^C(\theta_A, x) + \Phi_B^C(\theta_B, x) \tag{11}$$

Given (6), (11) and Proposition 2, we have:

Proposition 4 The optimal contract is such that:

$$\begin{cases} p_{x_S^C}(\theta_A, \theta_B) = 1 & \text{if} \quad \theta_B > r_B^C(\theta_A, x_S^C) \\ p_{\emptyset}(\theta_A, \theta_B) = 1 & \text{otherwise} \end{cases}$$

where $r_B^C(\theta_A, x_S^C)$ is such that $\Phi_A^C(\theta_A, x_S^C) + \Phi_B^C(r_B^C(\theta_A, x_S^C), x_S^C) = 0$. For all θ_A and θ_B, we have $x_S^C > x_S$. Moreover, for all θ_B, there exists $\tilde{\theta}_A(\theta_B) < \theta_B$ such that $x_S^C = N/2$. Last, for all θ_B there exists $\hat{\theta}_A(\theta_B) < \theta_B$ such that $x_S^C \gtrless x_F$ for all $\theta_A \gtrless \hat{\theta}_A(\theta_B)$.

The result $x_S^C > x_S$ is intuitive. Given that it is optimal to distort the location against high valuation agents and, given that the likelihood of a high valuation is now higher at location 0, the location is moved closer to agent B. Therefore, the range of valuations of agent A for which the good is located closer to B is increased ($x_C > N/2$ when $\theta_A > \tilde{\theta}_A(\theta_B)$). Also, compared to the first-best location, the distortion due to asymmetric information is not symmetric. Sometimes, the good is located closer to B, even if his realized valuation is higher than that of A.

In terms of the previous example, since the public system offers a poor education in science but a relatively better education in arts rather than mathematics, the principal attracts both types of parents by favoring the group with the best outside option, that is parents who think that a good education in arts is relatively more important. Parents who value mathematics more than arts enrol their children because this is still the best option to learn science.

Optimal locations over time

In the previous sections, we have analyzed a static game. This assumption is strong for applications such as sports tournaments which take place at regular intervals. To address this question, we consider a simple extension of the model of section 2. A sports tournament takes place at two periods $\tau = \{1, 2\}$ and two cities A and B compete at date $\tau = 0$ for hosting it at dates 1 and/or 2. The intrinsic demand for sports θ_i is private information of city i and it is the same across period. However, demand is increased by a positive and common knowledge shock δ the first time the game is organized in city i. This captures the fact that the event attracts consumers who would not ordinarily attend it. Or organizing the tournament requires a one time activity (e.g. repaving the streets) that increases the welfare of the city (e.g. it increases labor). Moreover, at each period, city i incurs a loss equal to $-N$ when the tournament is held in city j. Overall, the instantaneous utility of city i is $\pi(\theta_i + \delta)$ if the tournament takes place at city i for the first time, it is $\pi(\theta_i)$ if it takes place at city i for the second time and it is $\pi(\theta_i - N)$ if it takes place at the competing city. Agents discount the future at rate $\beta \in [0, 1]$ and the organizers must organize the tournaments at both periods.

Let us denote by $x_\tau \in \{0, N\}$ the location at date τ. The objective of the organizers is to design a mechanism specifying a probability $p_{x_1 x_2}(\tilde{\theta}_A, \tilde{\theta}_B)$ of locating the tournament at x_1 in the first period and x_2 in the second, conditional on the reported demands, as well as inter-temporal transfers $t_A(\tilde{\theta}_A, \tilde{\theta}_B)$ and $t_B(\tilde{\theta}_A, \tilde{\theta}_B)$ from city A and city B respectively. Formally, the analysis extends Proposition 1, which corresponds to the case $\beta = 0$ and $\delta = 0$.

Proposition 5 The optimal contract is such that:

$$\begin{cases} p_{0\,0}(\theta_A, \theta_B) = 1 & \text{if } \ \theta_A < \underline{z}_A(\theta_B) \\ p_{NN}(\theta_A, \theta_B) = 1 & \text{if } \ \theta_A > \overline{z}_A(\theta_B) \\ p_{0\,N}(\theta_A, \theta_B) = 1 & \text{if } \ \theta_A \in (\underline{z}_A(\theta_B), \theta_B) \\ p_{N\,0}(\theta_A, \theta_B) = 1 & \text{if } \ \theta_A \in (\theta_B, \overline{z}_A(\theta_B)) \end{cases}$$

where $\underline{z}_A(\theta_B) < \theta_B$ and $\overline{z}_A(\theta_B) > \theta_B$ increase in θ_B. Besides, if $\beta = 1$, it is optimal to alternate but the order does not matter. If $\delta = 0$, it is not optimal to alternate.

The logic of Proposition 1 extends to the dynamic setting. Suppose that $\theta_A < \theta_B$. Then, it is always optimal to let city A organize the tournament at least once and the principal strictly prefers to organize the tournament in city A in the first period if $\beta < 1$. Let us suppose this is the case, then the question is whether city A also organizes the second tournament.

First, it is optimal to organize both events in city A if the difference between the valuations is high enough so that the effect of the extra payoff is compensated. When the two valuations are relatively close, however, the principal might decide to organize the second event in city B.

Second, the region of valuations where alternating is optimal is affected by the size of the extra payoff δ. Alternating is more likely to occur when δ is high and there is no incentive to do so when δ tends to zero. Indeed, when δ is sufficiently small, any small positive difference between the two valuations compensates for the extra payoff.

Third, when agents are fully impatient ($\beta = 0$), only the first period matters and any location is optimal at date 2 from the perspective at date 0. On the other extreme, when agents are infinitely patient ($\beta = 1$), the inter-temporal payoff is the same when the tournament takes place first in city A or in city B. In that case, the principal requires to organize the tournament in city A at least once (but not necessarily in the first period). In that case, it is optimal to alternate when valuations are close enough but the order does not matter.

Fourth, private information does not modify these results qualitatively. The main difference is that the organizer must grant informational rents. Naturally, the incentives to reveal truthfully of each city are affected by the probabilities of hosting the event. Formally, suppose θ_A and θ_B are such that it is optimal under complete information to organize the event twice in city A. To disclose those valuations and achieve these locations, the organizer must grant the total second period rent $\frac{1-F(\theta_A)}{f(\theta_A)}\pi'(\theta_A) + \frac{1-F(\theta_B)}{f(\theta_B)}\pi'(\theta_B - N)$. However, if the seller decides to organize the event first at location 0 and then at location N, the second period rent becomes $\frac{1-F(\theta_A)}{f(\theta_A)}\pi'(\theta_A - N) + \frac{1-F(\theta_B)}{f(\theta_B)}\pi'(\theta_B + \delta)$. The organizer decides to alternate more (resp. less) often than under complete information if the second period rent is smaller (resp. higher) under that scenario.

6 Extensions

Optimal location selected by a social planner

It is difficult to reconcile the revenue-maximizing assumption with some applications, such as the decision to locate a public good (a hospital or a public school) between two communities. Suppose that the principal is a benevolent utilitarian regulator.[15] She offers a menu that specifies, for every pair $(\tilde{\theta}_A, \tilde{\theta}_B)$, a probability $p_x(\tilde{\theta}_A, \tilde{\theta}_B)$ of locating the good at x together with a

[15] We take a neutral approach even though in the case of local public goods, it is conceivable that the manager of the jurisdiction acts more on behalf of a certain type of residents. The objective function of local authorities as well as the subsequent effect on optimal provision of local public goods are addressed in Urban Economics. See for instance Hamilton (1975), Wildasin (1979) and Scotchmer (1994) among others. See also Scotchmer (2002).

subsidy $s_i(\tilde{\theta}_A, \tilde{\theta}_B)$ to agent i. Following the regulation literature we assume that subsidies are socially costly: \$1 transferred to an agent is raised through distortionary taxation and costs $\$(1+\lambda)$ to taxpayers, with $\lambda > 0$.[16] Let $\hat{u}(\theta_i, \tilde{\theta}_i)$ be the expected utility of agent i when his valuation is θ_i, his report is $\tilde{\theta}_i$, and agent j' report is truthful, then:

$$\hat{u}(\theta_i, \tilde{\theta}_i) = \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} \pi_i(\theta_i, x) p_x(\tilde{\theta}_i, \theta_j) + s_i(\tilde{\theta}_i, \theta_j) dF(\theta_j)$$

The utilitarian regulator maximizes social welfare W. Given the shadow cost λ of public funds, it is simply the payoff of the agents when the good is produced at x (π_A and π_B) minus the social costs of transferring s_A and s_B. Formally:

$$W = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Big[\pi_A(\theta_A, x) + \pi_B(\theta_B, x)\Big] - \lambda s_A(\theta_A, \theta_B) - \lambda s_B(\theta_A, \theta_B) dF(\theta_A) dF(\theta_B)$$

The regulator's optimization program is therefore:

$$\begin{aligned} \mathcal{P}_{\mathcal{W}} : \max_{p_x(\theta_A,\theta_B)} & \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Big[\Lambda_A(\theta_A, x) + \Lambda_B(\theta_B, x)\Big] dF(\theta_A) dF(\theta_B) \\ \text{s. t.} \quad & \text{(M)-(F)} \end{aligned}$$

$$\Lambda_i(\theta_i, x) = \pi_i(\theta_i, x) - \frac{\lambda}{1+\lambda} \frac{\partial \pi_i(\theta_i, x)}{\partial \theta_i} \frac{1 - F(\theta_i)}{f(\theta_i)}$$

Let us denote by $x_S^W(\lambda)$ the optimal second best location.

$$x_S^W(\lambda) = \arg\max_x \quad \Lambda_A(\theta_A, x) + \Lambda_B(\theta_B, x) \tag{12}$$

Given (2), (6), (12), $\mathcal{P}_{\mathcal{W}}$ and Proposition 2, we have:

Proposition 6 When a regulator chooses the location, the optimal contract is such that:

$$\begin{cases} p_{x_S^W}(\theta_A, \theta_B) = 1 & \text{if} \;\; \theta_B > r_B^W(\theta_A, x_S^W; \lambda) \\ p_{\emptyset}(\theta_A, \theta_B) = 1 & \text{otherwise} \end{cases}$$

where $r_B^W(\theta_A, x_S^W; \lambda)$ is such that $\Lambda_A(\theta_A, x_S^W) + \Lambda_B(\hat{r}_B(\theta_A, x_S^W; \lambda), x_S^W) = 0$. The location x_S^W is such that: $\frac{\partial x_S^W}{\partial \theta_A} > 0$, $\frac{\partial x_S^W}{\partial \theta_B} < 0$ and $x_S \gtreqless x_S^W \gtreqless x_F \gtreqless N/2$ for all $\theta_A \gtreqless \theta_B$. Furthermore, $\frac{\partial x_S^W(\lambda)}{\partial \lambda} \gtreqless 0$ for all $\theta_A \gtreqless \theta_B$, $x_S^W(0) = x_F$ and $x_S^W(\infty) = x_S$.

[16] The seminal analyses of regulation under asymmetric information are Baron and Myerson (1982) and Laffont and Tirole (1986). In the first paper society attaches a higher weight to consumers than to firms. In the second one, each party has equal weight but transfers are costly. Both models yield similar insights.

The characteristics of the optimal contract offered by a benevolent regulator and a privately interested party are very similar: location of the good closer to the agent with lowest valuation, distortion due to asymmetric information, possibility of not producing the good, etc. The main difference is that, in the regulation case, the relative weights of efficiency vs. rent extraction in the objective function of the principal are entirely determined by λ.

When transferring funds is costless ($\lambda = 0$), the regulator is interested exclusively in the efficiency of her action. Therefore, she takes the same decisions as under full information ($x_S^W(0) = x_F$ and $\hat{r}_i(\theta_j, x_S^W(0); 0) = \underline{\theta}$), even if it comes at the expense of a substantial subsidy. If subsidies from taxpayers to agents are prohibitively costly ($\lambda = \infty$), then the regulator's objective is formally equivalent to maximize welfare under the constraint that agents can be taxed but not subsidized ($s_i \leqslant 0$). This case is identical to the case of a privately interested principal, who trades-off efficiency and rents but will never choose to subsidize agents. The optimal decision coincides with that of Proposition 2: $x_S^W(\infty) = x_S$ and $\hat{r}_i(\theta_j, x_S^W(\infty); \infty) = r_i(\theta_j, x_S)$. When the cost of public funds is positive but finite $\lambda \in (0, \infty)$, the regulator is more concerned with increasing efficiency and less concerned with decreasing rents than a privately interested party. This is reflected in her choices: $x_S \gtreqless x_S^W \gtreqless x_F$ for all $\theta_A \gtreqless \theta_B$ and $\hat{r}_i(\theta_j, x_S^W(\lambda); \lambda) \in (\underline{\theta}, r_i(\theta_j, x_S))$.

Remark 3. The analysis shares some similarities with the optimal allocation of a public good studied in public economics (in the tradition of Clarke (1971) and Groves (1973)) and provides an alternative and complementary perspective. In the standard setting, each agent's valuation is a function $v(\theta, q)$ where θ is his type and q the quantity of public good. Given that *all agents prefer more quantity to less*, the main issue is to design a mechanism to prevent them from understating their type, getting away with a low payment while enjoying the public good (positive externality). In our setting, the valuation functions of agent A and B depend on the *location* x of the public good instead of the quantity provided. Here, *agents have opposite preferences over locations*. Also, given the positive externality, the incentives to underreport are present but the principal can use the location choice to mitigate them.

Optimal location when one agent is also the producer

Suppose now that agent A decides if he produces the good and where he locates it. This captures the fact that the planner (e.g. a parent/a tennis player) can also have a private interest in the project (e.g. a private school/a tennis club). In order to better isolate the changes in the incentives of the new decision-maker, we assume that B observes A's valuation θ_A for the good. Then, B does not have anything to infer from the mechanism proposed by A, and therefore A has no incentives to use the contract design to signal any information.[17]

Agent A offers a menu of contracts $\{p_x(\tilde{\theta}_B), t_B(\tilde{\theta}_B)\}$ such that, for each report $\tilde{\theta}_B$, agent B pays a transfer $t_B(\tilde{\theta}_B)$ and the good is located at x with probability $p_x(\tilde{\theta}_B)$. Denote by R_A the

[17] For a thoughtful analysis of contracting with an informed principal, see Maskin and Tirole (1990, 1992).

expected revenue of A (that is the sum of his own valuation and the expected transfer raised from agent B) and by $u_B^*(\theta_B, \tilde{\theta}_B)$ the utility of agent B with valuation θ_B and report $\tilde{\theta}_B$:

$$R_A = \int_{\underline{\theta}}^{\overline{\theta}} t_B(\theta_B) dF(\theta_B) + \pi_A(\theta_A, x)$$

$$u_B^*(\theta_B, \tilde{\theta}_B) = \sum_{x=0}^{N} \pi_B(\theta_B, x) p_x(\tilde{\theta}_B) - t_B(\tilde{\theta}_B)$$

The objective of agent A is to solve:

$$\mathcal{P_A}: \quad \max_{p_x(\theta_B)} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_B) \left[\pi_A(\theta_A, x) + \Phi_B(\theta_B, x)\right] dF(\theta_B)$$

$$\text{s. t.} \quad \sum_{x=0}^{N} \frac{\partial \pi_B}{\partial \theta_B} \times \frac{\partial p_x}{\partial \theta_B} \geqslant 0 \qquad (\mathrm{M_B})$$

$$p_x(\theta_B) \geqslant 0 \quad \forall x \quad \text{and} \quad \sum_{x=0}^{N} p_x(\theta_B) \leqslant 1 \qquad (\mathrm{F_B})$$

In $\mathcal{P_A}$, only θ_B is private information. Since it is only required to grant informational rents to B, the objective function is the sum of the *net surplus of agent A and the virtual surplus of agent B* ($\pi_A(\theta_A, x)$ and $\Phi_B(\theta_B, x)$ respectively). The monotonicity ($\mathrm{M_B}$) and feasibility ($\mathrm{F_B}$) constraints of agent B are the same as in Lemma 2, except that now the valuation θ_A is known. We denote by x_A the location that maximizes the surplus from agent A's perspective:

$$x_S^A \;=\; \arg\max_x \quad \pi_A(\theta_A, x) + \Phi_B(\theta_B, x) \qquad (13)$$

Given (2), (6), (13), $\mathcal{P_A}$ and Proposition 2, we have:

Proposition 7 When agent A chooses the location, the optimal contract is such that:

$$\begin{cases} p_{x_S^A}(\theta_B) = 1 & \text{if} \;\; \theta_B > r_B^A(\theta_A, x_S^A) \\ p_{\emptyset}(\theta_B) = 1 & \text{otherwise} \end{cases}$$

where $r_B^A(\theta_A, x_S^A)$ is such that $\pi_A(\theta_A, x_S^A) + \Phi_B(r_B^A(\theta_A, x_S^A), x_S^A) = 0$. The location x_S^A is such that: $\frac{\partial x_S^A}{\partial \theta_A} > 0$, $\frac{\partial x_S^A}{\partial \theta_B} < 0$ and $x_S^A > \max\{x_S, x_F\}$ for all θ_A and θ_B.

Again, depending on agent B's reported valuation, either the good is not produced ($e = \emptyset$) or it is situated at the location where the surplus is maximized ($e = x_S^A$). The novelty of this case is that agent A locates the good farther away from his own preferred location than in Proposition 2 ($x_S^A > x_S$), and also farther away than under full information ($x_S^A > x_F$). The idea is that, when agent A chooses the location, there is only one unknown parameter, B's valuation. In order to reduce B's rents, it is unambiguously better to bring the good closer to him. That same logic applies when we compare agent A's optimal choice with the full information case.

7 Concluding remarks

We have analyzed the optimal choice of a principal who decides whether to produce an indivisible good and which characteristics it contains. If the utility of agents is differentiated along two substitutable dimensions (an intrinsic willingness to pay for the good and a preference for characteristics), the principal offers a good with characteristics more on the lines of the preferences of the agent with lowest willingness to pay. Asymmetric information on the vertical dimension exacerbates this bias. If agents have different intensities of preferences for characteristics, it is optimal to bias the decision in favor of the agent who is the most sensitive to a deviation from his preferred characteristics. However, the inability to observe these intensities does not necessarily exacerbates the initial bias. The reason is that, when the intensities of preferences for characteristics are unknown, the principal can arbitrarily decrease the amount of asymmetric information with one agent by locating the good closer to him.

The analysis suggests that it is sometimes profitable to bias decisions against the preferences of the most interested parties. Coming back to the special case discussed in section 2, according to our analysis, the reason why the French schools adapt the program to the tastes of local citizens is simply that, although French parents are a priori more willing to pay for French education, the school must offer something of value to local citizens in order to attract them (for instance, local citizens might have a different educational culture). Given French parents are ready to give up some features of French education as long as the main philosophy is preserved, the school maximizes its revenue by adopting that strategy. Examples of such biases can be found in other economic situations. For example, operas generally schedule an important number of well known performances and only a few rare productions. This suggests that it is relatively easier to attract people who truly enjoy opera rather than people who attend it only on occasion.

The results rest on the assumption that individuals are differentiated along two dimensions. They assign an intrinsic valuation to the good but they have different preferences for its characteristics. Absent the second dimension, the principal takes a decision on the lines of the agent who values it most because it is the only way to generate a social value. In our setting, the principal generates a value also by choosing characteristics: the investor can locate the stadium in the city where there is already a high number of football supporters but also, she can locate it in a city in which residents go to football events only if they host them. Then, taking a decision on the lines of the agent who values the good most is not necessarily optimal. In other words, the optimal allocation of a non-excludable good is affected crucially by the characteristics it contains and how they are perceived by economic agents.

Appendix

Proof of Lemma 1. Denote by x_k each of the K locations available between 0 and N. Moreover, $x_0 = 0$, $x_K = N$ and there exists k^* such that $x_{k^*} = N/2$. The revenue of the seller is $\Pi(\theta_A, \theta_B, x) = \pi(\theta_A - cx) + \pi(\theta_B - c(N - x))$.

Consider x_k and θ_A such that $x_k = \arg\max \Pi(\theta_A, \theta_B, x)$. If the optimal location when agent A's type is $\theta'_A > \theta_A$ is $y < x_k$, then $\Pi(\theta_A, \theta_B, x_k) > \Pi(\theta_A, \theta_B, y)$ and $\Pi(\theta'_A, \theta_B, y) > \Pi(\theta'_A, \theta_B, x_k)$. Adding the two inequalities, we have $\Pi(\theta_A, \theta_B, x_k) - \Pi(\theta_A, \theta_B, y) > \Pi(\theta'_A, \theta_B, x_k) - \Pi(\theta'_A, \theta_B, y)$, which yields to a contradiction given $\Pi_{31}(\theta_A, \theta_B, x) > 0$. Therefore, the optimal location $x_F(\theta_A, \theta_B)$ is non-decreasing in θ_A. Similarly, and given $\Pi_{32}(\theta_A, \theta_B, x) < 0$, $x_F(\theta_A, \theta_B)$ is non-increasing in θ_B. Note that $\partial\Pi(\theta, \theta, x)/\partial x = 0$ at $x = N/2$. Given this location is available in the discrete version, $x_F(\theta, \theta) = N/2$. *Q.E.D.*

Proof of Lemma 2. We have $u_i(\theta_i, \tilde{\theta}_i) = u_i(\tilde{\theta}_i, \tilde{\theta}_i) + \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j)$ for all $i = \{A, B\}$. Adapting Myerson (1981), the agent reveals truthfully if and only if the two following conditions (M) and (LO) are satisfied:

$$\sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j) \int_{\tilde{\theta}_i}^{\theta_i} \frac{\partial \pi_i(s, x)}{\partial s} ds dF(\theta_j) \leqslant \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\theta_i, \theta_j) \int_{\tilde{\theta}_i}^{\theta_i} \frac{\partial \pi_i(s, x)}{\partial s} ds dF(\theta_j) \quad \forall\ \tilde{\theta}_i \leqslant \theta_i.$$

$$u_i(\theta_i) - u_i(\tilde{\theta}_i) = \int_{\tilde{\theta}_i}^{\theta_i} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds$$

The seller maximizes her expected revenue under constraints (M) and (LO), the individual rationality constraint and the feasibility constraints. To minimize the rent left to agents and satisfy the incentive compatibility constraint, the seller sets $u_i(\underline{\theta}) = 0$ (which ensures individual rationality is satisfied for all valuations). Using (LO) and integrating by parts, R is:

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \left[\Phi_A(\theta_A, x) + \Phi_B(\theta_B, x)\right] dF(\theta_A) dF(\theta_B).$$

The seller maximizes this expression under the remaining constraints (M) and (F). *Q.E.D.*

Proof of Proposition 1. Let $\Phi(\theta_A, \theta_B, x) = \Phi_A(\theta_A, x) + \Phi_B(\theta_B, x)$. Given the monotone hazard rate property and $\pi'' < 0$, $\Phi(\theta_A, \theta_B, x)$ is increasing in both θ_A and θ_B. Then, for all θ_B, there exists $r_A(\theta_B, 0)$ such that $\Phi(\theta_A, \theta_B, 0) \geqslant 0$ if $\theta_A \geqslant r_A(\theta_B, 0)$. Similarly, there exists $r_B(\theta_A, N)$ such that $\Phi(\theta_A, \theta_B, N) \geqslant 0$ if $\theta_B \geqslant r_B(\theta_B, N)$. Also, given $\pi''' \geqslant 0$, we have

$$\Phi_1(\theta_A, \theta_B, 0) - \Phi_1(\theta_A, \theta_B, N) = \left[\pi'(\theta_A) - \pi'(\theta_A - cN)\right] \left[1 - \frac{d}{d\theta_A}\left[\frac{1 - F(\theta_A)}{f(\theta_A)}\right]\right]$$

$$-\frac{1-F(\theta_A)}{f(\theta_A)}[\pi''(\theta_A)-\pi''(\theta_A-cN)]<0$$

and $\Phi(\theta_A,\theta_B,N)=\Phi(\theta_A,\theta_B,0)$ when $\theta_A=\theta_B$. Then, for all $\theta_A<\theta_B$, $\Phi(\theta_A,\theta_B,0)>\Phi(\theta_A,\theta_B,N)$ and for all $\theta_A>\theta_B$, $\Phi(\theta_A,\theta_B,0)<\Phi(\theta_A,\theta_B,N)$. Therefore, the allocation rule in proposition 1 maximizes the revenue of the seller and it is the optimal contract if it satisfies also (F) and (M). It is immediate that (F) holds. Differentiating $\Phi(r_A(\theta_B,0),\theta_B,0)=0$ with respect to θ_B yields $\frac{d}{d\theta_B}r_A(\theta_B,0)<0$. Similarly $\frac{d}{d\theta_A}r_B(\theta_A,N)<0$. Moreover, $r_A^{-1}(\theta_A,0)=r_B(\theta_A,N)$. There exist possibly many values θ^* such that $r_B(\theta^*,N)=r_A^{-1}(\theta^*,0)$. However $\frac{d}{d\theta_A}r_B(\theta_A,N)|_{\theta^*}=-\frac{\Phi_1(\theta^*,\theta^*,N)}{\Phi_2(\theta^*,\theta^*,N)}\leqslant-1$. which ensures that θ^* is unique. For all $\theta_A<\theta^*$, $r_B(\theta_A,N)>r_A^{-1}(\theta_A,0)$ and for all $\theta_A>\theta^*$, $r_B(\theta_A,N)<r_A^{-1}(\theta_A,0)$. For A, let us denote the probability of allocating the good at location x by $P_x(\theta_A)=\int_{\underline{\theta}}^{\overline{\theta}}p_x(\theta_A,\theta_B)dF(\theta_B)$, then

$$\text{If } \theta_A<\theta^*: \quad P_0(\theta_A)=1-F(r_A^{-1}(\theta_A,0)), \qquad P_N(\theta_A)=0;$$

$$\text{If } \theta_A>\theta^*: \quad P_0(\theta_A)=1-F(\theta_A), \quad P_N(\theta_A)=F(\theta_A)-F(r_B(\theta_A,N)).$$

It comes immediately that the allocation rule in Proposition 1 satisfies (M), and it is the optimal contract. Last, let $\phi(\theta_A,\theta_B,0;N)\equiv\Phi(\theta_A,\theta_B,0)$ and $\phi(\theta_A,\theta_B,N;N)\equiv\Phi(\theta_A,\theta_B,N)$. Both functions decrease in N. As a consequence $r_A(\theta_B,0)$ and $r_B(\theta_A,N)$ increase in N. Also, let $\chi(\theta_A,\theta_B,0;c)\equiv\Phi(\theta_A,\theta_B,0)$ and $\chi(\theta_A,\theta_B,N;c)\equiv\Phi(\theta_A,\theta_B,N)$. Both functions decrease in c. Therefore both $r_A(\theta_B,0)$ and $r_B(\theta_A,N)$ increase in c. *Q.E.D.*

Proof of Proposition 2. Using the technique of Appendix 1, we show that the optimal location $x_S(\theta_A,\theta_B)$ is non-decreasing in θ_A and non-increasing in θ_B. Then, for all θ_A there exists a subset of locations $\mathcal{X}(\theta_A)$ such that if k and $k+2$ are in $\mathcal{X}(\theta_A)$, then $k+1\in\mathcal{X}(\theta_A)$ (by continuity) and: (i) for all $k\in\mathcal{X}(\theta_A)$ there exist $h_{x_k}(\theta_A)$ and $h_{x_{k-1}}(\theta_A)>h_{x_k}(\theta_A)$ both increasing in θ_A such that $x_k=\arg\max\Phi(\theta_A,\theta_B,x)$ if $\theta_B\in[h_{x_k}(\theta_A),h_{x_{k-1}}(\theta_A)]$; (ii) for all $k\notin\mathcal{X}(\theta_A)$, the good is not located at x_k. Using the same argument as in Appendix 3, we can show that for any $k\in\mathcal{X}(\theta_A)$, we have $r_B(\theta_A,x_k)$ is decreasing in θ_A. Also $h_{x_{k-1}}(\theta_A)$ is such that $\Phi(\theta_A,h_{x_{k-1}}(\theta_A),x_{k-1})>\Phi(\theta_A,h_{x_{k-1}}(\theta_A),x_k)$. Then, if $h_{x_{k-1}}(\theta_A)>r_B(\theta_A,x_k)$, we have also $h_{x_{k-1}}(\theta_A)>r_B(\theta_A,x_{k-1})$. Combining the previous points, for all θ_A, there exists a subset $\tilde{\mathcal{X}}(\theta_A)=\{\underline{k}(\theta_A),...,\overline{k}(\theta_A)\}\subset\mathcal{X}(\theta_A)$ such that k is the optimal location when $\theta_B\in[g_{x_k}(\theta_A),g_{x_{k-1}}(\theta_A)]$ where $g_{x_{\overline{k}(\theta_A)}}(\theta_A)=r_B(\theta_A,x_{\overline{k}(\theta_A)})$ and $g_{x_k}(\theta_A)=h_{x_k}(\theta_A)$ for all $k<\overline{k}(\theta_A)$. The mechanism satisfies (F). We need to check it satisfies also (M).

Let $P_k(\theta_A)\equiv\int_{\underline{\theta}}^{\overline{\theta}}p_{x_k}(\theta_A,\theta_B)dF(\theta_B)$, we have $P_k(\theta_A)=F(h_{x_{k-1}}(\theta_A))-F(h_{x_k}(\theta_A))$ if $k\in\tilde{\mathcal{X}}(\theta_A)-\overline{k}(\theta_A)$ and $P_k(\theta_A)=F(h_{x_{\overline{k}(\theta_A)-1}}(\theta_A))-F(r_B(\theta_A,x_{\overline{k}(\theta_A)}))$ if $k=x_{\overline{k}(\theta_A)}$. It is 0 otherwise. We can rewrite (M) as $\sum_{k=0}^{N}\frac{d\pi_A}{d\theta_A}(\theta_A,x_k)\times\frac{dP_k}{d\theta_A}(\theta_A)\geqslant0$. Using the facts that $h_{x_k}(\theta_A)$ is increasing in θ_A and $r_B(\theta_A,x_k)$ is decreasing in θ_A, we have (M) satisfied.

If the number of locations tends to infinity, we have $x_S \gtreqless x_F$ when $\theta_A \gtreqless \theta_B$. The argument relies on the concavity of $\Phi(\theta_A, \theta_B, x)$ and $\Pi(\theta_A, \theta_B, x)$ in x. Moreover when $\theta_A < \theta_B$, we have $y^* = \text{argmax}\Phi(\theta_A, \theta_B, x) < \text{argmax}\Pi(\theta_A, \theta_B, x) = x^*$. Using this, we find that $x_S = x_F$ when $x_F < y^*$ and $x_S \leqslant x_F$ otherwise. By a symmetric argument, $x_S \geqslant x_F$ when $\theta_A > \theta_B$. *Q.E.D.*

Proof of Lemma 3. Using the technique of Appendix 1, $x_F^H(c_A, c_B)$ is decreasing in c_A and increasing in c_B. Also $\partial\Pi^H(c, c, x)/\partial x = 0$ at $x = N/2$ and $x_F^H(c, c) = N/2$. *Q.E.D.*

Proof of Lemma 4. Here the constraints (M) and (LO) are respectively:

$$\sum_{x=0}^{N} \int_{\underline{c}}^{\overline{c}} p_x(\tilde{c}_i, c_j) \int_{c_i}^{\tilde{c}_i} \frac{\partial \pi_i(s, x)}{\partial s} ds dF(c_j) \geqslant \sum_{x=0}^{N} \int_{\underline{c}}^{\overline{c}} p_x(c_i, c_j) \int_{c_i}^{\tilde{c}_i} \frac{\partial \pi_i(s, x)}{\partial s} ds dH(c_j) \quad \forall\ \tilde{c}_i \geqslant c_i.$$

$$u_i(c_i) - u_i(\tilde{c}_i) = -\int_{c_i}^{\tilde{c}_i} \sum_{x=0}^{N} \int_{\underline{c}}^{\overline{c}} p_x(s, c_j) \frac{\partial \pi_i}{\partial s}(s, x) dH(c_j) ds.$$

To minimize the rent left to agents and satisfy the incentive compatibility constraint, the seller sets $u_i(\overline{c}) = 0$. The rest of the proof is as in Appendix 2. *Q.E.D.*

Proof of Proposition 3. Let $\Psi(c_A, c_B, x) = \Psi_A(c_A, x) + \Psi_B(c_B, x)$. Using the same steps as in Appendix 5 and the facts that $\Psi_{31} < 0$ and $\Psi_{32} > 0$, the optimal location is non-increasing in c_A and non-decreasing in c_B. The rest of the proof follows Appendix 4. *Q.E.D.*

Proof of Proposition 5. Denote by $v_i^X(\theta_i)$ the inter-temporal gross surplus of agent i when the good is located at X where $X \in \{00, N0, 0N, NN\}$. For agent A, we have $v_A^{00}(\theta_A) = \pi(\theta_A + \delta) + \beta\pi(\theta_A)$, $v_A^{NN}(\theta_A) = \pi(\theta_A - N) + \beta\pi(\theta_A - N)$, $v_A^{0N}(\theta_A) = \pi(\theta_A + \delta) + \beta\pi(\theta_A - N)$ and $v_A^{N0}(\theta_A) = \pi(\theta_A - N) + \beta\pi(\theta_A + \delta)$. Under complete information, the seller extracts all the surplus and the total revenue when the good is located at X is $\Psi^X(\theta_A, \theta_B) = v_A^X(\theta_A) + v_B^X(\theta_B)$.

For all $\theta_A \lesseqgtr \theta_B$, $\Psi^{00}(\theta_A, \theta_B) \gtreqless \Psi^{NN}(\theta_A, \theta_B)$ and for all $\theta_A \lesseqgtr \theta_B$, $\Psi^{0N}(\theta_A, \theta_B) \gtreqless \Psi^{N0}(\theta_A, \theta_B)$. There exists $\underline{n}_A(\theta_B) < \theta_B$ increasing in θ_B and such that for all $\theta_A \lesseqgtr \underline{n}_A(\theta_B)$, $\Psi^{00}(\theta_A, \theta_B) \gtreqless \Psi^{0N}(\theta_A, \theta_B)$. There also exists $\overline{n}_A(\theta_B) > \theta_B$ increasing in θ_B such that for all $\theta_A \lesseqgtr \overline{n}_A(\theta_B)$, $\Psi^{NN}(\theta_A, \theta_B) \lesseqgtr \Psi^{N0}(\theta_A, \theta_B)$. Moreover, $\underline{n}_A(\theta_B)$ decreases in δ and $\overline{n}_A(\theta_B)$ increases in δ. Also, when $\beta = 1$, $\Psi^{0N}(\theta_A, \theta_B) = \Psi^{N0}(\theta_A, \theta_B)$ for all θ_A and θ_B. Last, it is always optimal to produce.

Under asymmetric information, if $V(\theta_i)$ denotes the expected utility of agent i, constraints (M) and (LO) are:

$$\sum_{X} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial p_X(s, \theta_j)}{\partial s} \frac{dv_i^X(s)}{\partial s} ds dF(\theta_j) \geqslant 0 \quad \forall\ \tilde{\theta}_i \leqslant \theta_i.$$

$$V(\theta_i) - V(\tilde{\theta}_i) = \int_{\tilde{\theta}_i}^{\theta_i} \sum_X \int_{\underline{c}}^{\overline{c}} p_X(s, \theta_j) \frac{dv_i^X}{ds}(s) dF(\theta_j) ds$$

The virtual surplus extracted from agent i when the good is located at X is therefore

$$\Phi_i^X(\theta_i) = v_i^X(\theta_i) - \frac{dv_i^X(\theta_i)}{d\theta_i} \frac{1 - F(\theta_i)}{f(\theta_i)}$$

and the total surplus is $\Phi^X(\theta_A, \theta_B) = \Phi_A^X(\theta_A) + \Phi_B^X(\theta_B)$. We have $\Phi^{00}(\theta_A, \theta_B) - \Phi^{NN}(\theta_A, \theta_B) \gtreqless 0$ for all $\theta_A \lesseqgtr \theta_B$. Moreover, for all $\theta_A \lesseqgtr \theta_B$, $\Phi^{0N}(\theta_A, \theta_B) \gtreqless \Phi^{N0}(\theta_A, \theta_B)$. There exists $\underline{m}_A(\theta_B)$ such that for all $\theta_A \lesseqgtr \underline{m}_A(\theta_B)$, $\Phi^{00}(\theta_A, \theta_B) \gtreqless \Phi^{0N}(\theta_A, \theta_B)$. Also, there exists $\overline{m}_A(\theta_B)$ such that for all $\theta_A \gtreqless \underline{m}_A(\theta_B)$, $\Phi^{NN}(\theta_A, \theta_B) \gtreqless \Phi^{N0}(\theta_A, \theta_B)$. *Q.E.D.*

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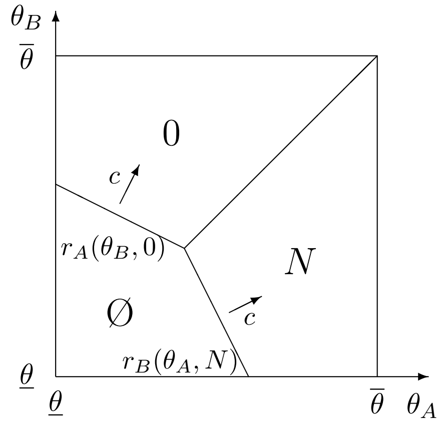

FIGURE 1: optimal location when $x \in \{0, N\}$

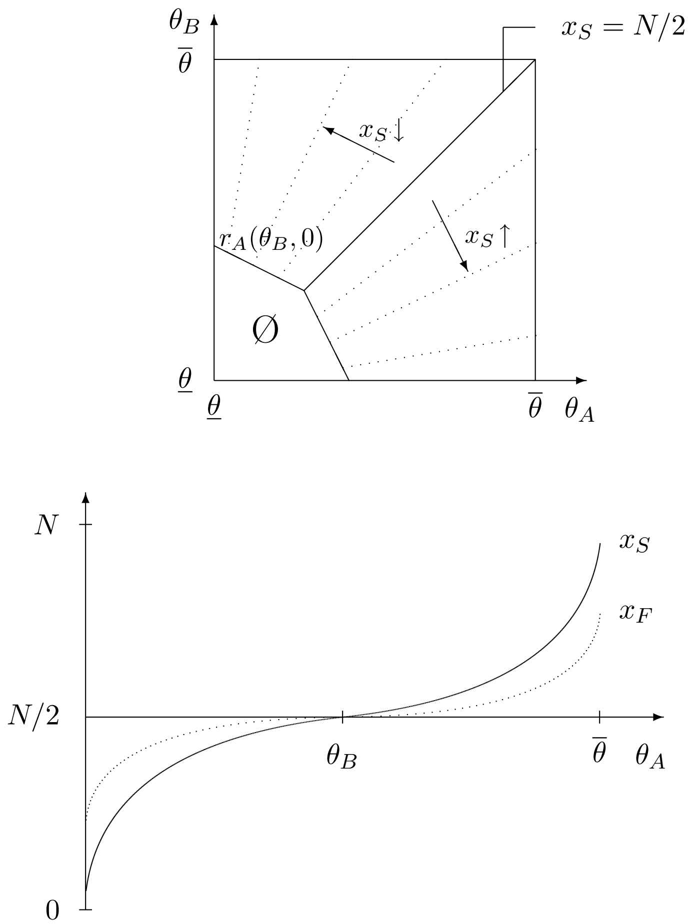

FIGURE 2: optimal location when $x \in \{0, 1, ..., N-1, N\}$