Self-awareness in time perception *

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Abstract

We investigate self-awareness in time perception using three time production tasks with different rewards structures, and collect self-assessments of performance. Participants had monetary incentives to target the true time in the first (baseline) task, not to exceed the true time in the second task and not to fall below the true time in the third task. We found that participants overestimated time in all tasks but responded correctly to incentives: they decreased their estimates in the second task and increased them in the third. Participants’ self-assessment in the baseline task was in line with their time perception biases, and their behavior in the other tasks was consistent with their (correct) beliefs. Self-perceived over-estimators decreased their estimates in the second task significantly more relative to self-perceived under-estimators, while in the third task they increased their estimates significantly less. We discussed the results within the context of recent studies on time perception.

Keywords: time perception, decision-making, self-awareness, metacognition.

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1 Introduction

*How did it get so late so soon?*

Dr. Seuss

The literature in psychophysics documents how we make time evaluations and demonstrates that our subjective assessments of the time that passes may be inaccurate. Discrepancies between objective time and subjective time are observed as a function of the time duration range – below or above 1 second –, but also the specific task design (Grondin (2010), Wearden and Lejeune (2008), Grondin (2014)). Furthermore, emotional states (Droit-Volet and Meck (2007), Droit-Volet and Wearden (2002), Fayolle et al. (2015)), attentional constraints (Buhusi and Meck (2006)) and drug usage (Wittmann et al. (2007)) also shape our perception of time. Even though the mechanisms of time-keeping are relatively well understood, little is known about how aware people are of their potential biases and how this knowledge affects their behavior when facing time-related incentives. This issue is closely related to our ability of being aware that we know something, a concept referred to as metacognition (Koriat (2007), Nelson (1996)). Metacognition has been applied to subjective time only recently (Sackett et al. (2010), Lamotte et al. (2012)) to show that individuals hold beliefs about distortions in their experience of time that affect their time-related judgements. However, it is unclear how such beliefs impact decision-making. For instance, if a person is typically late, is she bound to fail to meet important deadlines? Or is she conscious enough of her tendency to be able to correct it when the incentives are sufficiently high?

The objective of this study is to embed the classical time perception paradigm into a decision-making paradigm, where participants have to choose their time targets as a response to varying incentives.

Our study faced three main challenges. First, to isolate biases in time-keeping behavior, it was crucial to choose a task that relied as little as possible on orthogonal concerns. To prevent interference with memory processes, we avoided retrospective paradigms (Block and Zakay (1997), Fraisse (1984)) and participants were informed beforehand that they had to make a time related judgement. We also favored time production to time reproduction as the latter taxes working memory to process and store information on the time interval to reproduce (Baudouin et al. (2006)). We finally designed a novel task that discouraged chronometric counting (Grondin and Killeen (2009), Hinton and Rao (2004), Wearden (2003)) to better capture intrinsic timing properties.

Second, to be able to focus on how participants adjust their time targets to respond to incentives, it was critical to design a task in which we varied the structure of the rewards and paid participants as a function of their performance. This design departs
from traditional experiments on subjective time. Indeed, with a few exceptions (Wearden and Grindrod (2003)), participants are usually asked to produce accurate reports and are compensated with a fixed payment for their effort. Incentivized payments are, however, desirable in decision-making studies because the objective is to trigger cognitive processes that govern real-life choices. Concretely, we included three tasks, each featuring specific time-related incentives. In our baseline task, participants completed a time production task in the range of 31 to 41 seconds, and they were rewarded for accuracy. Formally, they earned maximal payoff ($20) when they correctly estimated the announced time and their payoffs was reduced symmetrically as their estimates were farther away from the announced time in either direction. We then asked participants to complete two time production tasks in which the reward structure was altered. In one of them, the announced time was a hard deadline: earnings increased as the estimate came close to the announced time and vanished afterwards. This reward scheme incentivized participants to avoid missing the deadline and to report lower estimates compared to the baseline task. In the other one, the announced time was a release time: earnings were nil for estimates below the release time, maximal at the release time and decreased afterwards. This scheme incentivized participants to wait past the announced time and to report higher estimates compared to the baseline.

Third, to assess whether participants were aware of their intrinsic timing attitudes, we needed to elicit their beliefs. There is a variety of methods of belief elicitations and recent studies in economics show that elicited beliefs are often meaningful and consistent with observed behavior in the laboratory (Schotter and Trevino (2014)). It is also often argued that the elicitation method should be incentivized to make sure that reported beliefs are accurate (Wright and Aboul-Ezz (1988), Gächter and Renner (2010)). Furthermore, the experimenter should minimize the chances that the procedure itself affected the behavior under study. In particular, we were not interested in feedback effects, which are known to affect timing behavior (Brown et al. (1995), Droit-Volet and Izaute (2005), Franssen and Vandierendonck (2002)). For these reasons, we opted for an incentivized method and a procedure that did not interfere with the main tasks. At the end of the experiment, we asked our participants to make a self-assessment regarding their performance in the baseline task and we rewarded them for accuracy.

Given the growing interest in the role of emotions on time perception (Droit-Volet and Meck (2007), Droit-Volet and Wearden (2002), Fayolle et al. (2015)) and decision-making (Damasio (1994), Schwartz (2000), Phelps et al. (2014)), we were also interested in the effect of emotions on the interaction between the two. Emotion, however, is a complex and rich concept and we opted for an exploration of the effect of physiological stress on time-related incentives. To this purpose, half of our population completed the Cold Pressor
Task, a method that increases the participants’ cortisol levels and has been previously shown to affect decision-making in other contexts (Porcelli and Delgado (2009), Lighthall et al. (2012)).

We derived hypotheses for a rational decision-maker who maximizes rewards, and fit those predictions to the data. We used time reports in the baseline task to assess the presence of a bias, for each individual and in the overall population. By comparing the responses between the baseline and each of the two other tasks, we assessed whether participants responded to incentives conditional on their biases. Finally, we used elicited beliefs to evaluate self-awareness and to further determine if behavior was contingent on beliefs.

2 Methods

2.1 Design and procedures

The experiment was conducted in the Los Angeles Behavioral Economics Laboratory (LA-BEL) at the University of Southern California.\(^1\) A total of 170 individuals (79 Male and 91 Female) participated in the study in 21 groups of 6 to 10 participants each. Among those, 83 individuals (36 Male and 47 Female) participated in a “Stress” session and underwent the Socially Evaluated Cold Pressure Task (CPT). Subjects completed a survey before entering the laboratory to ensure all criteria for cortisol sampling were satisfied. Sessions lasted for about 1h30min and all started at 3pm to control for the circadian variability in cortisol levels. Instructions were read out loud at the beginning of each task.

**Socially Evaluated Cold Pressure Task (CPT).** We induced stress by submerging the participant’s non-dominant hand in a bucket of ice water for 3 minutes. The procedure was coupled with a social pressure aspect. The level of physiological stress was assessed by collecting three saliva samples per participant (start, mid and end of experiment) in order to measure changes in cortisol levels. The details of the procedure are explained in Appendix A1.

**Time perception tasks.** The design was adapted from Brocas et al. (2016). Participants completed three tasks under three payoff variants. In each task, participants were asked to report 10 time intervals \( t \) of 31, 32, 33, 34, 35, 36, 37, 38, 39 and 41 seconds respectively, without knowing in advance the number or length of intervals to report.\(^2\) We designed a

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\(^1\)For information about the lab, please visit http://dornsife.usc.edu/label.

\(^2\)This is called a *prospective* time estimate in a *production* paradigm. Prospective (as opposed to retrospective) refers to a case where participants know in advance that they will be requested to estimate the elapsed time. Production occurs when participants are informed about the length of the interval they
Matlab-based program to implement the elicitation of the participants’ time perception. It presented the instructions on the screen and guided them to estimate time intervals. Participants were prompted the length of the interval \( t \) to be estimated. Then, participants marked the beginning and end of the interval by clicking on a button on the top right corner of the screen. The order for the 10 intervals was randomly selected but it was the same for all participants. At the beginning of the session, an experimenter explicitly asked participants to put away any time-keeping devices (watches, music players, cell phones, etc.) and made sure they complied. We used computer mice that produced no sound when clicked to ensure that participants could not use auditory cues to infer the behavior of others.

To make sure that participants did not count time, we asked them to solve a concurrent interfering task while estimating time intervals. In this task, they were sequentially presented a series of \( 4 \times 6 \) tables where each row and column had a name, and they were instructed to click on a specific cell. For example, in one table participants were asked: “Please click the cell where the column to the right of the column labeled athena intersects the row above the biology row.” The names of the rows and columns as well as the phrasing of which cell to click on changed from table to table, to make sure participants would pay attention to the interfering task and therefore could not count at the same time. There was a random and unspecified time limit to complete each task (between 10 and 15 seconds) and failure to complete it counted as an incorrect answer. Overall, we designed the interfering task in a way that it required sufficient effort to prevent participants from counting but was easy enough to make sure all participants could successfully complete it if they put attention. In Appendix A2 we provide a screenshot of one such table.\(^3\)

For each of the three tasks, the amount earned was a function of (1) the distance between the announced and reported intervals, and (2) the proportion of correct answers in the interfering task. More precisely, for each participant, one time interval of each task was randomly selected for payment. For that time interval, the participant earned money only if at least 75% of the interfering tasks were correctly answered.\(^4\)

In the “baseline” task, hereafter \( B \), participants earned $20 if the report coincided exactly with the announced interval \( t \). Participants lost $0.5 for each 1% that their report was above or below the announced interval. In the “under-report” task, hereafter \( U \),

\(^3\)The common task used in the interval timing literature consists in asking participants to repeat aloud digits presented on a computer screen (Wearden et al. (1997)). Given we organized sessions with several participants, it was not possible to use such method.

\(^4\)In 83% of the intervals (vs. 84% in Brocas et al. (2016)), participants answered correctly at least 75% of the interfering tasks and therefore were eligible for payment.
participants lost $0.5 for each 1% that their report was below the announced interval and they lost everything if the report was above the announced interval. Last, in the “over-report” task, hereafter O, participants lost $0.5 for each 1% that their report was above the announced interval and they lost everything if the report was below the announced interval. The three tasks are summarized in Fig.1. Task B was always performed first. The order of tasks U and O was randomized across sessions. The entire procedure was explained beforehand. While participants could potentially obtain as much as $60, earnings based on the three tasks averaged $16.9.

![Figure 1: Payments.](image)

**Figure 1:** Payments. In each task, the experimenter announced a time interval \( t \) and participants were paid according to the red function. In all tasks, payment was maximized ($20) when the report was exactly \( t \). Participants obtained positive payments for reports between 0.6 and 1.4 in task B (left), for reports between 0.6 and \( t \) in task U (center), and for reports between \( t \) and 1.4 in task O (right).

**Beliefs.** Participants were asked to assess their performance in task B by estimating how many of their reported intervals were below the announced intervals: 0 to 2, 3 to 7, or 8 to 10. This question was incentivized with $1 payment for a correct answer, and asked after tasks B, U and O were completed.

**Survey.** Finally, we conducted a survey also at the end of the session to collect demographic information such as gender, GPA and primary language spoken.

### 2.2 Theory and predictions

We start with a theoretical exercise to assess how participants who believe that their subjective perception may be noisy and biased should behave in our tasks. This exercise allows us to make testable predictions and to identify in our data whether behavior is driven by the existence of a bias that is not corrected, mistakes in decision-making or both.
**Model.** Consider participant $i$ who targets time interval $\tau$ and suppose that his perception $p_i^*(\tau)$ is biased by an amount $b^i$. For simplicity, let us assume it takes the linear form:\(^5\)

$$p_i^*(\tau) = \tau + b^i \tag{1}$$

To formalize the idea that a participant does not know his true bias and is therefore unable to correct for it, we model $b^i$ as a random variable with p.d.f. $g(b^i)$ and c.d.f. $G(b^i)$. Participant $i$ knows only that his own bias is drawn from the distribution of the bias in the population. Additionally, when participant $i$ wants to report his perception of an interval $\tau$, he does so noisily. Formally:

$$r_i = p_i^*(\tau) + \varepsilon_i \Leftrightarrow r_i = \tau + b^i + \varepsilon_i \text{ where } \varepsilon_i \text{ i.i.d. } N(0, \eta_i^2) \tag{2}$$

Overall, the difference between the report $r_i$ and the interval $\tau$ that the participant targets is due both to the perception bias $b^i$ and the individual noise $\varepsilon_i$ (with $\eta_i^2$ capturing the variance of that noise).

**Assumptions.** A desirable property of subjective time is mean accuracy, or the idea that reported time is on average unbiased with respect to the target ($E[r^i] = \tau$). We capture this property by assuming no expected bias in the population ($E[b^i] = 0$). Reported time is also usually modeled as being drawn from a normal distribution. We impose weaker assumptions, namely (i) a symmetric distribution of bias ($g(b^i) = g(-b^i)$) which implies that reports err symmetrically around the target ($Pr[r^i = \tau + z] = Pr[r^i = \tau - z]$) and (ii) log-concavity of the distribution of the random variable $a^i = b^i + \varepsilon_i$.\(^6\)

**Target.** When an interval $t$ is announced in task $k \in \{B, U, O\}$, participant $i$ chooses a target $\theta^*_k(t)$. Given the structure of payoffs, one could think that targeting $t$ is optimal since it results in maximum payoff. However, the participant is aware that his perception may be biased (1) and that his estimation is noisy (2). These affect his incentives to target $t$. In Appendix A3, we show that the optimal target $\theta^*_k(t)$ in task $k$ is:\(^7\)

$$\theta_B^*(t) = t; \quad \theta_U^*(t) < t; \quad \theta_O^*(t) > t \quad \text{with} \quad \theta_B^*(t) - \theta_U^*(t) = \theta_O^*(t) - \theta_B^*(t) \tag{3}$$

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\(^5\)We adopt an additive bias for technical simplicity. In our design, the specific functional form of $p^*(\tau)$ (additive, multiplicative, logarithmic) is not crucial since we have little variation in the announced intervals (between 31s and 41s). Ideally, we wanted to have multiple measures of only one interval but finally decided to have some small variation to add interest to the task. For an estimation of more general time perception functions over longer intervals (24s to 196s) we refer to Brocas et al. (2016).

\(^6\)Formally, let $f(a^i)$ be the p.d.f. of the random variable $a^i = b^i + \varepsilon_i$. Log concavity requires $\left(\frac{f'(a)}{f(a)}\right)' < 0$. It is a technical condition satisfied by many common distributions, including the normal distribution.

\(^7\)Trivially, as the variance of the bias and noise go to zero (e.g., because we allow time keeping devices), we get $\theta_U^*(t) \rightarrow t^-$ and $\theta_O^*(t) \rightarrow t^+$. 

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6
When payoffs are symmetric (Fig. 1, left), a participant who believes to be unbiased on average should target \( t \). This is not true anymore when payoffs are asymmetric. In task \( U \), participants have incentives to decrease their target because over-reporting \((r^i > t)\) is significantly more costly than under-reporting \((r^i < t)\). A lower target ensures that the final report remains within the range of positive rewards, that is, below \( t \) (Fig. 1, center).

Using an analogous argument, participants have incentives to increase their target in task \( O \) (Fig. 1, right). By the symmetry of the problem, the incentives to decrease the target in \( U \) and to increase the target in \( O \) are identical. This implies that targets in \( U \) and \( O \) are symmetric with respect to the announced time \( t \). Overall, the final report in task \( k \) and given the optimal target is:

\[
\begin{align*}
    r^i_B(t) &= \theta^*_B(t) + b^i + \varepsilon^i_B; \\
    r^i_U(t) &= \theta^*_U(t) + b^i + \varepsilon^i_U; \\
    r^i_O(t) &= \theta^*_O(t) + b^i + \varepsilon^i_O
\end{align*}
\]  

(4)

**Shading.** We call shading the amount by which targets, and henceforth reports, are decreased in task \( U \) and increased in task \( O \) with respect to task \( B \). For future reference, the shadings of participant \( i \) in tasks \( U \) and \( O \) respectively are denoted by \( h^i_U \) and \( h^i_O \):

\[
\begin{align*}
    h^i_U &= r^i_B - r^i_U \\
    h^i_O &= r^i_O - r^i_B
\end{align*}
\]

Using (4) it is immediate that \( h^i_O \sim N(\theta^*_O - \theta^*_U, 2\eta^2_i) \) and \( h^i_U \sim N(\theta^*_B - \theta^*_U, 2\eta^2_i) \). This means that while the report in each task crucially depends on the bias of the participant, shading across tasks does not. The reason is simply that the bias is unknown but constant across tasks. If the participant acts optimally, expected shadings \( h^*_{i_U} \) and \( h^*_{i_O} \) must satisfy:

\[
\begin{align*}
    h^*_{i_U} &= \theta^*_B - \theta^*_U \\
    h^*_{i_O} &= \theta^*_O - \theta^*_B
\end{align*}
\]

**Predictions.** The theory offers a number of testable predictions. In each task and in each trial, we observe the announced time interval \( t \) as well as the report \( r^i_k \) made by each participant. We will use that information to estimate the individual biases \( b^i \) and the individual shadings \( h^i_U \) and \( h^i_O \). Let \( I_k(r^i_k) \) be the empirical c.d.f. of reports in task \( k \). Also, let \( H_U(h^i_U) \) and \( H_O(h^i_O) \) be the empirical c.d.f. of shading in \( U \) and \( O \). We will test three main predictions of the model:

**Prediction 1.** There is no bias in the population in task \( B \): \( E[b^i] = 0 \).

**Prediction 2.** Participants shade downwards in task \( U \) and upwards in task \( O \). Formally, the distributions of reports across tasks satisfy first-order stochastic dominance: \( I_O(r) \leq I_B(r) \leq I_U(r) \) for all \( r \).

**Prediction 3.** Participants shade the same amount in \( U \) and \( O \). Formally, the distribution of shading between \( B \) and \( U \) is the same as between \( O \) and \( B \): \( H_U(h) = H_O(h) \) for all \( h \).
Notice that these predictions hold generically, that is, without having to specify a functional form for the distribution of biases in the population, $G(b^i)$.

**Calibration.** We can also obtain point predictions of the unobservable (optimal) targets in tasks $U$ and $O$ if we assume that the bias in the population is normally distributed, $b^i \sim N(0, \hat{\sigma}^2)$, where $\hat{\sigma}^2$ is the empirical variance of the bias of our participants in task $B$. By computing these optimal targets, we can further test the theory and determine whether the expected reports are in line with the targets. Details of the method are provided in Appendix A4.

### 3 Results

Consistent with previous studies, we found a 107% increase in cortisol level between the 1st and 2nd saliva samples among participants who completed the CPT compared to a 4% increase among those who did not (t-test, p-value < 0.001). Self-reported stress level was not significantly different between CPT and non-CPT participants (5.83 vs. 5.97, t-test p-value = 0.66), indicating that the stressor acted at the physiological level but not at the psychological level.

**Behavioral biases and shading.** For each participant, we estimated the bias in task $B$ across all 10 intervals. Note that the best estimate of the bias is simply the empirical mean of the difference between the observed report and the announced time. We found that, on average, participants overestimated time ($E[b^i] = 6.86$, p-value < 0.001). The density distribution of the bias in the population was symmetric around the mean (skewness = 0.48, median bias = 6.05). Out of our 170 participants (and using a 5% significance level), 88 had a significant positive bias, 22 had a significant negative bias and 58 had no significant bias in either direction. This result indicated that behavior was not consistent with *Prediction 1*. The distributions of biases were not significantly different between CPT and non-CPT participants (t-test for comparison of means (p-value = 0.67) or Kolmogorov-Smirnov test for comparison of distributions (p-value = 0.65)). The only noticeable treatment effect was that the variance of the distribution of biases was significantly higher under CPT (F test, p-value = 0.005). However, this was exclusively due to the fact that 7 CPT participants had extreme biases, outside the range of the non-CPT participants.

The data also revealed that the incentives to report in both tasks were well understood: consistent with *Prediction 2* and *Prediction 3*, participants shaded downwards in $U$ ($h_U^i \equiv r_B^i - r_U^i > 0$), upwards in $O$ ($h_O^i \equiv r_O^i - r_B^i > 0$) and by a similar amount in both cases ($h_U^i \simeq h_O^i$). The distribution of individual reports in $O$ first-order stochastically
dominated the distribution of reports in $B$, which itself first-order stochastically dominated the distribution of reports in $U$ (Fig.2, left: $I_O(r^i) \leq I_B(r^i) \leq I_U(r^i)$, Wilcoxon rank sum test, p-value < 0.001 for all pairwise comparisons). The average shading in tasks $U$ and $O$ was significantly above 0 ($E[h_U^i] = 4.62$ p-value < 0.001 and $E[h_O^i] = 6.55$ p-value < 0.001) and not significantly different from each other (p-value = 0.181). Finally, the distributions of shading were also comparable in both cases (Fig.2, right: $H_U(h) = H_O(h)$, Wilcoxon rank sum test, p-value = 0.07).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{CDF_of_reports_across_tasks.png}
\caption{Distributions of reports (left) and shading (right). Participants’ reports were significantly smaller in $U$ and significantly higher in $O$ compared to $B$ ($I_O(r) \leq I_B(r) \leq I_U(r)$). Shading was similar in magnitude in $U$ and $O$ ($H_U(h) = H_O(h)$).}
\end{figure}

Participants earned on average $8.4$ in $B$, $4.7$ in $U$ and $3.9$ in $O$ (Fig.3). The distributions of earnings was significantly smaller in $O$ than in $U$ (Wilcoxon signed rank test, p-value = 0.028). This was mostly due to a higher standard deviation in reports in task $O$ than in task $U$, and therefore a higher likelihood of falling outside the interval of positive rewards ($\hat{\sigma}_O = 12.34$ and $\hat{\sigma}_U = 10.57$, F-test for comparison of variance, p-value = 0.044; Fig.3, center and right). All the results were identical in the CPT and non-CPT populations. Combining the findings obtained so far, we concluded that although reported intervals were excessively high (resulting in a positive expected bias in the population), marginal changes in reports across tasks closely followed the predictions of theory at the aggregate level.

Participants with a smaller absolute bias were also less volatile (Pearson correlation coefficient between $|b^i|$ and $\eta_B^i$ was 0.43, p-value < 0.001) suggesting that, at the individual

\footnote{Naturally, earnings were highest in task $B$, but these are not comparable since the interval with positive rewards was twice as large in $B$ as in $U$ or $O$ (Fig.3). Standard deviations between $B$ and the other tasks are not comparable either, since task $B$ is always performed first.}
level, the ability to produce accurate intervals on average went hand in hand with the ability to not deviate too much from them. The relationship between bias and volatility was not modeled by our theory. It was however consistent with previous research (Brocas et al., 2016).

Last, we conducted a calibration exercise to obtain point estimates of optimal targets (see Appendix A4). Given the estimates of the variance of the sample ($\hat{\sigma}^2$) and the individual noise ($\hat{\eta}_i^2$), we found that, on average, participants should target $\theta^*_U = 30.88$ and $\theta^*_O = 40.32$. The empirical average reports were significantly higher than these optimal targets: 37.84 in $U$ and 49.02 in $O$. However, they were remarkably consistent if we account for the sample's positive bias. Indeed, given the bias estimated in task $B$, participants would be expected to report on average $\theta^*_U + E[b_i] = 37.74$ in $U$ and $\theta^*_O + E[b_i] = 47.18$ in $O$, which were not significantly different from the empirical average reports (t-test, p-value = 0.906 and p-value = 0.055). This calibration exercise confirmed that, on average, participants were acting optimally in tasks $U$ and $O$ given their (fixed) bias.

**Figure 3:** Distribution of earnings in $B$ (left), $U$ (center) and $O$ (right). The curves represent the p.d.f. of reports in each task. The dark area represents the region in which participants earned money in each task: $r^i_B \in [0.6,1.4]t$ (task $B$), $r^i_U \in [0.6,t]t$ (task $U$) and $r^i_O \in [t,1.4]t$ (task $O$).

**Heterogeneity.** Although shading followed the basic theoretical predictions at the aggregate level (downwards in $U$ and upwards in $O$), there was substantial heterogeneity across individuals. Indeed, only 96 participants shaded in the correct direction in both tasks. Among the rest, 37 shaded downwards in both cases, 36 shaded upwards in both cases and 1 shaded upwards in $U$ and downwards in $O$. When comparing the bias in task $B$ of these participants, we found that participants who shaded consistently with theory had a bias of 6.3 on average. Those who systematically decreased their report had a significantly higher bias (13.2 on average) while those who systematically increased their report had a significantly lower bias (1.9 on average). This finding is suggestive of a link
at the individual level between behavior in task $B$ and correction in the subsequent tasks.

**Bias awareness and correction.** Motivated by the heterogeneity in shading across individuals, we studied the relationship between behavior and self-assessed biases. If they were fully rational, every participant with a known bias should take the bias into account and end up producing accurate reports on average. Our analysis indicated that this was not the case, as we observed significant over-reporting. However, assuming that participants were not sophisticated enough to de-bias their behavior, were their beliefs consistent with their behavior ("I think I over-estimated and, indeed, I did over-estimate")? Also, did they act according to those beliefs in tasks $U$ and $O$ ("I think I over-estimated and therefore I adjusted my shading to it")?

We found that 45 participants (26%) perceived themselves as "Under-estimators," 78 participants (46%) perceived themselves as "Unbiased," and 47 participants (28%) perceived themselves as "Over-estimators." Although, as we already know, the majority of participants reported excessively high time intervals, they had on average correct beliefs about their biases in $B$ relative to the population: the distribution of biases among Over-estimators first-order stochastically dominated the distribution of Unbiased and Under-estimators (Wilcoxon rank sum test to compare Over-estimators with Unbiased and Under-estimators respectively, p-value = 0.021 and 0.016). Under-estimators had lower biases compared to Unbiased participants, although differences in the distributions were not statistically significant (Fig.4, left).

Participants also acted correctly upon their beliefs in tasks $U$ and $O$. In task $U$, Under-estimators shaded downwards less than Unbiased and Unbiased shaded downwards less than Over-estimators (Fig.4, center). Symmetrically, in task $O$ Under-estimators shaded upwards more than Unbiased and Unbiased shaded upwards more than Over-estimators (Fig.4, right). These differences were statistically significant, except for the difference between Unbiased and Under-estimators. Overall, while shading in $U$ and $O$ were of equal magnitude on aggregate, once we conditioned on the self-assessed bias of the participants they were not. Consistent with their beliefs, Over-estimators shaded significantly more in $U$ than in $O$ (Wilcoxon signed rank test, p-value = 0.042) and Under-estimators shaded significantly more in $O$ than in $U$ (Wilcoxon signed rank test, p-value = 0.005). Unbiased participants also shaded more in $O$ than in $U$ (Wilcoxon signed rank test, p-value = 0.041).

9 More precisely, "Under-estimators", "Unbiased" and "Over-estimators" are participants who reported in the post-tasks questionnaire that, according to their own estimates, 8 to 10, 3 to 7 and 0 to 2 of their reports in $B$ were below the announced interval, respectively.

10 In most cases, the differences were significant or close to significant after running a boxplot analysis and removing extreme outliers.
Figure 4: Distributions of biases in $B$ (left) and distributions of shading in $U$ (center) and $O$ (right) as a function of self-perceived beliefs. Biases in $B$ are statistically different across beliefs and consistent with them: Under-estimators are less biased than Unbiased who are less biased than Over-estimators. In $U$ and $O$, participants shade as a function of their belief.

The corrective behavior implied that while average reports were significantly different in task $B$ between Over-estimators and the other two groups (t-test, p-value = 0.009 and 0.012), they were not different between any two groups in tasks $U$ or $O$ (t-test, all p-values > 0.4), see Fig.5 (left). To sum up, our results indicated that participants had correct beliefs about their time perception bias relative to others in the population. Even though they were not able to anticipate their bias in task $B$, they correctly acted upon it in tasks $U$ and $O$. There was again no effect of the CPT procedure on the distribution of self-assessed biases or on the behavior of participants within groups.

**Behavioral adaptation model.** If participants were at least partially aware of their biases, why were they not correcting better for them in tasks $U$ and $O$? A possible explanation is that, even though they knew their tendency to misrepresent time in a given direction, they were unsure of the magnitude of their bias. Recall that an unbiased participant $i$ should target $t$ in task $B$, $\theta_{U}^{i*}(t)$ in task $U$ and $\theta_{O}^{i*}(t)$ in task $O$. Recall also that the optimal shading should be symmetric around $t$: $t - \theta_{U}^{i*}(t) = \theta_{O}^{i*}(t) - t$. Suppose that participant $i$ targeted the symmetric (but possibly suboptimal) intervals $t - \Delta^i$ and $t + \Delta^i$ in tasks $U$ and $O$, and that he believed he targeted $t + \bar{b}^i$ instead of $t$ in task $B$. In his mind:

$$t + \bar{b}^i - h_U^i = t - \Delta^i \quad \text{and} \quad t + \bar{b}^i + h_O^i = t + \Delta^i$$

Combining both equations, we could retrieve $\Delta^i$ (the target shading around the announced
time) and $\tilde{b}_i$ (the perceived bias):

$$\Delta^i = \frac{h^i_U + h^i_O}{2} \quad \text{and} \quad \tilde{b}_i = \frac{h^i_U - h^i_O}{2}$$

In words, a participant with a perceived positive bias ($\tilde{b}_i > 0$), an Over-estimator, would shade downwards in $U$ more than shade upwards in $O$ ($h^i_U > h^i_O$). The opposite would be true for a participant with a perceived negative bias, an Under-estimator.

We used this behavioral model to estimate the perceived bias of each individual based on the observed shadings. We found that the estimated perceived biases of the participants were consistent with their beliefs: -3.38 for Under-estimators (lowest and significantly negative, p-value = 0.008), -1.51 for Unbiased (not significantly different from zero, p-value = 0.053), and 2.94 for Over-estimators (highest and significantly positive, p-value = 0.045). Furthermore, participants acted as if they targeted on average $\Delta = 5.67$, with no significant differences across groups (mean between 5.4 and 5.9, t-tests all p-values > 0.4). This number was in the order of magnitude of our calibrated optimal shading $t - \theta^*_U(t) = \theta^*_O(t) - t \equiv 4.72$. Overall, even though we observed reports as represented in Fig.5 (left), participants acted as if they believed that their behavior was consistent with the perceived reports represented in Fig.5 (right). This behavioral model provided an as if representation of behavior that was both consistent with self-assessed biases and empirically observed biases.

![Figure 5](image)

**Figure 5:** Average reports (left) and average perceived reports (right) of self-perceived Under-estimators, Unbiased and Over-estimators. Reports across groups are different in task $B$, but beliefs are correct and participants shade according to them, so reports across groups are not statistically different in tasks $U$ and $O$.

**Demographics.** We did not find any effect of gender, gpa or language or any interaction between these and stress. In particular, stress equally affected the behavior in men and women.
It has been proposed that perceived time synchronizes with the ticking of an internal clock (Treisman (1963)), an idea mathematically represented by the scalar timing model (Gibbon (1977), Gibbon and Allan (1984)). This internal clock consists of a pacemaker that emits pulses (ticking) stored in an accumulator. Accuracy is obtained when the pacemaker is perfectly synchronized with objective time, and distortions occur when it speeds up or slows down. In our framework, when a time interval is announced, the participant chooses a target as a function of the reward (decision stage), and this target is implemented subject to noise and biases (time production stage). This time production stage corresponds to what an accumulator model would focus on. Instead, our main focus is on the decision stage.

In terms of experimental findings, the results obtained in our baseline task were comparable to previous studies of production of similar time intervals (Matell and Meck (2000), Wittmann et al. (2007)). Instead of asking directly participants to be as accurate as possible, we rewarded them for accuracy. As expected, this variant in design did not produce inconsistent results. On average, mean accuracy was violated as participants tended to overestimate time. However, the distributions of reports were symmetric, and the scalar property of interval timing was satisfied (see Appendix A5 for details). The observed tendency to overestimate was also consistent with findings in Brocas et al. (2016), where we used the same experimental procedure. In that study, we observed a form of Vierordt’s law over the second to minute range and intervals around 30s were, on average, over-estimated.

A key result of our analysis is that biases in time perception carry over across different incentives schemes while decision-making is optimal conditional on biases. This result has two implications. First, it suggests that time-keeping mechanisms are dissociated from decision-making mechanisms. Reports that do not agree with theoretical predictions in $U$ and $O$ result from errors in perception, not from the inability to determine the best course of action. An intuitive explanation lies on brain modularity (Bullmore and Sporns (2009), Chen et al. (2008)). Even though a large degree of overlap is observed in experimental paradigms, different areas and networks in the brain are specialized for different functions. Time perception has been found to involve specific brain networks in specific tasks (Grondin (2010), Penney and Vaitlilingam (2008)) and different aspects of decision-making also involve dedicated brain regions (Brocas and Carrillo (2014)). It is plausible that computations to determine targets are implemented first by a “decision” system and sent for implementation to the “time-perception” system. Second, optimality of decision-making indicates that the decision system is tuned to optimize behavior. These
two implications together provide support for modeling the brain as an organization of systems acting optimally within their range of action (Brocas and Carrillo (2008)).

Another key result is that participants had correct beliefs about their choices in B and took correct decisions given those beliefs in tasks U and O (even though beliefs were elicited after completing all three tasks). This suggests that awareness of their own time distortions determined how they made decisions regarding time estimates. However, this knowledge was imperfect. Indeed, participants were aware of their biases but (i) they were not able to anticipate them and correct for them in task B, and (ii) they missed the magnitude of the bias to fully adjust their behavior in tasks U and O. Decisions in the latter tasks suggest that participants possessed at that point specific knowledge about their performance in the previous task B, knowledge that helped them find their best course of action in subsequent tasks. Even in the absence of feedback, participants were able to access introspectively their experience of time and to measure it accurately.

The fact that biases were not anticipated beforehand, and not entirely corrected for afterwards is intriguing. It indicates in particular a limitation of metacognition. It is however consistent with results obtained in the context of chronic lateness in the workplace. It has been suggested that people are late because they fail to accurately plan how long a task will take (Kahneman and Tversky (1979), Buehler et al. (1994)). Even though the problem recurs, the same mistakes are made repeatedly. When targeting a time to act, a person may be aware of her tendency to be late but unable to anticipate it fully. In terms of the model previously suggested, the decision system acts upon information that accurately describes the direction of the bias but inaccurately its magnitude. Behavior is optimal conditional on the subjective bias but suboptimal conditional on the objective bias. This means that an individual who is typically late for appointments will correct his behavior when the meeting is particularly important. However, he will still be more likely to miss it than an individual who is rarely late.

Last, we expected that stress would affect reports. Indeed, recent research on time perception has shown that time perception is subject to triggers that affect dopamine release (Ivry and Spencer (2004), Meck (1996), Meck (2005)). It has also been shown that emotions shape our perception of time (Droit-Volet and Meck (2007)), in conjunction with alterations of dopamine concentrations (Meck (1983), Drevets et al. (2001)). A recent study showed that, in the case of psychological stress (induced with a mental arithmetic task), an increase in cortisol was correlated with an increase in DA release (Pruessner et al. (2004)). Anticipating that the CPT would generate a significant increase in cortisol levels, we were therefore expecting an (unobservable) effect on dopamine release generating an (observable) increase in time perception. However, although cortisol levels were significantly increased, time estimation was not significantly different between
the two populations. This absence of effect paralleled an absence of differences in self-assessments of stress levels. These unexpected findings suggest that physiological stress operates differently from psychological stress on time keeping mechanisms. They also indicate that changes in cortisol do not necessarily trigger changes in dopamine concentration. A possible interpretation is that different types of stress may affect information processing differentially, as suggested in Kogler et al. (2015).
References


Appendix

Appendix A1. Physiological stress and the Cold Pressure Task (CPT).

The Cold Pressure Task (CPT) is a well-established method to induce physical stress and has been safely used on adults and children since the 1930s (Hines and Brown, 1936). This task instructs participants to submerge their non-dominant hand in a bucket of ice water for a certain period of time (in our experiment, 3 minutes). To ensure that CPT was a salient stressor, we used a slight variant called the Socially Evaluated Cold Pressor Task (Schwabe, et al. 2008). It is the original CPT coupled with a social pressure aspect, where participants are instructed to look directly into a camera that will be video recording their facial expression during the hand submersion process. All our participants complied with the task.

The level of physiological stress was assessed by collecting three saliva samples per participant (start, peak and end of experiment) in order to measure changes in cortisol levels. We measured cortisol levels of all 170 participants (CPT and non-CPT) using the “passive drool” method. Although more demanding on participants, this is the safest method in biological testing, as it provides the purest sample possible and allows researchers to biobank samples for future testing (Granger et al., 2012). As standard in this type of experiments, participants were instructed not to use lip products for the entire day, drink anything other than water, eat, smoke, exercise, have caffeine, or chew gum for 60 minutes prior to the beginning of the experiment nor sleep for 120 minutes prior to the beginning of the experiment. Participants were asked at the beginning of the experiment whether they had complied with those instructions. Participants who had not complied were not allowed to participate. The samples were ice packed and sent for analysis to our partner lab ZRT laboratory (see http://zrtlab.com for details). Results were sent back within one week.

Appendix A2. Interfering task.

![Figure 6:](image)

**Figure 6:** In this example, the instructions read: “Please click the cell where the column to the right of the column labeled athena intersects the row above the biology row.” The correct answer required a click on the cell where the 1st row and 5th column intersect.

Appendix A3. Theory.
The payment $\Pi_k(r^i_k)$ for participant $i$ in task $k$ as a function of the report $r^i_k$ is:

$$
\Pi_B(r^i_B) = \begin{cases} 
\max \left\{ 0, \frac{r^i_B - (1-\alpha)t}{\alpha t} \right\} & \text{if } r^i_B \leq t \\
\max \left\{ 0, \frac{(1+\alpha)t - r^i_B}{\alpha t} \right\} & \text{if } r^i_B \geq t 
\end{cases}
$$

$$
\Pi_U(r^i_U) = \begin{cases} 
\max \left\{ 0, \frac{r^i_U - (1-\alpha)t}{\alpha t} \right\} & \text{if } r^i_U \leq t \\
0 & \text{if } r^i_U \geq t 
\end{cases}
$$

$$
\Pi_O(r^i_O) = \begin{cases} 
\max \left\{ 0, \frac{(1+\alpha)t - r^i_O}{\alpha t} \right\} & \text{if } r^i_O \geq t 
\end{cases}
$$

as graphically depicted in Fig.1. In the experiment, $G = $20 and $\alpha = 0.4$.

**Proposition 1** (i) $\theta^*_B(t) = t$; (ii) $\theta^*_U(t) \in ((1-\alpha)t, t)$; (iii) $\theta^*_O(t) \in (t, (1+\alpha)t)$; and (iv) $\theta^*_B(t) - \theta^*_U(t) = \theta^*_O(t) - \theta^*_B(t)$.

**Proof.** For expositional convenience we drop the announced interval $t$ from the target function $\theta_k$. We also drop the participant’s superscript $i$. Let $a \equiv b + \varepsilon$. Given a target $\theta_k$ in task $k$, we know from (4) that the report is:

$$
r_k = \theta_k + a \quad \text{where} \quad a \sim F(a)
$$

(i) **Baseline ($B$).** In task $B$, the expected payoff of the participant, $V_B$, is:

$$
V_B = \frac{G}{\alpha t} \left[ \int_{a=(1-\alpha)t-\theta_B}^{t-\theta_B} \left( a + \theta_B - (1-\alpha)t \right) f(a) da + \int_{a=t-\theta_B}^{(1+\alpha)t-\theta_B} \left( (1+\alpha)t - (a + \theta_B) \right) f(a) da \right]
$$

$$
\frac{\alpha t}{G} V_B = \left( (a + \theta_B) - (1-\alpha)t \right) F(a) \bigg|_{(1-\alpha)t-\theta_B}^{t-\theta_B} - \int_{(1-\alpha)t-\theta_B}^{t-\theta_B} F(a) da + \left( (1+\alpha)t - (a + \theta_B) \right) F(a) \bigg|_{t-\theta_B}^{(1+\alpha)t-\theta_B} + \int_{t-\theta_B}^{(1+\alpha)t-\theta_B} F(a) da
$$

$$
= \int_{t-\theta_B}^{(1+\alpha)t-\theta_B} F(a) da - \int_{(1-\alpha)t-\theta_B}^{t-\theta_B} F(a) da
$$

Optimizing over the target $\theta_B$, we can write the first-order condition as:

$$
\frac{\partial V_B}{\partial \theta_B} \bigg|_{\theta_B^*} = 0 \iff \left[ F(t - \theta_B^*) - F((1-\alpha)t - \theta_B^*) \right] - \left[ F((1+\alpha)t - \theta_B^*) - F(t - \theta_B^*) \right] = 0
$$

$$
\iff \int_{(1-\alpha)t-\theta_B^*}^{t-\theta_B^*} f(a) da = \int_{t-\theta_B^*}^{(1+\alpha)t-\theta_B^*} f(a) da \quad (5)
$$
The second-order condition can be written as:
\[
\frac{\alpha t}{G} \frac{\partial^2 V_B}{\partial (\theta_B)^2} = - \left[ f(t - \theta_B) - f((1 - \alpha)t - \theta_B) \right] + \left[ f((1 + \alpha)t - \theta_B) - f(t - \theta_B) \right]
\]
\[
= - \int_{(1-\alpha)t-\theta_B}^{t-\theta_B} f'(a)da + \int_{t-\theta_B}^{(1+\alpha)t-\theta_B} f'(a)da
\]

Log-concavity of \( f(a) \) implies that \( \frac{f'(a)}{f(t-\theta_B)} \leq \frac{f'(t-\theta_B^*)}{f(t-\theta_B^*)} \) for all \( a \geq t - \theta_B^* \). Therefore,
\[
\frac{\alpha t}{G} \frac{\partial^2 V_B}{\partial (\theta_B)^2} \bigg|_{\theta_B^*} < - \int_{(1-\alpha)t-\theta_B^*}^{t-\theta_B^*} f(a)\frac{f'(t - \theta_B^*)}{f(t - \theta_B^*)}da + \int_{t-\theta_B^*}^{(1+\alpha)t-\theta_B^*} f(a)\frac{f'(t - \theta_B^*)}{f(t - \theta_B^*)}da
\]
\[
< \frac{f'(t - \theta_B^*)}{f(t - \theta_B^*)} \left[ - \int_{(1-\alpha)t-\theta_B^*}^{t-\theta_B^*} f(a)da + \int_{t-\theta_B^*}^{(1+\alpha)t-\theta_B^*} f(a)da \right] = 0
\]
so \( \theta_B^*(t) \) is unique and a maximum. Finally, from (5) it is trivial to check that:
\( \theta_B^*(t) = t \)

since by the symmetry of \( f(a) \), we have:
\[
\int_{-\alpha t}^{0} f(a)da = \int_{0}^{\alpha t} f(a)da
\]

(ii) Under-report \( U \). In task \( U \), the expected payoff of the participant, \( V_U \), is:
\[
\frac{\alpha t}{G} V_U = \int_{a=(1-\alpha)t-\theta_U}^{t-\theta_U} \left( a + \theta_U - (1 - \alpha)t \right) f(a)da
\]
\[
= \left[ (a + \theta_U) - (1 - \alpha)t \right] F(a) \bigg|_{(1-\alpha)t-\theta_U}^{t-\theta_U} - \int_{(1-\alpha)t-\theta_U}^{t-\theta_U} F(a)da
\]
\[
= \alpha t F(t - \theta_U) - \int_{(1-\alpha)t-\theta_U}^{t-\theta_U} F(a)da
\]

Optimizing over the target \( \theta_U \), we can write the first-order condition as:
\[
\frac{\partial V_U}{\partial \theta_U} \bigg|_{\theta_U^*} = 0 \iff F\left( t - \theta_U^*(t) \right) - F\left( (1 - \alpha)t - \theta_U^*(t) \right) - \alpha t f\left( t - \theta_U^*(t) \right) = 0
\]
\[
\iff \int_{(1-\alpha)t-\theta_U^*}^{t-\theta_U^*} \left[ f(a) - f(t - \theta_U^*) \right]da = 0
\]

The second-order condition can be written as:
\[
\frac{\alpha t}{G} \frac{\partial^2 V_U}{\partial (\theta_U)^2} = -f(t - \theta_U) + f((1 - \alpha)t - \theta_U) + \alpha tf'(t - \theta_U)
\]
\[
= \int_{(1-\alpha)t-\theta_U}^{t-\theta_U} \left[ f'(t - \theta_U) - f'(a) \right]da
\]

Again, log-concavity of \( f(a) \) implies that
\[
\frac{f'(a)}{f(a)} > \frac{f'(t-\theta_\alpha)}{f(t-\theta_\alpha)}
\]
for all \( a < t - \theta_\alpha^* \). Therefore,
\[
\frac{\alpha t}{G} \left. \frac{\partial^2 V_U}{\partial (\theta_U)^2} \right|_{\theta_U^*} < \int_{(1-\alpha)t-\theta_\alpha^*}^{t-\theta_\alpha^*} \left[ f'(t-\theta_\alpha^*) - \frac{f'(t-\theta_\alpha^*)}{f(t-\theta_\alpha^*)} f(a) \right] da
\]
\[
< \frac{f'(t-\theta_\alpha^*)}{f(t-\theta_\alpha^*)} \int_{(1-\alpha)t-\theta_\alpha^*}^{t-\theta_\alpha^*} \left[ f(t-\theta_\alpha^*) - f(a) \right] da = 0
\]

Therefore \( \theta_\alpha^*(t) \) is unique and a maximum. Finally, from (6) it is trivial to check that:
\[
\theta_\alpha^*(t) \in ((1-\alpha)t, t)
\]
since:
\[
\frac{\alpha t}{G} \left. \frac{\partial V_U}{\partial \theta_U} \right|_{(1-\alpha)t} = \int_{0}^{\alpha t} [f(a) - f(\alpha t)] da > 0 \quad \text{and} \quad \frac{\alpha t}{G} \left. \frac{\partial V_U}{\partial \theta_U} \right|_{t} = \int_{-\alpha t}^{0} [f(a) - f(0)] da < 0
\]

(iii) **Over-report (O).** In task \( O \), the expected payoff of the participant, \( V_O \), is:
\[
\frac{\alpha t}{G} V_O = \int_{a=t-\theta_\alpha^*}^{(1+\alpha)t-\theta_\alpha^*} F(a) da - \alpha t F(t - \theta_\alpha^*)
\]

Optimizing over the target \( \theta_\alpha^* \), we can write the first-order condition as:
\[
\left. \frac{\partial V_O}{\partial \theta_\alpha^*} \right|_{\theta_\alpha^*(t)} = 0 \iff -F((1+\alpha)t - \theta_\alpha^*(t)) + F(t - \theta_\alpha^*(t)) + \alpha t f(t - \theta_\alpha^*(t)) = 0 \quad (7)
\]
\[
\iff \int_{t-\theta_\alpha^*}^{(1+\alpha)t-\theta_\alpha^*} [f(t-\theta_\alpha^*) - f(a)] da = 0
\]

The second-order condition can be written as:
\[
\frac{\alpha t}{G} \left. \frac{\partial^2 V_O}{\partial (\theta_\alpha^*)^2} \right|_{\theta_\alpha^*(t)} = f((1+\alpha)t - \theta_\alpha^*) - f(t - \theta_\alpha^*) - \alpha t f'(t - \theta_\alpha^*)
\]
\[
= \int_{t-\theta_\alpha^*}^{(1+\alpha)t-\theta_\alpha^*} [f'(a) - f'(t - \theta_\alpha^*)] da
\]

Log-concavity of \( f(a) \) implies that
\[
\frac{f'(a)}{f(a)} < \frac{f'(t-\theta_\alpha^*)}{f(t-\theta_\alpha^*)}
\]
for all \( a > t - \theta_\alpha^* \). Therefore,
\[
\frac{\alpha t}{G} \left. \frac{\partial^2 V_O}{\partial (\theta_\alpha^*)^2} \right|_{\theta_\alpha^*(t)} < \int_{t-\theta_\alpha^*}^{(1+\alpha)t-\theta_\alpha^*} \left[ \frac{f'(t-\theta_\alpha^*)}{f(t-\theta_\alpha^*)} f(a) - f'(t - \theta_\alpha^*) \right] da
\]
\[
< \frac{f'(t-\theta_\alpha^*)}{f(t-\theta_\alpha^*)} \int_{t-\theta_\alpha^*}^{(1+\alpha)t-\theta_\alpha^*} \left[ f(a) - f(t - \theta_\alpha^*) \right] da = 0
\]

Therefore \( \theta_\alpha^*(t) \) is unique and a maximum. Finally, from (7) it is trivial to check that:
\[
\theta_\alpha^*(t) \in \{t, (1+\alpha)t\}
\]
To calibrate the optimal targets in $U$ and $O$, we impose $b^i \sim N(0, \sigma^2)$. This means that:

$$a^i \equiv b^i + \varepsilon^i \sim N(0, z_i^2)$$

where $z_i^2 = \sigma^2 + \eta_i^2$

Notice that $E[b^i] = 0$, $g(b^i) = \frac{1}{2} \varphi\left(\frac{b^i}{\sigma}\right) = g(-b^i)$, and $f(a^i)$ is log-concave since the Normal distribution is log-concave, so the assumptions of the model are satisfied.

From Proposition 1(i) we know that $\theta_B^* = t$. Replacing $F(a)$ with $\Phi(\frac{a}{\sigma})$ in (6) and (7), we obtain that $\theta_U^*$ and $\theta_O^*$ solve the following first-order conditions:

$$\Phi\left(\frac{t - \theta_U^*}{z_i}\right) - \Phi\left(\frac{(1 - \alpha)t - \theta_U^*}{z_i}\right) - \frac{\alpha t}{z_i} \phi\left(\frac{t - \theta_U^*}{z_i}\right) = 0$$

(8)

$$-\Phi\left(\frac{(1 + \alpha)t - \theta_O^*}{z_i}\right) + \Phi\left(\frac{t - \theta_O^*}{z_i}\right) + \frac{\alpha t}{z_i} \phi\left(\frac{t - \theta_O^*}{z_i}\right) = 0$$

(9)

Form the data in task $B$ we can estimate $\tilde{\sigma}$, the standard deviation of the bias in the population. For each participant $i$ in task $B$, we can also estimate $\tilde{\eta}_i$, the standard deviation in the noise of his report. In our data, we have $\tilde{\sigma} = 12.16$ and $\tilde{\eta}_i \in [0.61, 21.14]$. Therefore, $\tilde{z}_i \in [12.77, 33.3]$. Inserting each value of $\tilde{z}_i$ in (8) and (9), and setting $\alpha = 0.4$ and $\tilde{t} = \sum t/10$, we get $\theta_U^* \in [30.86, 30.92]$ and $\theta_O^* \in [40.28, 40.34]$. Averaging over all individuals, we finally obtain $\theta_U^* = 30.88$ and $\theta_O^* = 40.32$. It is key to realize that despite the significant variability in the variance of noise across participants, optimal targets are extremely similar to each other. It is therefore with virtually no loss of generality that we can focus on the optimal average targets $\theta_U^*$ and $\theta_O^*$.

Appendix A5. Time estimation in task $B$ by interval.

Traditional theories of subjective time rely on two properties. The “mean accuracy” property postulates that people produce time intervals that are on average equal to the interval they are required to produce. The “scalar property of variance” requires that the sensitivity of time estimates is independent of the time to estimate (the variability to time ratio is constant). We present below the p.d.f. of reports for each announced interval $t$ (vertical line) in task $B$. We computed separately the distribution for CPT (blue) and non-CPT (green) participants. These distributions were not significantly different for any interval (KS-test, p-value $\gg 0.05$). Average reports in all intervals were significantly above the announced time $t$ (t-tests, p-value $< 0.006$) implying that mean accuracy was not satisfied by our data. We also tested for the scalar property of variance by
regressing the standard deviation on the mean of the reports. The R-squared of this regression was 0.55. We also computed the coefficient of variation (standard deviation / mean) for each announced time interval and regressed it on time. The slope coefficient was not significantly different from zero (p-value = 0.067). These findings indicate that the scalar property of variance was satisfied.