How long is a minute? *

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Abstract

Psychophysics studies suggest that our perception of time is different from the objective passage of time. Economics research emphasizes that the value of a reward depends on the delay involved. In this paper, we combine both strands and estimate time perception and time discounting functions at the individual level in an incentivized controlled laboratory environment. We find a negative and statistically significant correlation between time perception and time discounting: subjects for whom one unit of time feels longer are less willing to delay gratification by that amount of time. The result suggests that our ability to delay consumption is related to our mental representation of time delays.

Keywords: laboratory experiments, time perception, time discounting, time estimation.

JEL Classification: C91, D03, D91.

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Time is too slow for those who wait, too swift for those who fear, too long for those who grieve, too short for those who rejoice, but for those who love, time is not.

Henry Van Dyke (Music and Other Poems, 1904)

1 Introduction

Time is subjective. It flies when you enjoy and virtually stops when you suffer. Tomorrow is “in a very long time” for kids and “practically now” for seniors. The objective of this paper is to explore the relationship between time perception and time discounting. Our conjecture is simple: if one person perceives one week to be longer than another person, it seems natural that he will be less willing to delay a reward by that (objective) amount of time. Even though inter-temporal decisions likely depend on many different cognitive processes, we hypothesize that timekeeping mechanisms are partly responsible for observed choices. If the hypothesis is correct, it can help understand the paradoxical tendency of older adults to save more than younger adults (Banks et al., 1998) despite their shorter life expectancy. More generally, validating our hypothesis would suggest that eliciting discount rates is a valuable but incomplete measure to understand the intertemporal tradeoffs that different people make.

To address this question we ask our subjects to perform two tasks in a controlled laboratory environment. First, we elicit their time discount rates using the method proposed by Andreoni and Sprenger (2012a) (hereafter, [AS]), where subjects allocate a fixed amount of tokens between two dates. We use their convex time budget (CTB) method due to its robustness, and structurally estimate a quasi-hyperbolic discount function and curvature of utility. Second, we elicit their time perception estimates using a task adapted from the psychophysics literature and extended in several ways, where subjects reproduce intervals of lengths ranging between 20 seconds and 4 minutes. Formally, we ask them to click the start box to begin a time interval and click again when a predetermined amount of time (e.g., 2 minutes and 31 seconds) has passed. This task is performed in conjunction with a distractor task that prevents them from counting seconds. We estimate for each individual a time perception power function that maps true time intervals into perceived time intervals. Finally, we correlate the impatience or preference for the present derived from the time discounting task ($TD$) with the subjective evaluation of time obtained from...
the time perception task (TP).

We find substantial dispersion in the time discounting of our subjects. The estimated parameters in the TD task are in line with those found in [AS], with low levels of impatience, little evidence of present bias and some small (but positive) concavity in the utility function. Perception estimates in the TP task are also heterogeneous. Although a large fraction of individuals systematically underestimate time (around 40%), we also observe the opposite tendency in a number of subjects (around 30%). More generally, we find evidence of both concave and convex time perception functions.

The main novelty of the study is to analyze the relationship between time perception and time discounting. To this purpose, we correlate the estimated perception function with the estimated discount function. We first show that our subjects can be ranked consistently in their time perception and time discounting attitude for delays in the range of 1 hour to 1 week. We can then perform a correlation analysis at the individual level within this time range. For all intervals, we find a statistically significant negative correlation between the level of impatience estimated in the TD task and the subjective perception of time estimated in the TP task: the Pearson correlation coefficient (PCC) ranges between -0.20 and -0.30 with a p-value between 0.01 and 0.05 depending on the delays, conditions and functional specifications. In other words and consistent with our hypothesis, subjects who perceive 1 unit of time as a “long” interval are less likely to delay consumption by that amount of time than subjects who perceive it as a “short” interval. Said differently, timekeeping mechanisms are partly responsible for inter-temporal decision-making.

We then use this result to build a simple model of discounting based on time perception. Formally, we assume that subjects mentally represent the perceived time of a given true delay and apply a discount to this perceived delay. We find that the fit of this model is on aggregate equally good as that of the original model (though not strictly better), suggesting that time perception is a likely driver of mental discount computations when we evaluate future rewards.

Our paper relates to the growing literatures on time discounting and time perception. Time discounting has received much attention in economics. Researchers have proposed different parametric formulations of the time discounting function as well as different experimental designs to elicit them, and the empirical and experimental estimates vary significantly across studies. There are two main challenges for the elicitation of time discounting. First, subjects may not be time-consistent and place a greater value on immediate gratification compared to any future consumption. This has motivated hyperbolic specifications of time discounting as opposed to the standard exponential formulation.²

²The quasi hyperbolic formulation was first developed by Phelps and Pollak (1968) and later reintroduced by Laibson (1997). Lowenstein and Prelec (1992) propose a general hyperbolic specification.
Second, time is inherently uncertain and deciding to postpone consumption amounts to choosing a lottery. It is therefore important to be able to disentangle risk preferences from time preferences. Indeed, when choosing between consumption now and consumption in the future, a subject may choose the former because uncertainty about the future makes the present option more desirable. The recent literature proposes methods to jointly estimate time and risk preferences (Andersen et al. (2008); Andreoni and Sprenger (2012a, 2012b); Andersen et al. (2014)) and reports less or no evidence of a present bias. Our analysis relies on this last line of research, which allows us to better isolate time discounting. It does not improve the existing methods.

Time perception has also been extensively studied in the psychophysics literature. It is centered on prospective time evaluations, where subjects are informed beforehand that they have to make a time related judgment. These studies mostly focus on extremely short intervals (milliseconds, seconds) and use non-incentivized methods in which subjects have either to verbally assess a duration, reproduce or produce a time interval, or compare the duration of intervals presented successively (Lorraine (1979); Grondin (2010)). There are two major findings in this literature. First, prospective time evaluation is often consistent with Weber-Fechner’s law of human perception, implying that subjective time is not linear in true time but rather proportional to its logarithm (Grondin, 2001). Second, individual evaluations are qualitatively similar for time intervals of different lengths (Lewis and Miall, 2009), suggesting the existence of a single ‘internal clock’ mechanism that governs prospective timing. Our study draws on the concept developed in this literature and also focuses on prospective timing. However, we propose a new methodology that departs from the existing literature in several important respects. First, we focus on significantly longer time intervals than the majority of the literature (several minutes). Second, we introduce a new and incentivized elicitation method paired with a distractor task that prevents subjects from counting. Third, we provide a structural estimation of a two-parameter power function of time perception instead of imposing a logarithmic form. Indeed, although there is evidence of a concave relationship between true and perceived time for many subjects, there is also a significant fraction of individuals for which the opposite, convex relationship fits better. Finally, we estimate the perception function at the individual not the aggregate level. This allows us to study heterogeneity in perception across subjects.³

³Some studies investigate instead retrospective time evaluation, where subjects are not informed beforehand that they will have to make a time related judgment (Block and Zakay, 1997). By definition, under this approach only one measure can be obtained per individual. The studies find that retrospective time evaluations are usually shorter than prospective time evaluations (Fraisse, 1984) and they draw on different (memory) processes. We performed a one-shot retrospective time evaluation task and also found more underestimation than under prospective time evaluation (see Appendix A2).
Finally, we are not the first to argue the possibility of a relationship between time perception and time discounting. This has been theorized in several studies. Read (2001) tested for sub-additivity in time discounting and suggested that a possible reason for this effect was the subjective evaluation of time (due for example to differences in memory and attention). Lemoine (2015) built a model that captures the acceleration of time as we age and Capra and Park (2015) studied the effect of time distortions. However, most of the literature has focused on the potential relationship between time perception and present bias. In particular, Ray and Bossaerts (2011) proposed a theoretical approach to show that choices are present-biased with respect to calendar time if individuals discount the future exponentially with respect to biological time while the internal representation of time is stochastic and autocorrelated. Cui (2011) demonstrated that the scalar property of time perception also implies hyperbolic discounting. A few studies (Zauberman et al. (2009) and Han and Takahashi (2012) have used experimental techniques to relate present-bias and impulsivity to time perception. In the present study, we are interested in a more fundamental question: we want to test whether subjects who perceive objective time as subjectively longer will be less prone to delay consumption. We are not directly interested in time discounting biases, and there is in fact little evidence in our data in favor of hyperbolic discounting.

2 The experiment

2.1 Design and procedures

The experiment was conducted in the Los Angeles Behavioral Economics Laboratory (LA-BEL) at the University of Southern California. A total of 124 subjects participated in the study in 14 groups of 6 to 10 participants each. In order to participate in the experiment, subjects were required to be enrolled USC students with a USC Discretionary Card Account. Students frequently use their USC Card to pay in businesses on campus and the surrounding area. By special arrangement with the USC Card Department, we were able to deposit money into their accounts at our convenience. Sessions lasted for about 1h30min and started either at 10am or 12pm. They consisted of two main tasks, always administered in the same order: time discounting task followed by time perception task.

5These studies were not incentivized and produced unreasonably high discount rates (e.g., 160% annual rate for three-month delays). Importantly, they did not use a conventional time perception task. Instead, time perception was measured by marking how long future delays (three months, one year, three years) “felt” on a line scale. In our view, this approach is highly unsuitable to elicit time perception estimates.
6For information about the lab, please visit http://dornsife.usc.edu/label.
Instructions were read out loud at the beginning of each task. Since our time perception task is the most novel of the two, we will all along the paper discuss it first (but remember that it was administered second).

**Time perception task.** Participants were asked to produce 9 time intervals \( \tau \) of 24, 31, 41, 53, 69, 89, 116, 151 and 196 seconds respectively, without knowing in advance the number or length of intervals to produce.\(^7\) We designed a Matlab-based program to implement the elicitation of the participants’ time perception. It presented the instructions on the screen and guided subjects to estimate time intervals. Subjects were prompted the length of the interval \( \tau \) to be estimated. Then, subjects marked the beginning and end of the interval by clicking on a button on the top right corner of the screen. The order for the 9 intervals was randomly selected but it was the same for all subjects.\(^8\)

To make sure that subjects did not count time, we asked them to solve novel filler (distracting) tasks while estimating time intervals. In each of these tasks, subjects were presented a \( 4 \times 6 \) table where each row and column had a name and they were instructed to click on a specific cell. In the example of Figure 1, subjects were asked “Please click the cell where the column to the right of the column labeled athena intersects the row above the biology row”.

![Figure 1: Example of a filler task](image)

The names of the rows and columns as well as the phrasing of which cell to click on changed from table to table to make sure subjects would pay attention. There was a

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\(^7\)This is called a *prospective* time estimate in a *production* paradigm. Prospective (as opposed to retrospective) refers to a case where subjects know in advance that they will be requested to estimate the elapsed time. Production occurs when subject are informed about the length of the interval they must produce (Nichelli, 1996). This is different from reproduction, where subjects experience a time interval (without knowing its real length) and are then asked to reproduce a second interval of the same size.

\(^8\)Before coming to the laboratory, subjects were asked to put away any time-keeping devices such as watches, music players and cell phones. An experimenter made sure that subjects placed these items in their bag and monitored that they did not use any such device.
random and unspecified time limit to complete each task (between 10 and 15 seconds) and failure to complete it counted as an incorrect answer. The amount earned depended on the proportion of correct answers in the filler tasks and the distance between time estimates and true time intervals. For each subject, one time interval was randomly selected for payment. For this time interval, the subject earned money only if at least 80% of the filler tasks were correctly answered. The subject would then earn $25 if the estimate was within $\pm 5\%$ of the real length of the interval, $15 if it was within $\pm 10\%$ and $5 if it was within $\pm 20\%$. If less than 80% of the filler tasks were correctly answered, the subject did not earn anything no matter how good the estimation of the time interval was. The entire procedure was explained beforehand.¹⁰,¹¹

**Time discounting task.** Since the goal of the paper was not to provide an innovative way to elicit time discounting, we followed closely the CTB design and allocation procedure in [AS]. We provided subjects with a budget of experimental tokens to allocate either to a sooner time $t$ or a later time $t+k$, at different token exchange rates. The relative rate at which tokens translated into money determined the gross interest rate, $(1 + r)$. The amounts allocated at dates $t$ and $t+k$ were denoted by $c_t$ and $c_{t+k}$ respectively. We implemented a $3 \times 3$ design with three sooner payment dates starting from the experiment date ($t \in \{0, 7, 21\}$ in days) and three delay lengths ($k \in \{21, 42, 63\}$ also in days). For each of the 9 pairs of $(t, k)$, there were 5 different budgets and exchange rates for a total of 45 sooner-later token allocation tasks. Dates were selected to avoid holidays, vacations and examination dates. To avoid differential weekday effects, $t$ and $k$ were both multiples of 7, so that payments were scheduled to arrive on the same day of the week.

Subjects were given 10 tokens for each of the 45 allocation tasks. Tokens assigned to sooner and later payments had values $v_t$ and $v_{t+k}$, respectively. Since $v_{t+k}/v_t = (1 + r)$ is the gross interest rate over $k$ days, $(1 + r)^{1/k}$ is the daily interest rate. Values were

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Footnotes:

⁹The tasks required sufficient effort to prevent subjects from counting but were easy enough to make sure all subjects could complete them if they put attention. Participants were informed that if they reported the end of an interval during a task, that task would not count as correct or incorrect. Subjects also estimated a 10th time interval of 219 seconds that was not used for analysis. Here is why: participants clicked to report their time interval estimates and their answers to the filler task. All these clicks could be heard by other participants who could not disentangle between either types of clicks until almost the end of the experiment. As such, there were 9 relevant intervals during which subjects could hear all other subjects clicking to complete either task, and therefore could not make any inference about the time estimations of their peers. In the 10th trial, however, they could have used the absence of clicking as a cue that the others had finished their time estimations, which could have biased their own last time estimate.

¹⁰We chose this method because it is more intuitive and easier to explain than the (incentive compatible) quadratic scoring rule. Also, there is evidence that reports vary even among different proper scoring rules (Palfrey and Wang, 2009).

¹¹In 84% of the trials, subjects answered correctly at least 80% of the filler tasks and therefore were eligible for payment.
never multiples of $0.05 to avoid gravity point effects. To formally implement choices, we
provided paper booklets. Subjects had to circle their preferred token allocation among
the eleven possible combinations of tokens sooner vs. token later in each of the 45 tasks.
Appendix A1 shows the token rates, standardized daily interest rates and corresponding
annual interest rates for all 45 budget sets. It also shows the presentation of the first 5
tasks of the paper booklet, corresponding to $t = 0$ and $k = 21$ (Figure 7).

To avoid in-lab vs out-lab payments at different dates, all sooner and later payments,
including those for $t = 0$, were deposited into the subjects’ USC Card Accounts by 4pm on
the specified date.\textsuperscript{12} Subjects were described the payment method and the arrangement
made with the USC Card Department. They were told that they would receive a $4.64
thank-you payment for participating in two payments, $2.32 at the sooner and $2.32 at
the later date regardless of their choices, and that all experimental earnings were added
to these two payments. Subjects knew in advance that one of the 45 choices was going to
be selected for payment by drawing a numbered ball from a bingo cage. They were given
Professor Juan Carrillo’s business card and they were told to contact him if payments did
not reflect in their account, in which case a payment would be hand-delivered immediately.
Subjects were asked if they trusted the payment method at the end of the experiment and
95\% of respondents replied yes.\textsuperscript{13}

\textbf{Other tasks.} We conducted three peripheral tasks: a one-shot retrospective time estimate
task, a cognitive ability test, and a survey to collect demographic information. Details of
the procedures and results obtained in these tasks can be found in Appendix A2.

From the 124 subjects who participated in the study, four subjects were excluded due
to data recording issues, leaving us with a sample of 120 subjects.

\textbf{2.2 Challenges}

An experimental study of time perception and time discounting is subject to two chal-
lenge. First, the temporal horizons are different. We can realistically elicit multiple
prospective time estimates that are on the range of minutes whereas meaningful mon-
eyary tradeoffs must involve temporal delays that are on the range of weeks. We will

\textsuperscript{12}This removes the salience of immediate payment. It is likely to result in the later option being chosen
more frequently but it also makes the uncertainty and potential anxiety over payment similar whether it
is today or in the future (for a discussion, see Andreoni and Sprenger (2012a, 2012b)).

\textsuperscript{13}The full list of differences relative to [AS] are (first item refers to our design, second item to theirs): (i)
payment to USC card vs. payment by check; (ii) thank you payments of $2.32 vs. $5; (iii) slight differences
in $(r, t, k, m)$ but calibrated to equalize daily gross interest rates (see Figure 7); (iv) 11 vs. 101 choices per
budget; (v) pen and paper vs. computerized implementation; and (vi) 120 vs. 97 subjects.
therefore extrapolate our estimates upwards for time perception and downwards for time
discounting. We will also consider different horizons to check the robustness of our results.

Second and related, our goal is to compare perception and discounting across individuals. If some
time perception functions are not linear in true time and/or some time
discount functions are not exponential, rankings may depend on the horizon (for example,
a hyperbolic discounting subject may be more impatient in the short run and less impa-
tient in the long run than an exponential discounting subject). In the analysis, we will
put special emphasis in determining the time range for which the ranking of the estimates
between individuals is preserved.

3 Time perception ($TP$)

We first present the theoretical framework and experimental results of our time perception
task. Time perception refers to the fact that an objective length of time may be inaccu-
rately perceived, leading to under- or over-estimation of true delays. We consider a simple
model of time perception in which subject $i$ formulates a subjective duration $\theta_i$ of a true
time interval of length $\tau$ according to the function:

$$\theta_i(\tau) = a_i \tau^{b_i}$$  \hspace{1cm} (1)

where $a_i = b_i = 1$ corresponds to a correct time perception. This representation is
borrowed from Steven’s law, which posits a power relationship between the magnitude of
a physical stimulus (brightness of an image, level of a sound, sugar component of a meal,
etc.) and its perceived strength (Stevens, 1957). This theoretical relationship has been
applied to a variety of problems, including time perception (Stevens (1975); Luce (2002)).

We fitted this model to the data obtained from the time perception task ($TP$). For
each individual $i$, we estimated by non linear least squares (NLS) the parameters $a_i$ and
$b_i$ of the following regression:

$$y_{is} = a_i \tau_s^{b_i} + \epsilon_{is}$$  \hspace{1cm} (2)

where, for trial $s \in \{1, \ldots, 9\}$, the reported perception of individual $i$ is $y_{is}$, the true
length in seconds is $\tau_s \in \{24, 31, 41, 53, 69, 89, 116, 151, 196\}$, and the noise in the process
is $\epsilon_{is} \sim N(0, \sigma_i^2)$. We ran a boxplot analysis to identify extreme outliers and, among
those, we removed the most extreme and isolated values.\textsuperscript{14} Three subjects exhibiting a
very large $a_i$ parameter ($a_i > 8$) were excluded in this process. Figure 2 graphically depicts
the estimated parameters $(\hat{a}_i, \hat{b}_i)$ of the remaining 117 subjects. For illustrative purposes,\textsuperscript{A value is considered to be an outlier if it is at least 1.5 interquartile ranges below the first quartile
or at least 1.5 interquartile ranges above the third quartile. A coefficient of 3 (instead of 1.5) is usually
applied to detect extreme outliers. All the subjects we removed were extreme outliers.
Figure 3 presents the choices of three representative subjects (with \( \hat{b}_i > 1 \), \( \hat{b}_i \simeq 1 \) and \( \hat{b}_i < 1 \), respectively).

![Figure 2: Estimations of individual time perception parameters (\( \hat{a}_i, \hat{b}_i \)).](image)

We obtained three main findings. First, the power model explains remarkably well the data: the average \( R^2 \) is 0.98 and 106 out of 117 individuals have an \( R^2 \) greater than 0.95.\(^{15}\) In other words, subjects typically reported estimates that were very close to the best time perception power fit. Second, there is substantial heterogeneity across individuals in our sample. Indeed, 77 and 40 subjects had an estimated parameter \( \hat{a}_i \) greater and smaller than 1, respectively. Similarly, 81 and 36 subjects had an estimated parameter \( \hat{b}_i \) smaller and greater than 1, respectively. This suggests that crucial information is lost if we simply fit an aggregate perception function, and that constraining that function to be logarithmic (as in the majority of the psychophysics literature) severely undermines the quality of the estimates. Third, time perception parameters \( a_i \) and \( b_i \) are not independent across individuals. More precisely, we found a strong hyperbolic relation between the two parameters.\(^{16}\) This means that both parameters cannot be studied in isolation: an individual with a concave perception of time (\( \hat{b}_i < 1 \)) is extremely likely to exhibit a steep reaction to time (\( \hat{a}_i > 1 \)) and vice versa.

4 Time discounting (TD)

We now present the theoretical framework and experimental results of our time discounting task. We refer to [AS] for extra details of the theory and estimation. Subject \( i \) chooses at time 0 to allocate a budget \( m \) between consumption at \( t \), \( c_{i,t} \), and consumption at \( t+k \), \( c_{i,t+k} \), continuously along a convex budget set. Denoting \( (1+r) \) the gross interest rate,

\(^{15}\)\( R^2 \) is expected to be high since we fit two parameters with 9 observations. Still, since we impose \( \theta_i(0) = 0 \), our regression has 7 degrees of freedom. The subjects in Figure 3 are representative of the fit. We also tried a linear model but the fit dropped substantially.

\(^{16}\)The data is best fitted by the curve \( a = −3.64 + \frac{4.7}{b} \) (p-value < 0.001 for both parameters)
the budget constraint can be written as:

\[(1 + r)c_{i,t} + c_{i,t+k} = m\]  \hspace{1cm} (3)

We assume a time separable time discounting function \(\Phi_i(t)\) of time \(t\) from the perspective of time 0, and a CRRA utility of money:

\[U_0(c_{i,t}, c_{i,t+k}) = \Phi_i(t) \frac{1}{\alpha_i} (c_{i,t})^{\alpha_i} + \Phi_i(t + k) \frac{1}{\alpha_i} (c_{i,t+k})^{\alpha_i}\]  \hspace{1cm} (4)

where \(\alpha_i > 0\) is the curvature parameter. To estimate the inter-temporal utility function of each individual, we arbitrarily restrict attention to quasi-hyperbolic discount functions, that is, functions of the form:

\[\Phi_i(t) = \begin{cases} 
\beta_i \delta_i^t & t > 0 \\
1 & t = 0
\end{cases}\]

where \(\delta_i \in (0, 1)\) is the one period discount and \(\beta_i > 0\) the time inconsistency parameter. Note that \(\beta_i = 1\) corresponds to the standard exponential discounting model. A subject is time inconsistent when \(\beta_i \neq 1\), exhibiting a present-bias when \(\beta_i < 1\) and a future-bias when \(\beta_i > 1\). The subject chooses \(c_{i,t}\) and \(c_{i,t+k}\) by maximizing (4) subject to (3). The optimal amounts are:

\[c_{i,0}^* = \frac{m}{(1 + r) + (1 + r)\beta_i \delta_i^k} \frac{1}{1-\alpha_i}\]  \hspace{1cm} and  \hspace{1cm} \[c_{i,t}^* = \frac{m}{(1 + r) + (1 + r)\delta_i^k} \frac{1}{1-\alpha_i}\]  \hspace{1cm} (5)

**Figure 3:** Three examples of choices in time perception task
We fitted the model to the data obtained from the time discounting task (TD). For each individual, we estimated by NLS and MLE the parameters $\alpha_i$, $\delta_i$ and $\beta_i$ of the following regressions:

$$c_{i,0} = \frac{m}{(1 + r) + \left((1 + r)\beta_i\delta_i\right)^{1-\alpha_i}} + \epsilon_{i,0} \quad \text{and} \quad c_{i,t} = \frac{m}{(1 + r) + \left((1 + r)\delta_i\right)^{1-\alpha_i}} + \epsilon_{i,t}$$

where $\epsilon_{i,0} \sim N(0, \sigma_i^2)$ and $\epsilon_{i,t} \sim N(0, \sigma_i^2)$. Notice that variations in delay lengths $k$ and interest rates $(1 + r)$ allow for the identification of $\alpha_i$ and $\delta_i$. Variations in starting times $t$ allow for the identification of $\beta_i$.

From the 120 initial subjects, we removed 19 subjects who put all the tokens in the later option 44 or 45 times out of 45 (more than 97% of the time). We also ran a boxplot analysis and we excluded 6 extreme outliers whose estimates were $\hat{\alpha}_i \simeq 0$, $\hat{\alpha}_i > 2$ and/or $\hat{\beta}_i > 4$. Figure 4 presents the distributions of the $(\hat{\beta}_i, \hat{\delta}_i, \hat{\alpha}_i)$ estimated parameters for the 95 remaining subjects using MLE.

![Figure 4: Distribution of parameters in the time discounting task $(\hat{\beta}_i, \hat{\delta}_i, \hat{\alpha}_i)$](image)

We obtained reasonable estimates. The estimates are also similar (and generally consistent) with those in [AS] (see their Figure 3 in p. 3351). In particular, the vast majority of our $\hat{\beta}_i$ estimates are close to 1, implying no evidence of present-biased behavior. If anything, just like in [AS] we observe a future bias, although this is likely due to small measurement errors. As expected, the overwhelming majority of the $\hat{\delta}_i$ estimates are between 0.99 and 1.0 and most of the $\hat{\alpha}_i$ estimates are above 0.85 (but still below 1).

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17 Having non-reliable estimates for some subjects is not unusual in this type of exercise. For example, among the 97 subjects in [AS], 2 did not converge, 2 made automatic choices, 22 chose the later option more than 95% of the time and 7 were outliers according to our definition.

18 As expected, MLE and NLS give extremely similar estimates. The fit of the model is good. The average $R^2$ in our NLS estimation is 0.80.

19 Aggregate estimates are also similar to [AS]: $\alpha = 0.914$, $\delta = 0.9967$ and $\beta = 1.033$ in our sample compared to $\alpha = 0.897$, $\delta = 0.9991$ and $\beta = 1.007$ in [AS].
5 From time perception to time discounting

We have established that the time perception and time discounting of each individual $i$ are well summarized by $(a_i, b_i)$ and $(\beta_i, \delta_i, \alpha_i)$, respectively. In this section, we address the main question of the paper: the relationship between time discounting and time perception.

5.1 Main hypothesis

We hypothesize that objective delays between consumption dates are evaluated in a subjective manner, and the subjective estimates are used to choose between consumption options over perceived delays. The tendency to over- or under-represent true time will therefore be reflected in the willingness to delay gratification. Formally, we hypothesize that the discounting of an objective time interval for individual $i$, $\Phi_i(\cdot)$, corresponds to the time-weighting $f(\cdot)$ of the perceived length of that interval, $\theta_i(\cdot)$. Assuming for simplicity that $f(\cdot)$ is identical for all individuals, we have:

$$\Phi_i(t) = f(\theta_i(zt)) \quad \forall i$$  \hspace{1cm} (7)

where $z$ is the conversion rate between units of time in the discounting and perception tasks.\footnote{In our case, given we formulated the time perception task in seconds ($\tau = 1$ second) and the time discounting task in days ($t = 1$ day), we have $z = 60 \times 60 \times 24 = 86,400$.} According to (7), differences in discount functions across individuals are due to differences in their subjective perception of time. We impose the natural assumption that rewards at more distant perceived dates are valued less ($f'(\cdot) < 0$) but, for now, we do not posit any specific functional form on $f(\cdot)$. This alone immediately implies:

$$\theta_i(zt) \geq \theta_j(zt) \iff \Phi_i(t) \leq \Phi_j(t)$$  \hspace{1cm} (8)

The relationship in (8) formalizes the intuitive idea that if an objective length of time $zt$ is subjectively perceived as a longer interval by subject $i$ than by subject $j$ ($\theta_i(zt) > \theta_j(zt)$), then subject $i$ is less willing than subject $j$ to postpone a reward by that amount of time ($\Phi_i(t) < \Phi_j(t)$).

**Hypothesis** Subjects for whom one unit of time is perceived as longer are less willing to delay gratification by that amount of time.

5.2 Main result

Recall that our time perception task elicits perceived durations in the range of minutes while our time discounting task elicits inter-temporal choices in the range of weeks. Ideally,
we would like to determine whether the tendency to under or over-evaluate time in the estimated range of the \( TP \) task (minutes) is related to the patience level in the estimated range of the \( TD \) task (weeks).

As mentioned in section 2.2, the non-linearity of the time perception function is potentially challenging: a subject with \( \hat{b}_i < 1 \) may over-estimate short intervals and under-estimate long intervals whereas a subject with \( \hat{b}_j > 1 \) may exhibit the opposite pattern. Fortunately, given a power functional form \( \theta(\cdot) \) and the empirical relationship between the estimates \( \hat{a}_i \) and \( \hat{b}_i \), it is possible to determine a time interval after which our subjects can be stably ranked in terms of their perception of time. To find such time interval in our data, we performed the following exercise. For each subject \( i \) and given his estimated parameters \((\hat{a}_i, \hat{b}_i)\), we evaluated his perception of an interval of length \( \tau_x \) (that is, \( \hat{\theta}_i(\tau_x) = \hat{a}_i \tau_x^{\hat{b}_i} \)) and then ordered all subjects from \( \max_i \{ \hat{\theta}_i(\tau_x) \} \) to \( \min_i \{ \hat{\theta}_i(\tau_x) \} \). We repeated the same exercise for an interval of length \( \tau_x' \). Finally, we asked by how much this ranking changed between \( \tau_x \) and \( \tau_x' \). We found that the ranking of 28% of our subjects changed by more than 5 positions between 588 seconds (3 times the highest estimated interval) and 1 hour. This percentage dropped to 20% between 1 hour and 1 day, to 1% between 1 day and 7 days and to 0% thereafter. Overall, subjects in our sample can be ranked with stability regarding their perception of time for intervals above 1 hour.

We used a similar methodology to determine for which time intervals we can rank subjects by their level of impatience. We took the estimated \((\hat{\beta}_i, \hat{\delta}_i)\) parameters to determine for each individual \( i \) the value at date 0 of one unit of consumption at different dates \( t_x: 1 \) hour, 6 hours, 12 hours, 18 hours, 1 day and 7 days. We then ranked our subjects by their level of impatience at each of these dates. We found that 27% of the subjects changed ranks by more than 5 positions between 1 day and 7 days and none changed ranks between any of the shorter intervals. Subjects can then be ranked steadily in terms of their time discounting for all time horizons below 1 day.

The overall conclusion of this exercise is that subjects in our sample can be ranked reasonably consistently in terms of time perception and impatience when true time intervals lie between 1 hour and 1 day. We can then meaningfully compare directly time perception and time discounting in those time intervals.

Given the exclusion criteria considered earlier for our estimations, we kept for the analysis only the 92 subjects for whom we obtained accurate and reasonable estimates in both the \( TP \) and \( TD \) tasks.\(^{21}\) We then considered the smallest upward time extrapolation for which subjects’ time perception can be ranked steadily, namely 1 hour (1h). Similarly,

\(^{21}\)Recall that we excluded 3 extreme outliers in \( TP \), 6 extreme outliers in \( TD \) and 19 subjects with insufficient variance in \( TD \). Removing 23% of the sample is not ideal. In section 5.3, we discuss the robustness of our results to other (more or less stringent) sample specifications.
we considered the smallest downward extrapolation for which subjects’ time discounting can be ranked steadily, namely 1 day (1d). We then determined the relationship between time discounting and time perception at the individual level over that range. The results are summarized in Table 1 and in the next result.

<table>
<thead>
<tr>
<th>Measure</th>
<th>PCC</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d ((\text{TP})) - 1d ((\text{TD}))</td>
<td>-0.22</td>
<td>0.038</td>
</tr>
<tr>
<td>1h ((\text{TP})) - 1d ((\text{TD}))</td>
<td>-0.26</td>
<td>0.013</td>
</tr>
<tr>
<td>1h ((\text{TP})) - 1h ((\text{TD}))</td>
<td>-0.26</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table 1: Correlation between time perception estimates \((\hat{a}_i(zt)^{\hat{\beta}_i})\) and time discounting estimates \((\hat{\beta}_i\hat{\delta}_i)\).

**Result** There is a negative and statistically significant correlation between time perception and time discounting in the 1 hour to 1 day range.

The result provides support for our Hypothesis. Impatience is associated with the perception that time passes slowly, as predicted by our simple model. Subjects who produce a higher time interval in the time perception task (overestimate time) tend to consume earlier. Stated differently, subjects who feel that 1 unit of time is long are less willing to delay gratification simply because their opportunity cost of waiting is perceived as being big.\(^{22}\) Figure 5 represents the scatterplot of predicted time perception at 1 day and predicted time discounting of 1 day.\(^{23}\)

Overall, the data provides support for the link between subjective perception of time and impatience, and the existence of a time-weighting function \(f(\cdot)\) that transforms perceived time into discount rates. From Figure 5 it seems that the negative relationship is convex and that there is substantial heterogeneity across individuals in the valuation of future rewards and the subjective evaluation of delays. Notice also that the discounting is above 1 for many subjects. Taken literally, it would mean that individuals strictly prefer 1 unit of consumption tomorrow to 1 unit today. This is obviously unreasonable. The reason for such result is that many \(\hat{\beta}_i\)-estimates are above 1, so the downward extrapolation of the

\(^{22}\)We perform the same analysis using a Bayesian method and find similar results (bayesian PCC = -0.11 (respectively -0.13 and -0.08) with a likelihood that the correlation is negative of 0.82 (respectively 0.86 and 0.75) for the correlations in the order displayed in Table 1.

\(^{23}\)The reader may wonder whether the result is driven by observations that are “visually distant” from others in the graph. If (for ad hoc reasons) we decide for example to exclude from our sample the 3 subjects who made the most extreme choices, we obtain the same correlation magnitude and significance. More rigorous robustness checks are presented in section 5.3.
parameters to one day results in unrealistically high patience levels. Remember, however, that the main objective of our analysis is not to estimate levels of time discounting and time perception but to be able to perform comparisons across individuals.\footnote{If we do not extrapolate the discount function (e.g., we compute the discounting of 5 weeks), then more than 90\% of subjects will exhibit reasonable levels of preference for the present.}

It is important to emphasize that the estimates $(\hat{\alpha}_i, \hat{\beta}_i)$ on one hand and $(\hat{\beta}_i, \hat{\delta}_i)$ on the other are obtained from independent datasets, $\mathcal{T}\mathcal{P}$ and $\mathcal{T}\mathcal{D}$. There is a priori no reason (other than the endogenous relationship emphasized by our model) why the measures constructed from these two dataset should correlate. And yet, there is evidence that the mechanisms underlying time perception and time discounting are linked. Furthermore, the data collected in each task is noisy, which weakens the correlation between perception and discounting. This means that the correlation obtained is likely to be underestimated.\footnote{Experimental data is intrinsically noisy and subject to measurement errors. Given this noise, the true coefficient of correlation between time perception and time discounting is by construction higher than the coefficient of correlation between their noisy measurements (see Gillen et al. (2015) for a statistical method that corrects for measurement errors).}

To better investigate the relationship between time perception and time discounting, we run a regression between time discounting of 1 day and time perception of 1 day, including several measures of relevance. First, subjects who over-estimate time should in principle complete more filler tasks. A simple correlation exercise indicated that subjects who reported longer durations also completed successfully more filler tasks (PCC = 0.3, p-value = 0.001). However, they also made more mistakes and their performance was not significantly better. We used the number of filler tasks completed successfully ($\text{Filler score}$) as a control. Second, it may also be the case that cognitive abilities, as measured

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Time perception and time discounting estimates for $T = 1$ day.}
\end{figure}
by IQ and GPA scores affect discounting attitudes. We therefore included the GPA score reported by subjects (GPA score) and the results of our cognitive ability test (IQ score). We also included a dummy for gender (Male) as well as $\hat{\alpha}_i$ estimates to control for intrinsic preference attitudes (Preference). Table 2 reports our results. Model 1 corresponds to the simple correlation analysis (and yields the same significance). Model 2 includes the various control measures.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d ($TP$)</td>
<td>-4.205 e-07 (*)</td>
<td>-4.431 e-07 (*)</td>
</tr>
<tr>
<td>Filler score</td>
<td>-</td>
<td>1.695 e-03</td>
</tr>
<tr>
<td>GPA score</td>
<td>-</td>
<td>-5.983 e-02</td>
</tr>
<tr>
<td>IQ score</td>
<td>-</td>
<td>2.055 e-02</td>
</tr>
<tr>
<td>Male</td>
<td>-</td>
<td>-2.917 e-02</td>
</tr>
<tr>
<td>Preference</td>
<td>-</td>
<td>-3.117 e-01</td>
</tr>
<tr>
<td>constant</td>
<td>1.105 (***)</td>
<td>1.325 (***</td>
</tr>
</tbody>
</table>

Adjusted $R^2$: 0.04 0.06

significance levels: * = 5%, ** = 1%, *** = 0.1%

**Table 2:** Regression analysis of 1 day ($TD$).

Even though the linear model is only very partially explaining the variance in the data, it allows to identify significant and insignificant correlations between our measures. In this case, the results indicate that time perception contributes to discounting significantly. However, there is no evidence that any of the other measures has an impact in our sample.

5.3 Robustness

We conducted a number of checks to test the robustness of our main result.

**Analysis with and without outliers.** To check whether our results are sensitive to the exclusion criteria we used to retain our 92 subjects, we construct two new samples. First, a large sample, where we reintroduce subjects who exhibit little variance in behavior in the discounting task. Given some of them also had extreme estimates, the new sample is composed of 106 subjects (88% of participants). Second, a small sample that excludes outliers which are less extreme compared to the main sample. We found again that time discounting and time perception were correlated in the 1 hour to 1 day range. The main

---

26. The same regression analysis can be performed with 1h ($TP$) and / or 1h ($TD$). The results are very similar except that significance is increased when shorter time intervals are used.

27. The exclusion criteria were $a_i < 0.16$, $a_i > 8$, $a_i \simeq 0$, $a_i > 1$, $\beta_i < 0.3$, $\beta_i > 4$, $\delta_i > 1$ and subjects who put all the tokens in the later option 44 or 45 times out of 45. We also omitted subjects for whom
correlations are reported in Table 3 and the results of the regression analysis are collected in Table 4. Overall, the presence of outliers does not seem to amplify or diminish the main finding obtained earlier.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Small sample (78)</th>
<th>Large sample (106)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCC</td>
<td>p-value</td>
</tr>
<tr>
<td>1d (TP) - 1d (TD)</td>
<td>-0.25</td>
<td>0.027</td>
</tr>
<tr>
<td>1h (TP) - 1d (TD)</td>
<td>-0.23</td>
<td>0.043</td>
</tr>
<tr>
<td>1h (TP) - 1h (TD)</td>
<td>-0.23</td>
<td>0.043</td>
</tr>
</tbody>
</table>

**Table 3:** Correlation between time perception estimates and time discounting estimates varying the exclusion criteria.

<table>
<thead>
<tr>
<th></th>
<th>Small sample</th>
<th>Large sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d (TP)</td>
<td>-3.712 e-07 (*)</td>
<td>-5.351 e-07 (*)</td>
</tr>
<tr>
<td>Filler score</td>
<td>7.210 e-04</td>
<td>1.168 e-03</td>
</tr>
<tr>
<td>GPA score</td>
<td>5.169 e-02</td>
<td>-1.463 e-02</td>
</tr>
<tr>
<td>IQ score</td>
<td>1.508 e-03</td>
<td>2.072 e-02</td>
</tr>
<tr>
<td>Male</td>
<td>1.665 e-02</td>
<td>-4.133 e-02</td>
</tr>
<tr>
<td>Preference</td>
<td>2.284 e-02</td>
<td>-1.322 e-01</td>
</tr>
<tr>
<td>constant</td>
<td>0.821 (***</td>
<td>1.078 (***)</td>
</tr>
<tr>
<td># obs.</td>
<td>78</td>
<td>106</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.064</td>
<td>0.020</td>
</tr>
</tbody>
</table>

significance levels: * = 5%, ** = 1%, *** = 0.1%

**Table 4:** Regression analysis of 1 day (TD) varying the exclusion criteria.

**Weibull specification.** Since the quasi-hyperbolic discount function does not always capture accurately the discounting pattern of individuals, we follow Andersen et al. (2014) and consider the parametric specification proposed by Read (2001) to account for time subadditivity, namely the Weibull function:

$$\Phi^W_i(t) = \exp(-\mu_i t^{1/s_i})$$

with $\mu_i > 0$ and $s_i > 0$. This is comparable to quasi-hyperbolic discounting in that it is also a two-parameter function that boils down to time-consistent exponential discounting.

we had convergence problems in the estimation. This small sample is composed of 78 subjects (65% of participants). It corresponds to the dataset analyzed in detail in the previous version of the paper.
when one of the parameters takes a specific value (in our formulation, when \( s_i = 1 \)). For each individual, we fitted the parameters of the new discounted utility function and we obtained estimates for \( \alpha_i \), \( \mu_i \) and \( s_i \). We found that the Weibull and quasi-hyperbolic models perform on aggregate very similarly according to the Akaike Information Criterion (AIC). Since the Weibull model performs marginally better for some subjects and marginally worse for others, we assigned each subject his best model and conducted the same correlation analysis as we did before with the 92 subjects of the main model. Table 5 reports the same information as in Table 1 with the best fitted time discounting model for each individual. As we can see, both the correlation and the statistical significance remain in the same range when we add flexibility in the parametric form of the discount function (marginally better for 1 day and marginally worse for 1 hour intervals).

<table>
<thead>
<tr>
<th>Measure</th>
<th>PCC</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d (( TP )) - 1d (( TD ))</td>
<td>-0.23</td>
<td>0.026</td>
</tr>
<tr>
<td>1h (( TP )) - 1d (( TD ))</td>
<td>-0.26</td>
<td>0.011</td>
</tr>
<tr>
<td>1h (( TP )) - 1h (( TD ))</td>
<td>-0.23</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Table 5: Correlation between time perception estimates \( (\hat{a}_i(zt)\hat{b}_i) \) and the best fitted time discounting estimates \( (\hat{\beta}_i \hat{\delta}_t) \) or \( \exp(-\hat{\mu}_t(t^{1/s}_i)) \).

Time perception and discounted utility. Our structural approach implicitly assumes that the utility model is correct. If, contrary to our specification, utility and discount are not fully separable in the “true model” that generates the data, \( \hat{\alpha}_i \) estimates may capture some elements of discount while \( \hat{\beta}_i \) and \( \hat{\delta}_i \) estimates may capture some elements of utility. To address this possibility, we correlate time perception with the discounted utility of consumption of several units. Formally, we take the CRRA utility representation of our model and extend the relationship in equation (8) to:

\[
\theta_i(zt) \geq \theta_j(zt) \iff \beta_i \delta_{ti} \frac{1}{\alpha_i} (c)^{\alpha_i} \leq \beta_j \delta_{tj} \frac{1}{\alpha_j} (c)^{\alpha_j}
\]  

In words, the hypothesis is that if a subject perceives one unit of time as a longer interval than another subject, he will value less the consumption of \( c \) units after that amount of time. Table 6 reports the same correlation exercise as in Tables 1 and 5 for the valuation of 5 units of consumption.²⁹

The results are more significant compared to Table 1, suggesting that the perception of time affects the evaluation of delayed rewards rather than simply the evaluation of delays.

²⁸To be precise, the Weibull model performed slightly better. However, this was due to two subjects who were significantly better captured by that model. After removing these two subjects, the average performance of the two models was almost indistinguishable (the mean AIC was 237 for the quasi-hyperbolic
<table>
<thead>
<tr>
<th>Measure</th>
<th>PCC</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d (TP) - 1d (TD)</td>
<td>-0.23</td>
<td>0.026</td>
</tr>
<tr>
<td>1h (TP) - 1d (TD)</td>
<td>-0.28</td>
<td>0.008</td>
</tr>
<tr>
<td>1h (TP) - 1h (TD)</td>
<td>-0.28</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Table 6: Correlation between time perception estimates ($\hat{a}_i(zt)^{\hat{b}_i}$) and discounted utility estimates ($\hat{\beta}_i \hat{\delta}_t \frac{1}{\hat{\alpha}_t} (c)^{\hat{\alpha}_t}$).

Non-parametric test. Our data does not allow us to make meaningful non-parametric tests based on general patterns of consumption (such as the percentage of tokens allocated to early consumption) and general tendency to underestimate time (such as the proportion of times the subject underestimated the true time). Here is why. The non-linearity of the perception function strongly indicates that the estimated curvature is more important to predict over- vs. under-estimation of long intervals than the observations in the computed range. Also, the decision to allocate tokens early depends on both the degree of patience and the concavity of the utility function and the percentage allocated to the earliest date is not capturing well the attitude of a person towards delays. However, one simple analysis we can conduct is to divide our sample between those who underestimate 1 day and those who overestimate 1 day. We find that the average discount of 1 day is different for these two groups (1.10 vs. 0.99, p-value of a two-sample t-test = 0.007, the difference of the means is greater than 0 with probability 0.936 with a Bayesian t-test). As expected, the average discount function in the first group is significantly higher than in the second, indicating more patience from subjects who underestimate how long 1 hour is.

As a general conclusion, all these specifications suggest that the relationship between perceived time and preference for the present is stable: people who feel that time passes relatively more slowly (i.e., they overestimate the true length of objective time) are more inclined to consume early.

6 Individual time-weighting function

From the $\mathcal{TP}$ dataset, we showed that individual $i$’s time perception is well summarized by $\theta_i(zt) = a_i(zt)^{b_i}$. From the $\mathcal{TD}$ dataset, we found that individual $i$’s discount function can

---

Naturally, units matter for this exercise. We retained 5 units because this roughly corresponds to the dollar amounts they had to evaluate in $\mathcal{TD}$. We tried different units around that number and found similar results.
be approximated by $\Phi_i(t) = \beta_i \delta_i^t$. Our analysis revealed that the discount function can be interpreted as the time weighting of perceived time. Even though a given perceived time interval is reached for different true time intervals for different individuals, its weighting is similar across individuals. Overall, we have shown that the data can be summarized reasonably well by a common weighting function $f(\cdot)$, that transforms perceived times into discount rates $\Phi_i(t) = f(\theta_i(zt))$. Still, we noted some individual differences. The purpose of this section is to investigate this heterogeneity in more detail.

To do so, we posit that the weighting function that transforms perceived time into discount has the same structure for every participant but it is parametrized individually. More precisely, we assume:

$$\Phi_i(\theta_i(zt)) = \beta_i' d_i' a_i(zt)^{\delta_i}$$

This quasi-hyperbolic formulation is the same as the one we used to estimate discounts, except that participants are now assumed to discount payoffs with respect to their perception of time rather than the true time. It also uses one second as the unit interval of time, so $d_i'$ can be interpreted as the discount per second. Notice that if we set $\delta_i' = d_i$, then we can rewrite the previous function using one day as the (standard) unit of time:

$$\Phi_i(\theta_i(t)) = \beta_i' \delta_i a_i(t)^{\delta_i}$$

Our objective is to revisit the $TD$ data and to propose a new discounting model driven by perceived time rather than true time. Following the very same optimization procedure as in section 4, the optimal consumption of individual $i$ at date $t$ is:

$$c_{i,0}^{**} = \frac{m}{(1 + r) + \left((1 + r)\beta_i' d_i' a_i(zt)^{\delta_i}\right) \frac{1}{1-\alpha_i}}$$

and

$$c_{i,t}^{**} = \frac{m}{(1 + r) + \left((1 + r)\delta_i a_i(zt+k)^{\delta_i} - a_i(zt)^{\delta_i}\right) \frac{1}{1-\alpha_i}}$$

To make the estimation comparable to that in section 4, we do not estimate all 5 parameters again. Instead, we import the time perception parameters $\hat{a}_i$ and $\hat{b}_i$ estimated from the dataset $TP$ and we estimate by MLE the remaining three parameters ($\hat{\beta}_i', \hat{\delta}_i', \hat{\alpha}_i'$) in dataset $TD$ exactly as we did before. Figure 6 presents the distributions of the ($\hat{\beta}_i', \hat{\delta}_i', \hat{\alpha}_i'$) estimated parameters.

A comparison between Figure 4 and Figure 6 suggests that the distribution of estimated parameters are remarkably similar when we consider perceived time rather than true time. We find that $\hat{\beta}_i$ and $\hat{\beta}_i'$ are positively correlated (PCC = 0.77, p-value < 0.001) and so are $\hat{\alpha}_i$ and $\hat{\alpha}_i'$ (PCC = 0.91, p-value < 0.001). Said differently, participants have very similar time inconsistency and curvature estimates in both models. Interestingly, even though the distributions of $\hat{\delta}_i$ and $\hat{\delta}_i'$ are very similar, the parameters are not significantly correlated (PCC = 0.13, p-value = 0.228). This is not surprising because $\hat{\delta}_i'$
is now applied to perceived time and individuals are highly heterogeneous in their time perception. The new parameter is therefore adjusting for the individual perception biases. Formally, the counterpart of $\delta_t^i$ is now $\hat{\delta}'_i$ and is adjusted so, unlike for parameters $\beta$ and $\alpha$, correlations of $\delta$ across models cannot be studied independently of $t$ (for that very same reason, $\hat{\delta}'_i$ cannot be interpreted as the daily discount factor). Overall, the model introduced here is a reinterpretation of the standard quasi-hyperbolic discounting model in terms of perceived time. According to AIC, both models perform very similarly (AIC = 236 for the initial model and AIC = 237 for the new model), suggesting that information regarding time perception is useful to describe their discounting attitude.

The main conclusion of this section is that a model in which time perception affects the way we perceive delays and evaluate future rewards is very plausible. Note however that the heterogeneity we observe also suggests that other mechanisms may be at play. In other words, we cannot predict with certainty how a subject will discount future rewards based on how he reports experienced time.

7 Concluding remarks

This paper provides experimental evidence on the relationship between time perception and time discounting. Our data reveals a negative correlation between the two: subjects for whom one unit of time feels longer are less willing to delay gratification by that objective amount of time. This result suggests that our ability to delay consumption is related to our mental representation of time delays between now and the future. Our result also indicates the existence of an underlying internal clock that governs time representations irrespective of the unit of time.\textsuperscript{30}

\textsuperscript{30}This conclusion is strengthened by the fact that, according to our retrospective task, there is also a relationship between retrospective and prospective time evaluation (see Appendix A2). Overall, we
Our result is consistent with a growing body of the literature that studies the underlying mechanisms of time related evaluations. Prospective timing has been associated with working memory, a function performed by the dorsolateral prefrontal cortex (dlPFC) (Grondin (2010); Lewis and Miall (2006)). Furthermore, time representation has been shown to involve the striatum and basal ganglia (Ivry and Spencer (2004); Meck (2005)). Interestingly, recent evidence in neuroscience supports the idea that the dlPFC and the striatum are also implicated in time discounting (Van den Bos et al., 2014). This provides a rationale for why time perception and time discounting should be related, as indicated by our data.

The result is also in line with findings obtained in the time discounting and time perception literatures over the life cycle. It has been shown that the subjective perception of the passing time tends to speed up with age, so that people increasingly underestimate time as they age. Ordinary days appear longer for children and shorter for older adults (Block et al. (1999); Coelho et al. (2004)). In parallel, children succumb to immediate gratification while older adults are typically willing to wait for rewards (Green et al. (1999); Lockenhoff et al. (2011)). In other words and consistent with our findings, children are impatient and overestimate time whereas older adults are patient and underestimate time. Interestingly, the dlPFC, which has been shown to be at the core of time related judgments, is late to develop in children (Casey et al., 2005) and early to age (Raz et al., 2005). These points taken together suggest that the relationship between time perception and time discounting and the changes over the life cycle are no coincidence.

Last, we would like to note that the correlation between perceived time and discounting indicates that subjects judge future delays based on current experiences. Said differently, there is a relationship between how long time feels when experienced and how long time is expected to feel. While our result does not prove causality between time perception and time discounting, it suggests that manipulating the current perception of time may affect inter-temporal decisions. The literature in psychophysics has already shown that time perception can be altered by a long series of stressors including changes in body temperature and environmental factors (Droit-Volet and Meck (2007); Meck and MacDonald (2007)). On the discounting side, Ebert and Prelec (2007) have demonstrated that time preferences can also be affected by pressure and attention manipulations. A natural alley for future research is to investigate whether the relationship between time perception and time discounting holds with manipulations and whether it is possible to induce patient choices in an efficient and ecologically valid way. Indeed, most of the policies that target behavior rely on information transmission (tell people that saving for the future is good) or exploit behavioral biases (changing the default option). An alternative is to design environments conjecture that a common mechanism is involved in all time related evaluations.
that promote beneficial decision-making. If time perception is an important driver of
time-related decisions and if it can be manipulated, creating environments in which better
time evaluations are made should improve the way people make inter-temporal trade-offs.
References


Appendix

Appendix A1. Time discounting tasks

First 5 time discounting tasks: \((t, k) = (0, 21)\)

Figure 7: Parameters and presentation of the time discounting task
Appendix A2. Other tasks in the experiment

1. Description of tasks.

The timing was the same in all sessions. We started with the sound of the first bell. We then proceeded to the time discounting task. At the end of the time discounting task, we rang the second bell and asked subjects to estimate the time that passed between the two bells (the one-shot retrospective time estimate task). We then moved to the time perception task. We ended the experiment with the cognitive ability test and the survey. Details of the tasks are provided below.

a. Retrospective time estimate task. This task began with the sound of a bell. Subjects were told “the bell you just heard marked the beginning of the experiment. From now on we would like to have your undistracted attention.” A second bell sound rang later during the experiment. Subjects were then told to estimate the interval of time between the two bells and that they would earn $5.00 paid in cash at the end of the experiment if their estimate was within the real length of the interval ±10%.

b. Cognitive ability test. At the end of the experiment we conducted an incentivized cognitive ability test. We used a short version of Raven’s IQ test, namely Set I of the Raven’s Advanced Progressive Matrices (APM) with a five minute time control, as developed by Raven et al. (1998). This set consists of 12 non-verbal multiple choice questions that become progressively more difficult. Each test item consists of a pattern with a missing element. From the eight choices below the pattern, the subject is to identify the piece that will complete the pattern. Instructions for the test were read directly from the script provided. Subjects were made familiar with the format of the test and method of thought required through two practice problems preceding the test. During this time, they were allowed to ask questions from the experimenters. We incentivized subjects by paying $5 in cash to the top 2 scorers in the test.

c. Demographic survey. We also administered a survey to collect demographic information such as gender, GPA and primary language spoken.

2. Results of the retrospective time evaluation task

The retrospective time estimate task is interesting in that it gives a measure of time perception for intervals longer than a few minutes. However, the data is extremely volatile as it contains a single observation. In our experiment, the time interval between the two bells, \( \nu \), varied between 23min 35s and 41min 36s depending on the sessions.

There is a fundamental difference between the prospective time production task (the one studied in the main paper) and the retrospective time evaluation task. In the prospective task, participants are informed that they have to make a time related judgment and
they base such judgment on their experienced duration. Prospective timing problems have been demonstrated to involve attention (Block and Zakay (1997); Brown (2008)). By contrast, in the retrospective task, participants receive no prior warning that they will have to make a time related judgment and their report relies on remembered duration, a piece of information retrieved from memory. Retrospective timing problems are therefore associated with memory processes and they do not involve attention (Block, 2003).

Research on time perception has mostly focused on prospective timing and in particular on the properties of the ‘internal clock’ – a central mechanism responsible for estimating time – as well as the relationship between time perception and attention (Brown (2008); Grondin (2010)). Even though retrospective and prospective timing are likely to involve different processes, they should also share some. Indeed, estimating a length of time retrospectively or keeping track of a starting time to produce a length is likely to involve similar abilities to “travel in time.” We investigate this possibility by looking at the relationship between the results obtained in both tasks.

For each subject \(i\), we computed the percentage difference between the reported interval between the two bells, \(r_i(\nu)\), and the real interval \(\nu\) in the corresponding session. This gives a measure of the perception bias in the retrospective task: \(PB = \frac{r_i(\nu) - \nu}{\nu}\). Figure 8 presents a histogram with the perception bias of all subjects, after removing four outliers whose bias was below -0.70.

![Figure 8: Distribution of “perception bias”](image)

We notice from Figure 8 that the majority of subjects underestimate time. The average \(PB\) is -0.196 (st. error = 0.025). Next, we computed the percentage difference between the reported interval between the two bells, \(r_i(\nu)\), and the interval we predicted the individual would report based on his estimates (\(\hat{a}_i, \hat{b}_i\)) in the prospective time production task. This gives a measure of the excess bias, above and beyond our subjective time estimate: \(EB = \frac{r_i(\nu) - \hat{a}_i(\nu)\hat{b}_i}{\nu}\). Again, reports are typically lower than the predicted estimates. The average
is -0.095 (st. error = 0.055). It is about one-half of the average PB but also more dispersed. The results on PB and EB are consistent with the existing literature, which suggests that retrospective time is subjectively perceived as shorter than both prospective time and real time (Zakay and Block (2004); El Haj et al. (2013)).

Interestingly, we also found that the excess bias (EB) was strongly correlated with the parameter $b_i$ (PCC = -0.78, p-value = 0.000). Figure 9 depicts this relationship.

![Figure 9: Correlation between $b_i$ and “excess bias”](image)

Subjects with an approximately linear time perception in the prospective task ($b_i \simeq 1$) were reasonably well predicted in the retrospective task. By contrast, subjects with a convex (respectively concave) evaluation of time, that is $b_i > 1.05$ (respectively $b_i < 0.95$), had a lower (respectively higher) estimate of the time delay between the two bell sounds compared to what we predicted. These findings suggest that time evaluation in both tasks are related, indicating that retrospective and prospective time evaluation rely on a common subset of processes. They also point to systematic differences between the two and a specific pattern: subjects with a convex time evaluation perceive time as longer than it truly is and remember past events as passing relatively faster whereas subjects with a concave time evaluation perceive time as shorter than it truly is and remember past events as passing relatively slower.\(^{31}\)

3. Results of the IQ test and survey

We found almost no associations between our results and the answers to the questionnaire. In particular, we did not find any gender effect. At the aggregate level, we found

\(^{31}\)The relationship between prospective and retrospective time evaluation is consistent with the findings in El Haj et al. (2013). The authors found systematic differences for both time evaluations between Alzheimer’s disease patients and healthy controls, indicating the existence of a correlation between the processes underlying both. They also administered a mental time travel task and found that the results obtained in that task were strongly correlated with both retrospective and prospective time evaluation. This indicates that some processes are involved in both mental time travel and time evaluation.
that GPA scores and performance in the IQ test were positively correlated (PCC = 0.28, p-value = 0.01). However, none of them were found to have an effect on time perception or discounting, and their distributions were similar across clusters.

We also investigated the effect of language. Chen (2013) suggests that the way we represent time and we allocate consumption over time might be associated with factors such as culture or language. We found only a small difference between subjects who reported to use English (N = 49) and Chinese (N = 21) as their primary spoken language (6 subjects reported “other” as their primary language). Chinese speakers had higher time perception estimates than English speakers, driven mostly by a higher $b_t$-parameter: $\bar{b}_t = 1.01$ vs. $\bar{b}_t = 0.92$, p-value = 0.025. However, they were not allocated differently across clusters 1 and 2.\footnote{Notice that our subjects are either domestic students or international students living in the US, so cultural differences are likely to be less pronounced than if we compare populations living in different countries (as in Chen (2013) for example).}

\textbf{Appendix A3. Cluster analysis}

The aggregate analysis shows that differences in time perception are associated with differences in impatience but it also suggests substantial heterogeneity in behavior. The objective of this appendix is to investigate in more detail the differences across subjects.

To study heterogeneity, we use the time perception and discount estimates to group individuals with the objective of finding patterns of behavior. We focus on the 92 subjects of the main model, and the time interval $T = 1$ day for both perception and discounting.\footnote{We conducted the same analysis with the other time intervals reported in Table 1 and obtained consistent results. Only a few subjects shifted from one group to another as we shifted the time interval, and none of the differences were significant.} We consider a model-based clustering method to identify the clusters present in our population. A wide array of heuristic clustering methods are commonly used but they typically require the number of clusters and the clustering criterion to be set ex-ante rather than endogenously optimized. Mixture models, on the other hand, treat each cluster as a component probability distribution. Thus, the choice between numbers of clusters and models can be made using Bayesian statistical methods (Fraley and Raftery, 2002). We implement our model-based clustering analysis with the Mclust package in R (Fraley and Raftery, 2006). We consider ten different models with a maximum of nine clusters each, and determine the combination that yields the minimum Bayesian Information Criterion (BIC).\footnote{Hierarchical agglomeration first maximizes the classification likelihood and finds the classification for up to nine clusters for each model. This classification then initializes the Expectation-Maximization algorithm which does maximum likelihood estimation for all combinations of models and number of clusters. Finally, the BIC is calculated for all combinations with the Expectation-Maximization generated parameters.} For our data, the diagonal model with varying volume and shape that endoge-
nously yields three clusters minimizes the BIC. Table 7 presents the average statistics of the time perception and time discounting parameters for subjects within each cluster. Figure 10 provides the same scatterplot as Figure 5, except that subjects are coded by cluster (the ellipses superimposed on the plot correspond to the within-cluster covariances and the mean of each cluster is marked with a * sign).

<table>
<thead>
<tr>
<th>Perception (1 day)</th>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.40 (0.11)</td>
<td>2.17 (0.30)</td>
<td>0.50 (0.16)</td>
</tr>
<tr>
<td>$b$</td>
<td>0.94 (0.01)</td>
<td>0.84 (0.03)</td>
<td>1.23 (0.04)</td>
</tr>
<tr>
<td>Discounting (1 day)</td>
<td>1.02 (0.01)</td>
<td>1.29 (0.09)</td>
<td>0.82 (0.14)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.03 (0.01)</td>
<td>1.30 (0.09)</td>
<td>0.83 (0.14)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.997 (0.00)</td>
<td>0.996 (0.00)</td>
<td>0.997 (0.00)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.95 (0.01)</td>
<td>0.82 (0.06)</td>
<td>0.92 (0.02)</td>
</tr>
<tr>
<td># subjects</td>
<td>68</td>
<td>19</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 7: Summary statistics by cluster for $T = 1$ day

Cluster 1 is close to what we would expect of rational economic agents. Their subjective perception of time is almost linear and reasonably close to the true time, with a slight underestimation on average (which is magnified with the extrapolation). They are also very patient and time-consistent. Cluster 2 is a group of subjects exhibiting a concave time perception function. They tend to significantly underestimate time and, as a consequence, they are more willing to delay consumption than subjects in cluster 1. This is reflected by an extremely high patience (and a future bias). Cluster 3 is a small group of subjects exhibiting a strongly convex time perception, extreme overestimation of time and the lowest discount. They are the only subjects to exhibit a present bias.35

To investigate the significance of differences across clusters, we ran a series of t-tests. We found that the 1 day time discounting and the 1 day time perception are all significantly different across clusters.

35 Against what one may think upon casual inspection, these subjects do not drive the correlations. In fact, correlations are stronger if we remove them from the sample.
Figure 10: Time perception and time discounting by cluster