

Collusion with two privately informed agents¹

Short overview of Laffont and Martimort - Collusion under asymmetric information (1997)

1 The setting

- One Principal
- Two agents A_1 and A_2 producing each q_1 and q_2 with respective marginal costs θ_1 and θ_2 .
- Preferences of A_i are summarized by $U_i = t_i - \theta_i q_i$. Outside option is 0.
- Preferences of Principal: $S(q) - (t_1 + t_2) + \alpha(U_1 + U_2)$ with $q = q_1 = q_2$ (one-to-one technology), $S' > 0$ and $S'' < 0$.
- Third party cares about $U_1 + U_2$
- Timing:
 - Date 0: T offers ex-ante agreement to share equally any asymmetric transfers received by the group. If at least one firm refuses, the agreement is not enforced.
 - Date 1: Nature draws types. Each remains private information of the agent.
 - Date 2: Principal offers grand contract.
 - Date 3: Firms decide to participate. If one refuses, game is over.
 - Date 4: T offers side-contract. If one refuses, the grand contract is played non cooperatively.
 - Date 5: If side-contract accepted by all, message game in the side-contract (non-cooperative play).
 - Date 6: Side transfers as well as manipulated reports to the principal are decided. Message game in the grand contract (cooperative play).
 - Date 7: Grand contract enforced.
- Grand mechanism specifies $(q(\theta'_1, \theta'_2), t_1(\theta'_1, \theta'_2), t_2(\theta'_1, \theta'_2))$
- Side-contract specifies $(\phi(\theta''_1, \theta''_2), y_1(\theta''_1, \theta''_2), y_2(\theta''_1, \theta''_2))$ where $\phi(\cdot)$ is a function that manipulated the reports to determine what to report in the grand mechanism.
- Side-contract must be balanced $y_1(\theta_1, \theta_2) + y_2(\theta_1, \theta_2) = 0$ for all pairs (θ_1, θ_2) .

2 Optimal contracts

- T chooses instruments to maximize the expected welfare under incentive compatibility constraints $U_i(\theta_i) > U_i(\theta'_i) - (\theta_i - \theta'_i)E(q(\phi(\theta'_i, \theta_j)))$; individual rationality constraints $U_i(\theta_i) >$

¹This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

$V_i(\theta_i)$ where $V_i(\theta_i)$ is what is obtained non-cooperatively in the grand contract; and the budget constraint.

- A grand contract giving $V_i(\theta_1)$ and $V_2(\theta_2)$ is collusion-proof if the equilibrium corresponding side-contract is the null side contract.
- The Principal can always obtain with a collusion proof contract what she would obtain in a contract with coalition formation. We can restrict to collusion-proof contracts.

Note: compared to AGV mechanisms, we have an additional IR constraint. Getting efficiency for the team is not ensured.

2.1 Anonymous contracts

- The equilibrium entails pooling because of a tension between the ICs. To separate, the principal needs to increase payments to the high type but this affects the constraints in the coalition.
- The equilibrium entails distortions in quantities.
- At equilibrium, there is an increased rent for the efficient agent.
- There are many equilibria.

2.2 Non anonymous contracts

- Possible if we remove date 0.
- Third best=second best in that case.
- In general, designing non anonymous contracts is not enough to destroy the stake of collusion.