Elements of bargaining theory*

1 Nash Bargaining

Suppose there are 2 players. The utility of each player is denoted by u_i $i = \{1, 2\}$. Players can agree on an allocation of utility, but if they fail, there exist a disagreement point, or status quo $\hat{u}_i$. The Nash bargaining solution consists utility levels u_1 and u_2 such that a series of axioms are satisfied. In particular, the solution must (i) be Pareto optimal; (ii) satisfy the property of independence of irrelevant alternatives (if we remove allocations that are not optimal, the optimal point remains the same in the new set of allocations); (iii) be symmetric (if all agents are identical, then gain is split equally); (iv) be individually rational (cooperative solution makes agents better-off than at the threat point); (v) depend only on cardinal characteristics of the utility functions. The Nash solution is the pair (u_1, u_2) such that $(u_1 - \hat{u}_1) \times (u_2 - \hat{u}_2)$ (the Nash product) is maximized. Nash defines then a non-cooperative game that reaches that outcome. This is a one shot game. Either agents reach an agreement (that is agree on a point in the feasible set), or they get the status quo outcome (threat point). This can be generalized to the case where agents have different bargaining strength.

2 Rubinstein bargaining (alternating offers game)

Two players must agree on how to share a pie of a fixed size, and they bargain over time. This is an infinite horizon game and information is complete. This reflects the fact that bargaining is dynamic and consists of offers and counteroffers. Formally, in even periods, player 1 proposes a sharing rule of the pie and player 2 accepts or rejects. If it is accepted, the game ends and the sharing rule is implemented. Otherwise, player 2 (who plays in odd periods) makes an offer. The discount factor is δ. Suppose the size of the pie is 1, then payoffs are as follows. When players agree on a sharing rule $(x, 1-x)$ at date t, then the payoffs of player 1 and player 2 are respectively $\Pi_1(x, \delta, t) = \delta^t x$ and $\Pi_2(x, \delta, t) = \delta^t(1-x)$. In this game, the players will reach an agreement in period 1 and $x^* = \frac{1}{1+\delta}$.

(Roughly, this is NOT a proof, let v be the equilibrium continuation payoff of player 2 in the game starting tomorrow. If player 1 makes an offer today that is rejected by

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player 2, then player 2 gets δv. Then, player 1 keeps $1 - \delta v$ and offers δv. Given the game is stationary, v is also the equilibrium continuation payoff of player 1 in the game starting today. Therefore we must have $v = 1 - \delta v$, that is $v = \frac{1}{1+\delta}$.)

Note also that the Nash Bargaining Solution is same as the solution to the symmetric alternating offers game when $\delta \to 1$

3 Incomplete Information and bargaining

In the non-cooperative approach of bargaining, it is in general possible to find a game form to reach a particular split of the pie (with complete and incomplete information). Also, there are many equilibria under incomplete information: this is reminiscent of signaling games. The analysis is often restricted to one-sided-offer (only one player makes the offer) games with complete information on that side.