

Asymmetric information and dynamics¹

1 Setting

- Agent privately informed about type $\theta \in \{\underline{\theta}, \bar{\theta}\}$;
- Principal is not informed;
- There are two periods;
- Case of price discrimination.

Period 1, agent's utility is $U_1 = \theta V(q_1) - T_1$

Period 2, agent's utility is $U_2 = \theta V(q_2) - T_2$

Inter-temporal utility is $\theta V(q_1) - T_1 + \delta[\theta V(q_2) - T_2]$ where δ is the discount factor.

2 Static setting

- Complete information: the solution is $(\bar{q}^*, \bar{\theta}V(\bar{q}^*))$ for type $\bar{\theta}$. and $(\underline{q}^*, \underline{\theta}V(\underline{q}^*))$ for type $\underline{\theta}$.
- Asymmetric information: the seller offers

$$(\bar{q}^*, \bar{\theta}V(\bar{q}^*) - \Delta\theta V(\underline{q}^{**})) \quad \text{and} \quad (\underline{q}^{**}, \underline{\theta}V(\underline{q}^{**}))$$

with $\underline{q}^{**} < \underline{q}^*$. Type $\underline{\theta}$ gets $U = 0$ and type $\bar{\theta}$ gets $U = \Delta\theta V(\underline{q}^{**})$.

3 First Best and Dynamics

Replicate the first best of the static case: $\bar{q}_t = \bar{q}^*$ and $\underline{q}_t = \underline{q}^*$ for all t . Also $\bar{T}_t = \bar{\theta}V(\bar{q}^*)$ and $\underline{T}_t = \underline{\theta}V(\underline{q}^*)$ for all t .

4 Long-term contracts under asymmetric information

- The firm offers the second best contract of the static case for both periods. Type θ gets no rent, type $\bar{\theta}$ gets rents twice.
- It is not efficient for the Principal because type is revealed at the end of the first period. In the second period, the principal knows the type but still leaves rents. Besides it is not Pareto efficient because the low type would be indifferent in the second period between the contract he gets and a renegotiated agreement in which he consumes the efficient quantity and get no rent.

¹This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

5 Renegotiation

- Find a mutually beneficial agreement.
- Suppose type is revealed at the end of the first period: if type is $\underline{\theta}$, make him consume \underline{q}^* and leave no rent. The principal is better-off and the agent is indifferent. No renegotiation for type $\bar{\theta}$. This has the property to eliminate ex-post inefficiency.
- Incentives of type $\bar{\theta}$ to reveal his type: if he chooses \bar{q}^* in the first period, then he gets $(1 + \delta)\Delta\theta V(\underline{q}^{**})$. If he chooses \underline{q}^{**} in the first period, then the principal's naive inference is that the type is $\underline{\theta}$ and renegotiation occurs. The agent gets:

$$\begin{aligned} & \bar{\theta}V(\underline{q}^{**}) - \underline{\theta}V(\underline{q}^{**}) + \delta[\bar{\theta}V(\underline{q}^*) - \underline{\theta}V(\underline{q}^*)] \\ &= \Delta\theta V(\underline{q}^{**}) + \delta\Delta\theta V(\underline{q}^*) > \Delta\theta(1 + \delta)V(\underline{q}^{**}) \end{aligned}$$

given $\underline{q}^* > \underline{q}^{**}$.

- The principal anticipates it and designs a renegotiation-proof contract (renegotiation does not take place at equilibrium) such that $\bar{\theta}$ is induced to reveal at some point. The ex-post inefficiency is reduced.

6 Breach of contract (no commitment)

- If the agent's type is revealed to be $\bar{\theta}$, make him consume \bar{q}^* and pay $\bar{\theta}V(\bar{q}^*)$. This eliminates ex-post inefficiency and there is no rent at date 2. However, agents anticipate it.
- Type $\bar{\theta}$ prefers to choose \underline{q}^{**} in the first period.

$$\Delta\theta V(\underline{q}^{**}) + \delta \times 0 < \bar{\theta}V(\underline{q}^{**}) - \underline{\theta}V(\underline{q}^{**}) + \delta[\bar{\theta}V(\underline{q}^*) - \underline{\theta}V(\underline{q}^*)]$$

To make him reveal, we may pay him at date 1 the rent he would get if the Principal would commit, that is give him $\delta\Delta\theta V(\underline{q}^{**})$, but then,

- Type $\underline{\theta}$ prefers to choose \bar{q}^* also:

$$\underline{\theta}V(\bar{q}^*) - [\bar{\theta}V(\bar{q}^*) - \Delta\theta V(\underline{q}^{**})] + \delta\Delta\theta V(\underline{q}^{**}) = \Delta\theta[(1 + \delta)V(\underline{q}^{**}) - V(\bar{q}^*)] > 0$$

when δ is sufficiently large. The Principal must anticipate it.