The dynamics of regulatory contracts[1]

Short overview of Laffont and Tirole - A theory of incentives in regulation and procurement (Handbook)

1 Setting

• Two periods $t = 1, 2$.
• One risk neutral regulator.
• One project per date.
• One risk neutral firm.
• Firm's cost to realize project $C_t = \beta - e_t$ where $\beta \in \{\underline{\beta}, \overline{\beta}\}$ is firm's private information, and e_t is firms's privately selected (non observable) cost-reduction effort at date t.
• The regulator's beliefs at date t is $\nu_t(\underline{\beta})$
• C_t observable ex post.
• Disutility of effort $\psi(e_t)$, with $\psi(\cdot)$ increasing, convex, $\psi(0) = 0$ and $\psi'''(\cdot) > 0$.
• Regulator designs a contract specifying a payment contingent on the observed cost. We take the convention that costs are reimbursed, so the payment is simply an "extra subsidy".
• Firm's profit at each period is $U_t = t_t - \psi(e)$ where t_t is subsidy at date t.
• Regulator's welfare at each period is $W_t = S - (1+\lambda)[t_t + C_t] + U_t$ where λ is the shadow cost of public funds.

2 Static problem

• Under complete information, $t = \psi(e)$, $U = 0$, and e is set to maximize $S - (1+\lambda)[\psi(e) + \beta - e]$, yielding e^* such that $\psi'(e^*) = 1$ for all β.
• Under incomplete information, the problem is an adverse selection problem because if C is observed, β is revealed, then e can be deduced. The regulator offers $(t(\underline{\beta}), C(\underline{\beta}))$ and $(t(\overline{\beta}), C(\overline{\beta}))$. So at equilibrium, type $\underline{\beta}$ will exert $\underline{e}$ and type $\overline{\beta}$ will exert $\overline{e}$. Denote by $\underline{U}$ and $\overline{U}$ the equilibrium utility of type $\underline{\beta}$ and $\overline{\beta}$ respectively. The objective of the principal can be rewritten as

$$S - (1+\lambda)\nu[\psi(\underline{e}) + \underline{\beta} - \underline{e}] - (1+\lambda)(1-\nu)[\psi(\overline{e}) + \overline{\beta} - \overline{e}] - \lambda\nu\underline{U} - \lambda\nu\overline{U}$$

Individual rationality constraints are simply $\underline{U} \geqslant 0$ and $\overline{U} \geqslant 0$. Incentive compatibility constraints are $\underline{U} \geqslant \overline{U} + \psi(\overline{\beta} - C(\overline{\beta})) - \psi(\underline{\beta} - C(\overline{\beta}))$ and $\overline{U} \geqslant \underline{U} + \psi(\underline{\beta} - C(\underline{\beta})) - \psi(\overline{\beta} - C(\underline{\beta}))$.

[1]This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

Solving the optimization program yields $\overline{U} = 0$, $\underline{U} = \psi(\overline{\beta} - C(\overline{\beta})) - \psi(\underline{\beta} - C(\overline{\beta})) \equiv \psi(\overline{e}) - \psi(\overline{e} - \Delta\beta) = \phi(\overline{e})$, $\underline{e} = e^*$ and $\overline{e} < e^*$. Precisely, $\overline{e}$ is such that $\psi'(\overline{e}) = 1 - \frac{\lambda\nu}{(1+\lambda)(1-\nu)}\phi'(\overline{e})$.

3 Regulation with Commitment

• Regulator offers mechanism $(t_1(\tilde{\beta}), C_1(\tilde{\beta}), t_2(\tilde{\beta}), C_2(\tilde{\beta}))$. Firm accepts or refuses. Firm announces $\tilde{\beta}$. Projects is realized as well as payoffs (as specified).

• First Best: as in the static case, the optimal effort is $e_t = e^*$ with $\psi'(e^*) = 1$ for all t; and the payment $t_t = \psi(e^*)$.

• Second Best: $\underline{e}_t = e^*$, $t(\underline{\beta}) = \phi(\overline{e}) + \psi(e^*)$, $\overline{e}_t < e^*$, $t(\overline{\beta}) = \psi(\overline{e})$. Note that $\overline{e}$ will be denoted sometimes $\overline{e}(\nu)$ where ν is the prior belief, and it is a decreasing function of the prior.

• At date 2, $\overline{\beta}$ exerts less effort than first best: the optimal contract with commitment is inefficient.

4 Regulation without Commitment

4.1 The case of short-term contracts

• First Best. As in the static case and the dynamic case with commitment.

• Revelation principle: the regulator wants to renegotiate and the Revelation principle does not apply. She offers mechanism $(C_1, t_1(C_1))$, in period 1 and $(C_2, t_2(C_2; t_1(C_1), C_1)))$ in period 2.

• Second Best. If the agent reveals truthfully, the regulator extracts all rents in the second period. Then $\underline{e}_2 = \overline{e}_2 = e^*$, $t_1 = t_2 = \psi(e^*)$. Type $\overline{\beta}$ does not have incentives to lie, but type $\underline{\beta}$ does. To induce truth-telling, he must be paid future rents upfront (receive rent he will not get tomorrow if he reveals). However, this creates now an incentive for type $\overline{\beta}$ to lie, take the money and run.

• Possible types of equilibria are: separation (with distortions), semi-separation, pooling.

• In the second period, when firm realizes C_1, then regulator learns about his type. Let $x_1(C_1, \beta)$ be the probability that type β choose C_1. The posterior beliefs at date 2 are (by Bayes'rule):

$$\nu_2(\underline{\beta}, C_1) \equiv \nu_2(C_1) = \frac{x_1(C_1, \underline{\beta})\nu_1(\underline{\beta})}{x_1(C_1, \underline{\beta})\nu_1(\underline{\beta}) + x_1(C_1, \overline{\beta})(1 - \nu_1(\underline{\beta}))}$$

and the second-period contract is the static optimal contract given beliefs ν_2 and $(1-\nu_2)$. We have $\underline{e}_2 = e^*$; $\overline{e}_2(\nu_2) < e^*$, $t_2(\underline{\beta}) = \psi(e^*) + Z(\nu_2)$, $t_2(\overline{\beta}) = \psi(\overline{e}_2(\nu_2))$ where $Z(\nu_2) = \phi(\overline{e}_2(\nu_2))$.

• In the first period, separation occurs if there exists a contract $(\underline{C_1}, t_1(\underline{C_1}))$ and $(\overline{C_1}, t_1(\overline{C_1}))$ such that $x_1(\underline{C_1}, \underline{\beta}) = 1$ and $x_1(\overline{C_1}, \overline{\beta}) = 1$. We need

$$t_1(\underline{C_1}) - \psi(\underline{\beta} - \underline{C_1}) \geq 0 \quad (\underline{\text{IR}})$$
$$t_1(\underline{C_1}) - \psi(\underline{\beta} - \underline{C_1}) \geq t_1(\overline{C_1}) - \psi(\underline{\beta} - \overline{C_1}) + \delta Z(\nu_2(\overline{C_1})) \quad (\underline{\text{IC}})$$
$$t_1(\overline{C_1}) - \psi(\overline{\beta} - \overline{C_1}) \geq 0 \quad (\overline{\text{IR}})$$
$$t_1(\overline{C_1}) - \psi(\overline{\beta} - \overline{C_1}) \geq t_1(\underline{C_1}) - \psi(\overline{\beta} - \underline{C_1}) \quad (\overline{\text{IC}})$$

When $\delta \to 0$, contract tends to the optimal static contract. As $\delta \uparrow$ rents $\uparrow$ and the IC of $\overline{\beta}$ is less likely to be satisfied (take-the-money-and-run). Still, as long as δ is small enough, there is full separation. Equilibria can be compared and ranked.

5 Commitment and Renegotiation

• Long-term (two-period) contract with possibility of renegotiation at the end of first period if regulator and firm agree.

• First period: regulator offers long term contract $t_1(C_1), t_2^0(C_1, C_2))$ which yields second-period rents $\underline{U}_2^0$ for $\underline{\beta}$ and $\overline{U}_2^0$ for $\overline{\beta}$. Suppose that $\overline{U}_2^0 = 0$.

• Second period: after observation of C_1, regulator updates belief ($\nu_2(C_1)$) and regulator offers new contract $t_2(C_2)$.

• Second period game: the revelation principle applies, the contract must satisfy:

$$t_2(\underline{\beta}) - \psi(\underline{e}_2) \geq \underline{U}_2^0$$
$$t_2(\underline{\beta}) - \psi(\underline{e}_2) \geq t_2(\overline{\beta}) - \psi(\overline{e}_2 - \Delta\beta)$$
$$t_2(\overline{\beta}) - \psi(\overline{e}_2) \geq \overline{U}_2^0 = 0$$
$$t_2(\overline{\beta}) - \psi(\overline{e}_2) \geq t_2(\underline{\beta}) - \psi(\underline{e}_2 + \Delta\beta)$$

The regulator chooses contract that maximizes social welfare given beliefs $\nu_2(C_1)$ under these constraints.

• A **renegotiation-proof contract** is a long-term contract in which the negotiation game is anticipated. The renegotiation-proof contract is rent-constrained in period 2. It entails separation or pooling.

• When δ is small, the optimal renegotiation-proof contract is a separating one. When δ increases, it tends towards pooling.

• Optimal contract with commitment is not renegotiation-proof.

• Renegotiation-proof long-term contracts perform better than short-term contracts: avoid take-the-money-and-run strategy.

• Even if commitment, revelation principle does not apply. Indeed, there is no full commitment since there is no commitment to not renegotiate.