Asymmetric information and the informed principal[1]

1 Game theoretic approach of signaling

• Sender (S) informed about type θ and chooses a_s at stage 1;

• Receiver (R) uninformed about θ and prior belief is a function $p(\theta)$;

• R chooses a_r at stage 2, after observing a_s;

• Payoffs are $u_s(a_s, a_r, \theta)$ and $u_r(a_s, a_r, \theta)$ respectively for S and R;

• At stage 2, R observes a_s, and updates his belief. The posterior is a function $\pi(\theta|a_s)$ obtained by guessing that S chooses a strategy at state 1 (guess must coincide with actual strategy at equilibrium), and posterior is computed using Bayes rule. R chooses a_r such that

$$a_r^*(a_s) = \text{argmax}_{a_r} E_\theta[\pi(\theta|a_s)\, u_r(a_s, a_r, \theta)]$$

• At stage 1, S anticipates how R will play at stage 2, and chooses a_s such that:

$$a_s^*(\theta) = \text{argmax}_{a_s} u_s(a_s, a_r^*(a_s), \theta)$$

• Bayesian Rationality condition. The receiver makes inferences that must be correct.

• Sequential rationality condition. The sender chooses an action to maximize his own payoff: the action must be incentive compatible for the sender.

• Separating equilibrium: type is fully revealed at equilibrium. It is necessary to specify out-of-equilibrium beliefs that sustain this equilibrium.

2 Setting for contracting

• The Sender is Principal, the Receiver is Agent. The action of the sender is a contract.

• One Principal, informed about his type α

• One agent informed about his type θ.

Principal designs a contract consisting of an allocation x and a payment t.

• Private values: the agent is not affected directly by α. His utility is $u_A(x, t, \theta)$.

• Common values: the agent is affected directly by α. His utility is $u_A(x, t, \theta, \alpha)$

3 Optimal contracting with Private values

• Suppose α is known, then the Principal designs a contract contingent on reports on θ, and must satisfy IR and IC:

[1]This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

$$u_A(x(\theta), t(\theta), \theta) \geqslant 0, \qquad u_A(x(\theta), t(\theta), \theta) \geqslant u_A(x(\theta'), t(\theta'), \theta)$$

We call this case full information and the optimal contracts are denoted by $(x^{\alpha}(\theta), t^{\alpha}(\theta))$.

• When α is unknown, the Principal does not need to restrict herself to contracts of the form $(x(\theta), t(\theta))$. She can hold information until the message game and submit a report. Then , a contract takes the form $(x(\theta, \alpha), t(\theta, \alpha))$. When designing it, she must satisfy IR and IC:

$$E_{\alpha} u_A(x(\theta, \alpha), t(\theta, \alpha), \theta) \geqslant 0, \qquad E_{\alpha} u_A(x(\theta, \alpha), t(\theta, \alpha), \theta) \geqslant E_{\alpha} u_A(x(\theta', \alpha), t(\theta', \alpha), \theta)$$

Overall, she must satisfy constraints only on average.

• Principal with type α can always offer the full information contract $(x^{\alpha}(\theta), t^{\alpha}(\theta))$. Given values are private, beliefs about α have no role to play and given it is IC and IR (by construction), it is feasible. The Principal can therefore secure the full information outcome.

• Hiding information in the proposal stage is optimal. All principals offer the same menus of contract (and no inference can be made on α) and move in the message game.

4 Optimal contracting with Common values

• Suppose α is known, then the Principal designs a contract contingent on reports on θ, and must satisfy IR and IC:

$$u_A(x(\theta), t(\theta), \theta, \alpha) \geqslant 0$$

$$u_A(x(\theta), t(\theta), \theta, \alpha) \geqslant u_A(x(\theta'), t(\theta'), \theta, \alpha)$$

optimal contracts are denoted $(x^{\alpha}(\theta), t^{\alpha}(\theta))$.

• If Principal's type is α and she offers the full information contract $(x^{\alpha}(\theta), t^{\alpha}(\theta))$, the utility of the agent still depends on beliefs. For instance suppose there are two types $\underline{\alpha}$ and $\overline{\alpha}$ and optimal full information contracts are such that $u_A(x^{\underline{\alpha}}(\theta), t^{\underline{\alpha}}(\theta), \overline{\alpha}) \leqslant 0$. If the principal offers $(x^{\underline{\alpha}}(\theta), t^{\underline{\alpha}}(\theta))$ and the agent believes for some reason that type is $\overline{\alpha}$, he rejects the contract.

• Signaling type is beneficial. However, the full information solution is not incentive compatible for the Principal. If low type Principal could induce the agent to accept the full information contract of a high type Principal, she would be better-off.

$$w(x^{\overline{\alpha}}(\theta), t^{\overline{\alpha}}(\theta), \overline{\alpha}) \geqslant w(x^{\underline{\alpha}}(\theta), t^{\underline{\alpha}}(\theta), \underline{\alpha})'$$
$$w(x^{\underline{\alpha}}(\theta), t^{\underline{\alpha}}(\theta), \underline{\alpha}) \leqslant w(x^{\overline{\alpha}}(\theta), t^{\overline{\alpha}}(\theta), \underline{\alpha}).$$

It is possible to achieve a separating equilibrium: the agent infers the true type and there is no uncertainty when he decides to participates. Then, IC and IR must hold pointwise (not in expectation only).