Optimal contracting with moral Hazard[1]

Short overview of Holmstrom (1979)

1 Setting

- One agent with objective $u(t,a) = v(t) - c(a)$ where t is a transfer.
- One principal with objective $w(y-t)$.
- Outcome is of the form $y = a + \epsilon$ where ϵ is a noise.
- Contract specifies t as a function of observable outcome y.

2 Optimal contracting

- Program of the Principal

$$\max_{t(y)} \int w(y - t(y)) dF(y|a)$$

$$\begin{aligned} &\int [v(t(y)) - c(a)] dF(y,a) \geqslant \int [v(t(y)) - c(a')] dF(y,a') \qquad \forall\, a' \neq a \\ &\int [v(t(y)) - c(a)] dF(y,a) \geqslant 0 \end{aligned}$$

- We do not know whether the utility of the agent is concave. A method consists in assuming it is, replacing the MH constraint by the FOC $\int v(t(y)) f_a(y,a) = c'(a)$.
- Apply multipliers μ and λ to the two constraints and consider the pointwise optimization of the Lagrangian $\int L(t(y)) f(y,a) dy$ where

$$L(t(y)) = w(y - t(y)) + \mu \tfrac{1}{f(y,a)} [v(t(y)) f_a(y,a) - c'(a)] + \lambda [v(t(y)) - c(a)].$$

- Optimal $t(y)$ satisfies $-w'(y - t(y)) + \mu v'(t(y)) \frac{f_a(y,a)}{f(y,a)} + \lambda v'(t(y)) = 0$ that is:

$$\frac{w'(y - t(y))}{v'(t(y))} = \mu \frac{f_a(y,a)}{f(y,a)} + \lambda \tag{1}$$

The first term reflects the distortion caused by moral hazard. If we differentiate this equation with respect to y, assume $v'' < 0$, $v' > 0$, $w'' < 0$, $w' > 0$ and make specific assumptions on the distribution, we can show that $t'(y) > 0$.

- Standard assumptions. (i) First order stochastic dominance: $F_a \leq 0$, the higher a, the higher the probability that y is high. (ii) Monotone likelihood ratio property: $\frac{\partial}{\partial y} \frac{f_a(y,a)}{f(y,a)} > 0$, production is positively related to action, if action increases, then high output relatively more likely.

[1] This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

3 Moral Hazard and risk aversion of parties

• First Best and risk neutral Principal: transfer satisfies $\frac{1}{v'(t(y))} = \lambda$. If the agent is risk-averse, then it has to be the case that $t(y) = k$. The Principal bears all the risk. If agent is risk neutral, payment must be such that IR is satisfied at optimal effort, but wage does not need to be constant.

• Moral hazard and risk neutral Principal: if agent is risk neutral, it is possible to implement the first best by giving a payment equal to the value of the output up to a constant. If agent is risk averse, agent requires higher payments to implement the same thing. At equilibrium, there will be distortions.