

Asymmetric information and the uninformed principal¹

1 The setting

- One Principal with full bargaining power.
- Contract or mechanism denoted by \mathcal{M} .
- One or several agents. Suppose only one.
- Agent is privately informed about a characteristics, called type θ .
- Principal knows θ is drawn from common knowledge distribution $F(\cdot)$ with density $f(\cdot)$ on $\Theta = [\underline{\theta}, \bar{\theta}]$.

Typical situation can be summarized by the following three-stage game.

Stage 1: the Principal announces \mathcal{M} as a function of what may happen in the relationship.

Stage 2: the agent “responds” to this mechanism by making a choice. By doing this, he sends a *message* to the principal. This stage is called the message game.

Stage 3: the Principal implements the choice of the agent, conditional on the message sent.

- Principal’s objective: $w(\mathcal{M}, \theta)$.
- Agent’s objective: $u(\mathcal{M}, \theta)$.
- Agent’s outside option: normalized to 0.
- Principal wants to induce the agent to participate. Then, \mathcal{M} must be such that $u(\mathcal{M}, \theta) \geq 0$, called individual rationality constraint (IR).

2 The revelation principle

It is optimal to design \mathcal{M} as a function of all possible messages (see complete contract).

Denote by $y(m)$ the “allocation” contingent on message m

- Agent with type θ sends $m^*(\theta) = \operatorname{argmax}_m u(y(m), \theta)$ and gets $y(m^*(\theta))$.
- Direct mechanism such that $y(m^*(\theta)) = y^*(\theta)$ achieves the same payoffs.
- When offered $y^*(\theta)$, by construction type θ prefers to send message θ .

\Rightarrow Restrict attention to direct mechanisms that satisfy incentive compatibility (IC): $u(y(\theta), \theta) \geq u(y(\theta'), \theta)$ for all θ, θ' .

¹This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.

3 Optimal contracting

The problem of the Principal boils down to:

$$\begin{aligned} \max_{y(\theta), \theta \in \Theta} E_{\theta}(w(y(\theta), \theta)) \\ u(y(\theta), \theta) \geq u(y(\theta'), \theta) \quad \forall \theta' \neq \theta, \forall \theta \\ u(y(\theta), \theta) \geq 0 \end{aligned}$$

Remarks on Incentive compatibility

- (IC) is equivalent to two necessary and sufficient conditions denoted by (IC1) and (IC2). (IC1) states how equilibrium utilities vary with θ . (IC2) is a ‘monotonicity’ condition: it says how the allocation varies with θ . See Myerson (1981) for details.
- Differential approach: it is sometimes possible to write IC as $\frac{\partial}{\partial \theta'} u(y(\theta'), \theta) = 0$ at $\theta' = \theta$.
- Spence-Mirrlees condition (or sorting or single crossing condition). Suppose that the contract consists of a quantity q and a payment t . It says that the isocurves of types θ and type θ' cross only once.
- Bunching. Occurs when the optimal solution of the unconstrained problem does not satisfy (IC2).
- Countervailing incentives. It happens when IR depends on the type. This modifies the incentives to report truthfully.

4 Implementation of the first best solution

- Suppose two agents, agent 1 and agent 2 with types θ_1 and θ_2 .
- Utility of agent i is $v_i(x, \theta_i) + t_i$ where x is an allocation that affects all agents (e.g. a public good) and t_i an individual transfer.
- Budget constraint $t_1 + t_2 \leq 0$ (BB).
- Look for solution that maximizes welfare $v_1(x, \theta_1) + t_1(\theta) + v_2(x, \theta_2) + t_2(\theta)$.
- Efficiency (or first-best): budget is balanced and solution is $f^*(\theta) = \operatorname{argmax} v_1(x, \theta_1) + v_2(x, \theta_2)$
- Problem: how to implement this solution under asymmetric information.
- Incentive compatibility:

(1) Telling the truth as a dominant strategy: $u_i(x(\theta_i, \tilde{\theta}_{-i}), \theta_i) \geq u_i(x(\tilde{\theta}_i, \tilde{\theta}_{-i}), \theta_i)$. See Clarke (1971) and Groves (1973). Possible to implement with externality payments (agents and planner have same interests). But, BB fails

(2) Telling the truth on average (bayesian incentive compatibility):

$E_{\theta_{-i}}[u_i(x(\theta_i, \theta_{-i}), \theta_i)] \geq E_{\theta_{-i}}[u_i(x(\tilde{\theta}_i, \theta_{-i}), \theta_i)]$. See D’Aspremont and Gérard-Varet (1979). Again implementation with externality payments, and here (BB) is satisfied. However, it may conflict with (IR).