Optimal Auctions[1]

Short overview of Myerson (1981)

1 Setting

• One risk neutral seller o

• One single object

• n risk neutral bidders. v_i is bidder i' s valuation. It is private information.

• v_i s independently drawn from distribution $F(\cdot)$ with density $f(\cdot)$ on $V = [\underline{v}, \overline{v}]$.

• outside option of buyer is 0.

Also, v_o is seller's valuation and is common knowledge. Normalize to 0.

• Mechanism is an allocation rule X and payments t_i for all $i = 1, ...n$.

2 Optimal mechanism design

• Revelation principle: restrict to direct mechanism that satisfy Bayesian IC (BIC).

• Objective: expected payments.

• Feasibility: mechanism is feasible if BIC, IR, as well as $\sum X_i \leq 1$ and $X_i \geq 0$.

• IC can be rewritten as combination of (IC1) and (IC2):

(IC1) $\quad U_i(v_i, v_i) - U_i(\tilde{v}_i, \tilde{v}_i) = \int_{\tilde{v}_i}^{v_i} E_{v_{-i}} x_i(v_{-i}, s) ds$

(IC2) $\quad E_{v_{-i}} X_i(v_{-i}, v_i) \geq E_{v_{-i}} X_i(v_{-i}, \tilde{v}_i)$;

• Combining (IC1) and (IR), $U_i(v_i, v_i) = \int_{\underline{v}}^{v_i} E_{v_{-i}} X_i(v_{-i}, s) ds$, which can be replaced in the objective of the seller. The objective writes now as: $\int_V \sum_{i=1}^n E_{v_{-i}} X_i(v)[v_i - \frac{1-F(v_i)}{f(v_i)}] f(v_i) dv_i$

• Regular case

If $g(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$ is monotone strictly increasing in v_i (monotone hazard rate property), the optimal auction mechanism is:

$$X_i^*(v_i, v_{-i}) = \begin{cases} 1 & if\ v_i > \max_{j \neq i}\ v_j > r_o \\ 0 & otherwise \end{cases}$$

where $r_o > 0$ and is such that $g(r_o) = 0$.

• Inefficiency: if $\max v_i \in [v_o, r_o]$, the seller does not allocate the object.

• The optimal auction mechanism can be implemented by simple auctions procedures with reserve price r_o.

[1] This document is intended to provide only a few take-home messages. It is not a substitute for attending class and taking notes.