Hiring under asymmetric information

Sketch of solution

Q1: $\max \ \Pi(q) - t$ s.t. $t - \theta c(q) \geq 0$ $\Rightarrow$ $\max\limits_{q} \ \Pi(q) - \theta c(q)$

q_1^*: $\Pi'(q_1^*) = \theta_1 c'(q_1^*)$ $\qquad t_1^* = \theta_1 c(q_1^*)$

q_2^*: $\Pi'(q_2^*) = \theta_2 c'(q_2^*)$ $\qquad t_2^* = \theta_2 c(q_2^*)$.

Q2: $\max \ p\left(\Pi(q_1) - t_1\right) + (1-p)\left(\Pi(q_2) - t_2\right)$

s.t.
$$t_1 - \theta_1 c(q_1) \geq 0$$
$$t_2 - \theta_2 c(q_2) \geq 0$$
$$t_1 - \theta_1 c(q_1) \geq t_2 - \theta_1 c(q_2)$$
$$t_2 - \theta_2 (q_2) \geq t_1 - \theta_2 c(q_1)$$

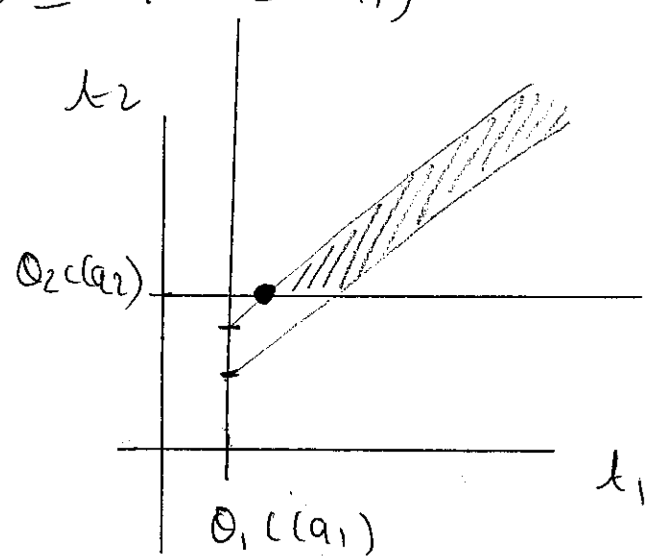
<u>Step 1</u>

$t_1 \geq \theta_1 c(q_1)$

$t_2 \geq \theta_2 c(q_2)$

$t_2 \leq \theta_1 (c(q_2) - c(q_1)) + t_1$

$t_2 \geq \theta_2 (c(q_2) - c(q_1)) + t_1$



$t_2^{**} = \theta_2 c(q_2)$

$t_1^{**} = \theta_1 c(q_1) + \underbrace{(\theta_2 - \theta_1)}_{=\Delta\theta} c(q_2)$

step2

$$\max \quad p\left(\pi(q_1) - \theta_1 c(q_1) - \Delta\theta c(q_2)\right) + (1-p)\left(\pi(q_2) - \theta_2 c(q_2)\right)$$

$$q_2 \le q_1$$

FOC q_1 $\qquad \pi'(q_1) - \theta_1 c'(q_1) = 0 \qquad q_1^{**} = q_1^*$

FOC q_2 $\qquad \pi'(q_2) - \theta_2 c'(q_2) = \overbrace{\frac{p}{1-p}\Delta\theta c'(q_2)}^{+} \qquad q_2^{**} < q_2^*$