University of Southern California
Graduate Program

ECON 537
Contracts, Organizations and Institutions
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Problem sets

The following problems will be solved in class

Price discrimination in the wine market

Consider a local monopoly on the wine market. The cost of one bottle is 20. The utility of each consumer is represented by $\theta V(q)-t$ where θ is a taste parameter, q the quantity he buys and t the payment he makes. Assume that $V(q)=0.25[1-(1-q)^2]$. In other words, the utility represents the surplus of the consumer after payment and consumption. The monopoly does not know the true value of θ but knows that it is either $\theta_1=200$ with probability 3/4 or $\theta_2=600$ with probability 1/4. Last, in case a consumer does not buy a bottle of wine, he has to drink water which gives him a utility equal to 0.

(a) Suppose the monopolist knows exactly the tastes of his clients. What is the optimal pricing strategy. Draw the allocation in a graph quantity/transfer.

(b) Suppose now that the monopolist only knows the distribution of types but cannot deduce the type of each consumer. If the monopolist offers the allocation he chooses under complete information, what can we expect consumers will choose?

(c) characterize the optimal pricing policy under the assumption of question (b). Explain.

Does the auction format matter?

There are two symmetric agents bidding for a good. The willingness to pay (or valuation) of agent 1 (resp. agent 2) is v_1 (resp. v_2) and is private information. The willingness to pay is drawn from a uniform distribution on (0,1).

(a) Suppose the auctioneer sells the good through a second price sealed bid auction. Agent 1 guesses that agent 2 will bid according to an increasing bid function b(.), namely he expects him to bid $b_2=b(v_2)$.

What is the expected utility of agent 1 in the auction $u(v_1,b_1)$ if he bids b_1?

Characterize the optimal bid b_1.

What is the equilibrium of the game (the bidding strategy of agent 2 must be compatible with the bidding strategy of agent 1, it is a Nash equilibrium).

What is the expected payment of agent 1?

(b) Suppose the auctioneer sells the good through a first price sealed bid auction. Agent 1 guesses that agent 2 will bid according to an increasing bid function b(.).

What is the expected utility of agent 1 if he bids b_1?

In fact, agent 1 guesses the bidding function of agent 2 is of the form $b(v_2)=v_2\ (1-\alpha)$ with α in (0,1). Characterize the optimal bid b_1.

If agent 1 makes that bid, what is the optimal bid of agent 2?

Characterize the Nash equilibrium.

What is the expected payment of agent 1?

Compare your results in questions (a) and (b). Explain the differences and/or similarities.

How to make employees work?

The employer (she) asks the employee (he) to perform a task. The employee will exert some effort $0 \leq e \leq 1$ which affects his probability of success in the task: when the employee exerts e, he is successful with probability $p(e)=e$ and fails with probability $1-p(e)=1-e$. The value of success to the employer is 1, while failure is valued 0. The employer designs a contract to provide incentives for effort. More precisely, she offers wages w_S in case of success and w_F in case of failure. Therefore, from the perspective of the employer, the net value of success is $1-w_S$, and the net value of failure is $0-w_F$. When the employee exerts e, he incurs a disutility $C(e)=0.5\ e^2$. Last, if the employee does not accept the incentive scheme offered by the employer, his reservation utility is 0.

(a) Write the expected utility of the manager.
(b) Write the expected utility of the employee.
(c) If the effort is observable, what is the maximization program of the employer?
(d) If the effort is observable, what expected utility is left to the employee in equilibrium?
(e) Characterize the optimal effort when effort is observable, as well as equilibrium wages.
(f) Suppose now effort is not observable. What additional constraint the employer must take into account? Write that constraint.
(g) What is the maximization program of the employer when the effort is not observable?
(h) Consider an effort e. Determine the wage w_S as a function of w_F such that the agent prefers to select that given effort rather than any other level of effort. What is the expected utility of the agent in that case? Under which condition on w_F the agent accepts the contract?
(i) Suppose there is no restriction on the wages: they can be negative if needed, provided that the agent finds profitable to accept the incentive scheme. Is it possible for the employer to make the employee choose the same effort as in (e)? What are the wages in that case?
(j) Suppose the employer cannot punish the employee in case of failure and must set $w_F = 0$. Determine the wage w_S such that the agent prefers to select a given effort e rather than any other level of effort. What is the expected utility of the agent in that case? Is the contract accepted? Characterize the optimal effort, as well as the equilibrium w_S.

Signaling quality

1- A seller is offering one product. A consumer thinks the product is of good quality with probability ¼ and of bad quality with probability ¾. The consumer values the good quality product 100 and the bad quality product 40. The seller knows that he can sell the good quality product (resp. bad quality product) at price 50 (resp. at price 20) in the future (that is in case the consumer does not buy today). Given this, what quality is sold on the market and at what price?

2- In fact, it costs 10 to produce a bad quality product and 15 to produce a good quality product. The seller is also the producer and maximizes his profit. Suppose the consumer keeps the same beliefs as in question 1. Which product is produced in equilibrium and what is the profit per unit sold? Are the probabilities assigned by the consumer correct then? If not, what are the correct probabilities, equilibrium price on the market and profit of the seller? Explain.