Product Differentiation

Horizontal differentiation

- Continuum of consumers located uniformly between 0 and 1
- Each consumer wants to buy 1 unit of a good
- Disutility from distance, transportation cost: $t(d) = td^2$

d is distance between consumer's location and point of purchase

- Two firms, each with cost $C(q) = c \cdot q$
- Game:

  - Stage 1: Firms locate simultaneously:

  firm 1 chooses a and firm 2 chooses $1 - b$.

  - Stage 2: Firms choose prices simultaneously:

  firm 1 chooses p_1 and firm 2 chooses p_2.

  - Stage 3: Each consumer chooses a firm and buys

Stage 3

Each consumer observes locations and prices.

Consider consumer located at point x:

Total price incurred from purchasing from firm 1 is

$$p_1 + t(x-a)^2$$

Total price incurred from purchasing from firm 2 is

$$p_2 + t(1-b-x)^2$$

Consumer is indifferent if $p_1 + t(x-a)^2 = p_2 + t(1-b-x)^2$

$\Rightarrow$ there exists $x^*(p_1, p_2, a, b) = \frac{p_2 - p_1}{2t(1-a-b)} + \frac{1-b+a}{2}$ such that

for all $x < x^*$, consumers purchase from firm 1 and

for all $x > x^*$, they purchase from firm 2.

$\Rightarrow$ The demand faced by firm 1 is $x^*(p_1, p_2, a, b)$

The demand faced by firm 2 is $1 - x^*(p_1, p_2, a, b)$.

Stage 2

Each firm observes locations and anticipates how consumers will act at stage 3.

• Firm 1 $\max_{p_1}(p_1 - c)x^*(p_1, p_2, a, b)$. The first-order condition is

$$\frac{p_2 - p_1}{2t(1-a-b)} + \frac{1-b+a}{2} - (p_1 - c)\frac{1}{2t(1-a-b)} = 0$$

Therefore, the best response is

$$p_1^*(p_2, a, b) = 1/2[t(1-b+a)(1-a-b) + p_2 + c]$$

• Firm 2 $\max_{p_2}(p_2 - c)[1 - x^*(p_1, p_2, a, b)]$. Its best response is

$$p_2^*(p_1, a, b) = 1/2[t(1-a+b)(1-a-b) + p_1 + c]$$

• The equilibrium prices are $p_1^*(a, b)$ and $p_2^*(a, b)$ where $p_1^*(a, b) = p_1^*(p_2^*(a, b), a, b)$ and $p_2^*(a, b) = p_2^*(p_1^*(a, b), a, b)$. Formally,

$$p_1^*(a, b) = c + t(1-a-b)(1 - \frac{b-a}{3})$$
$$p_2^*(a, b) = c + t(1-a-b)(1 - \frac{a-b}{3})$$

The consumer indifferent between the two options is located at $x^*(a, b) = x^*(p_1^*(a, b), p_2^*(a, b), a, b)$.

$\Rightarrow$ If firms have same location ($a = 1 - b$): $p_1 = p_2 = c$
$\Rightarrow$ If $t = 0$: $p_1 = p_2 = c$

Stage 1

Each firm anticipates how consumers will act at stage 3 and how prices will be set at stage 2.

- Firm 1 $\max_a (p_1^*(a,b) - c)x^*(a,b) \equiv \max_a \frac{t}{18}(1-a-b)(\frac{3-b+a}{3})^2$.

Profit is decreasing in a therefore $a^* = 0$.

- Firm 2 $\max_b (p_2^*(a,b) - c)(1 - x^*(a,b))$ and similarly $b^* = 0$.

$\Rightarrow$ There is maximum differentiation

$\Rightarrow$ Equilibrium prices are $p_1^* = p_2^* = c + t$

$\Rightarrow$ $x^* = 1/2$: firms share the market.

Vertical differentiation

• Continuum of consumers with tastes parameters on $[\underline{\theta}, \overline{\theta}]$

Assume that $\overline{\theta} - 2\underline{\theta} > 0$.

• Each consumer wants to buy only one unit of a good

• s represents the quality of the good

• Utility of a consumer with taste θ buying a good of quality s at price p is

$$\theta s - p$$

• Two firms, each with cost $C(q) = c \cdot q$

• Game:

Stage 1: Firms choose qualities s_1 and s_2 simultaneously.

Stage 2: Firms choose prices p_1 and p_2 simultaneously.

Stage 3: Each consumer chooses firm and buys

Stage 3

Each consumer observes qualities and prices.

• The consumer indifferent between the two options has a taste such that $\theta s_1 - p_1 = \theta s_2 - p_2$

$$\theta^*(p_1, p_2, s_1, s_2) = \frac{p_1 - p_2}{s_1 - s_2}$$

$\Rightarrow$ The demand faced by firm 1 is $\theta^*(p_1, p_2, s_1, s_2) - \underline{\theta}$.

$\Rightarrow$ The demand faced by firm 2 is $\overline{\theta} - \theta^*(p_1, p_2, s_1, s_2)$.

Stage 2

Firms observe quality and anticipate stage 3 moves.

- Firm 1 $\max_{p_1}(p_1 - c)[\theta^*(p_1, p_2, s_1, s_2) - \underline{\theta}]$. Best response is

$$p_1^*(p_2, s_1, s_2) = 1/2[\underline{\theta}(s_1 - s_2) + p_2 + c]$$

- Firm 2 $\max_{p_2}(p_2 - c)[\overline{\theta} - \theta^*(p_1, p_2, s_1, s_2)]$. Best response is

$$p_2^*(p_1, s_1, s_2) = 1/2[-\overline{\theta}(s_1 - s_2) + p_1 + c]$$

- The equilibrium prices are $p_1^*(s_1, s_2)$ and $p_2^*(s_1, s_2)$ where $p_1^*(s_1, s_2) = p_1^*(p_2^*(s_1, s_2), s_1, s_2)$ and $p_2^*(s_1, s_2) = p_2^*(p_1^*(s_1, s_2), s_1, s_2)$:

$$p_1^*(s_1, s_2) = \frac{s_2 - s_1}{3}(\overline{\theta} - 2\underline{\theta}) + c$$
$$p_2^*(s_1, s_2) = \frac{s_2 - s_1}{3}(2\overline{\theta} - \underline{\theta}) + c$$

The consumer indifferent between the two options has type $\theta^*(s_1, s_2) = \theta^*(p_1^*(s_1, s_2), p_2^*(s_1, s_2), s_1, s_2)$.

$\Rightarrow$ If $s_1 = s_2$: $p_1 = p_2 = c$.

Stage 1

Firms anticipate moves at stages 3 and 2.

- Firm 1 $\max_{s_1}(p_1^*(s_1, s_2) - c)[\theta^*(s_1, s_2) - \underline{\theta}] \equiv \frac{s_2 - s_1}{9}(\overline{\theta} - 2\underline{\theta})^2$.

The profit is decreasing in s_1. The best response is to choose the minimum quality no matter what firm 2 is doing.

- Firm 2 $\max_{s_2}(p_2^*(s_1, s_2) - c)[\overline{\theta} - \theta^*(s_1, s_2)]$.

The best response is to choose the maximum quality no matter what firm 1 is doing.

$\Rightarrow$ There is maximum differentiation.